JOINT SOURCE CHANNEL CODING OVER CHANNELS WITH INTERSYMBOL INTERFERENCE USING VECTOR CHANNELS AND DISCRETE MULTITONE

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ABSTRACT

We investigate joint source channel coding for channels with intersymbol interference (ISI) where the source coder is a vector quantizer. In our previous work we used a block MAP equalizer, which takes into account the residual correlation between the VQ outputs and also provides for soft decisions to improve the performance. In this paper we propose the use of a vector channel approach and discrete multitone modulation for joint source channel coding on channels with ISI. Using these modulation procedures, intersymbol interference can be eliminated and the problem of joint source channel coding for ISI channels is reduced to the problem of coding for vector Gaussian channels. Optimization of the signal set is performed through optimal power allocation to the subchannels. Simulation results are presented for both vector and discrete multitone channels and compared to the results obtained by using the block MAP equalizer and to the OPTA (optimum performance theoretically attainable) bound.

1. INTRODUCTION

Under complexity constraint, joint source channel coding results in lower distortion than the separate source and channel coding. Recently, there has been an increased interest in channel optimized vector quantization (COVQ), in which the design of a vector quantizer and the associated receiver incorporate known channel error characteristics. It reduces the code design to an optimization problem and results in simple decoding procedures. Both discrete memoryless channels (DMC) [3, 4] and Gaussian channels [5] have been considered. In our previous work [1, 2, 6, 11] we have addressed the transmission of a vector quantized Gauss-Markov source over discrete channels with intersymbol interference. We considered soft decision decoding using a MAP symbol by symbol detector [7] that accounts for residual source redundancy.

In [2] we derived a recursive block MAP algorithm for multidimensional modulation implemented as time division multiplexing (TDM) over a channel with intersymbol interference. In [1] and [2] we designed COVQ using channel error matrices obtained through simulating the MAP decoder. We have also implemented constrained VQ techniques for channels with ISI, such as a channel-matched tree structured vector quantizer in conjunction with a reduced complexity block MAP algorithm [6].

Recently, joint source channel coding for an orthogonal frequency division multiplexing (OFDM) [8, 9] system has been reported [10]. BPSK modulation is used, and the subchannels are modeled as binary symmetric channels; ISI is not considered. The transmitted power is distributed among subchannels to minimize the channel distortion, providing unequal protection to vector indices. We have used the FDM approach with a small number of subchannels to design COVQ in [11], where we again used a block MAP equalizer and also performed an optimal power allocation to the subchannels.

Holsinger [12] showed that orthogonality of the subband signals through a distorted channel can be achieved by using the eigenfunctions of a channel auto-correlation function as "carriers". This approach is attracting considerable interest [13, 14], since DSP has enabled the implementation of this procedure in the discrete-time domain using matrix decomposition methods. The theory behind the continuous time approach has been extensively explained in [15]. The problem of transmitting a signal through a frequency selective channel is equivalent to transmission through a vector Gaussian channel in which the eigenfunctions of the channel correlation function form a basis for the signal space and the channel scales each dimension in this signal space by the square root of the corresponding eigenvalue. The receiver consists of a bank of filters matched to the output eigenfunctions of the channel correlation function. Slightly different is the discrete multitone modulation (DMT), where the basis functions are complex exponentials, and the DFT of the channel impulse response coefficients are used instead of the square roots of the eigenvalues. Both approaches eliminate ISI, but not interblock interference (IBI), which is removed using a preamble or a decision feedback equalizer.

In this paper we use a vector channel model which provides a solution to joint source channel coding for frequency selective channels, reducing the problem to that of joint source channel coding on Gaussian vector channels. We derive this channel model using singular value decomposition (SVD). We then study a DMT channel model, which can be also considered as a Gaussian vector channel. We develop optimal encoder, signal set and receiver designs for both channel models under a total transmitted power constraint. To reduce complexity, we design a number of COVQ's and signal sets for groups of subchannels. We perform an optimal power allocation to the subchannels in each group, in order to use the "good" subchannels more efficiently.

2. OPTIMAL ENCODER AND DECODER DESIGN

We assume a zero-mean stationary discrete time Gauss-Markov source given by an auto-regressive model. Let $\mathbf{x}_n$ be a k-dimensional vector obtained from this source with probability density function denoted by $p(\mathbf{x}_n)$. The vector $\mathbf{x}_n$ is mapped into a VQ codeword chosen from the set of k-dimensional vectors $\mathbf{c}_1, \ldots, \mathbf{c}_N$. The number of codebook vectors is $N = 2^{kR}$ where R is the source rate. Denote Ω_i as the VQ encoding region: $\mathbf{s}_i$ will be transmitted if $\mathbf{x}_n \in \Omega_i$. Here $\mathbf{s}_i$ is a point in a $2p$-dimensional constellation, given as $\mathbf{s}_i = [s_i^1, \cdots, s_i^p]$, where $s_i^j, j = 1, \cdots, p$ are signal points in M-ary two-dimensional subconstellations. The transmitted signal at time n is $\mathbf{u}_n = [u_n^1, u_n^2, \ldots, u_n^p]$.

Denote the signal used by the decoder at time n as $\mathbf{z}_n = [z_n^1, \ldots, z_n^p]$. The receiver computes the minimum mean-squared error (MMSE) estimate $\hat{\mathbf{x}}_n$ of $\mathbf{x}_n$, which is the conditional expectation of $\mathbf{x}_n$, given $\mathbf{z}_n$

$$\begin{aligned}\hat{\mathbf{x}}_n &= E[\mathbf{x}_n|\mathbf{z}_n] = \sum_{i=1}^{N} \mathbf{c}_i p(\mathbf{u}_n = \mathbf{s}_i|\mathbf{z}_n) \\ &= \frac{\sum_{i=1}^{N} \mathbf{c}_i P(\mathbf{u}_n = \mathbf{s}_i) p(\mathbf{z}_n|\mathbf{u}_n = \mathbf{s}_i)}{\sum_{i=1}^{N} P(\mathbf{u}_n = \mathbf{s}_i) p(\mathbf{z}_n|\mathbf{u}_n = \mathbf{s}_i)} \qquad (1)\end{aligned}$$

where $\mathbf{c}_i$ are the centroids of the region Ω_i:

$$\mathbf{c}_i = \frac{\int_{\Omega_i} \mathbf{x}_n p(\mathbf{x}_n) d\mathbf{x}_n}{\int_{\Omega_i} p(\mathbf{x}_n) d\mathbf{x}_n} \qquad (2)$$

To design the COVQ we use the process described in [2, 6], which is based on [3]. In this process we approximate the vector channel as a DMC whose error probabilities are denoted $P_c(j|i) = P(\hat{\mathbf{u}}_n = \mathbf{s}_j|\mathbf{u}_n = \mathbf{s}_i)$, where $\hat{\mathbf{u}}_n$ is the maximum likelihood estimate of $\mathbf{u}_n$; $P_c(j|i)$ can be computed from the vector channel model. The encoder design is performed by iterating between

$$\hat{\mathbf{c}}_j = \frac{\sum_{i=1}^{N} P_c(j|i) \int_{\Omega_i} \mathbf{x}_n p(\mathbf{x}_n) d\mathbf{x}_n}{\sum_{i=1}^{N} P_c(j|i) \int_{\Omega_i} p(\mathbf{x}_n) d\mathbf{x}_n} \qquad (3)$$

for $j = 1, \ldots, N$ and

$$\Omega_i = \{\mathbf{x} : \sum_{j=1}^{N} P_c(j|i) \|\mathbf{x}_n - \hat{\mathbf{c}}_j\|^2 \leq \sum_{j=1}^{N} P_c(j|l) \|\mathbf{x}_n - \hat{\mathbf{c}}_j\|^2, \forall l\} \qquad (4)$$

for $i = 1, 2, \ldots, N$, until convergence. These equations are obtained by minimizing the distortion

$$D = \frac{1}{k} \sum_{i=1}^{N} \sum_{j=1}^{N} P_c(j|i) \int_{\Omega_i} p(\mathbf{x}_n) \|\mathbf{x}_n - \hat{\mathbf{c}}_j\|^2 d\mathbf{x}_n \qquad (5)$$

which approximates the actual distortion with the accuracy of the DMC approximation to the vector channel.

3. VECTOR CHANNELS

The input-output relationship of a discrete-time linear time-invariant dispersive channel corrupted by additive noise is

$$y_k = \sum_{m=0}^{\nu} h_m v_{k-m} + w_k \qquad (6)$$

where $\{h_m\}_{m=0}^{\nu}$ are the channel impulse response coefficients. The noise samples w_k come from a proper complex zero mean Gaussian process with variance σ^2.

By considering transmission of blocks of size L and omitting the block index n, we have

$$\mathbf{y} = \mathbf{H}\mathbf{v} + \mathbf{w} \qquad (7)$$

$\mathbf{y} = [y_{L-1}, y_{L-2}, \ldots, y_0]'$, $\mathbf{v} = [v_{L-1}, v_{L-2}, \ldots, v_0, \ldots, v_{-\nu}]'$, $\mathbf{w} = [w_{L-1}, w_{L-2}, \ldots, w_0]'$ and $\mathbf{H}$ is the $L \times (L+\nu)$ channel matrix:

$$\mathbf{H} = \begin{bmatrix} h_0 & h_1 & \cdots & h_\nu & \cdots & 0 \\ 0 & h_0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & h_0 & \cdots & h_\nu \end{bmatrix} \qquad (8)$$

It is easy to show that the rank of this matrix is L. We perform a SVD [16] on $\mathbf{H}$:

$$\mathbf{H} = \Phi \begin{bmatrix} \Sigma & 0 \end{bmatrix} \Psi^H \qquad (9)$$

where Φ is unitary matrix of rank L and the columns of Φ form an orthogonal basis of eigenvectors for $\mathbf{H}\mathbf{H}^H$. The eigendecomposition of $\mathbf{H}\mathbf{H}^H$ is $\mathbf{H}\mathbf{H}^H = \Phi\Sigma^2\Phi^H$. Define $\Psi = [\Psi_1, \Psi_2]$, and $\Psi_1 = \mathbf{H}^H\Phi\Sigma^{-1}$, so that the columns of Ψ_1 are orthonormal to each other. We obtain $\Phi^H\mathbf{H}\Psi_1 = \Sigma$. Notice that the matrix Σ^2 is a diagonal matrix of the eigenvalues of $\mathbf{H}\mathbf{H}^H$. The matrix $\Sigma = \text{diag}(\sigma_0, \cdots, \sigma_{L-1})$ is the matrix of the positive square roots of these eigenvalues. We also have

$$\mathbf{H}^H\mathbf{H} = \Psi \text{diag}(\sigma_0^2, \ldots, \sigma_{L-1}^2, 0, \ldots, 0)\Psi^H \qquad (10)$$

The mutual information $I(\mathbf{V}; \mathbf{Y})$ is maximized when the eigenvectors of $\mathbf{R}_v = E[\mathbf{v}\mathbf{v}^H]$ are matched to those of $\mathbf{H}^H\mathbf{H}$. If we use the columns of Ψ as the waveforms for signaling, and the conjugates of the columns of Φ for demodulation in a correlation receiver, which uses biorthogonality, the effect of the channel is just to scale each element of the input by σ_i. Denoting $\mathbf{u}_e = [u_{L-1}, u_{L-2}, \cdots, u_{-\nu}]'$, $\mathbf{u} = [u_{L-1}, u_{L-2}, \cdots, u_0]'$ and $\mathbf{z} = \Phi^H\mathbf{y}$ as a column vector of processed samples used for detection

$$\begin{aligned}\mathbf{z} &= \Phi^H\mathbf{y} = \Phi^H(\mathbf{H}\mathbf{v} + \mathbf{w}) = \Phi^H(\mathbf{H}\Psi\mathbf{u}_e + \mathbf{w}) \\ &= \Phi^H\Phi[\Sigma\ 0]\Psi^H\Psi\mathbf{u}_e + \Phi^H\mathbf{w} \\ &= \Sigma\mathbf{u} + \Phi^H\mathbf{w} \qquad (11)\end{aligned}$$

The components of $\Phi^H\mathbf{w}$ are uncorrelated and with variances equal to the variance (power) in w_k since the matrix Φ is unitary. Figure 1 summarizes this vector channel structure. Note that a preamble of length ν is necessary to eliminate interblock interference.

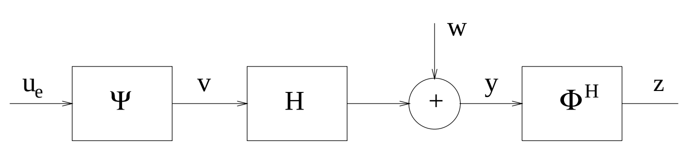

Figure 1: Block diagram of the vector channel

The DMT channel is similar to the vector channel; for DMT we use complex exponentials as waveforms to transmit the information symbols. We modify the notation used for the vector channel: $\mathbf{u}_e = \mathbf{u} = [u_0, \ldots, u_{L-1}]'$, $\mathbf{v} = [v_0, \ldots, v_{L-1}]'$, $\mathbf{y} = [y_0, \ldots, y_{L-1}]'$, $\mathbf{w} = [w_0, \ldots, w_{L-1}]'$, $\mathbf{z} = [z_0, \ldots, z_{L-1}]'$. The block of channel outputs is obtained by multiplying $\mathbf{v}$ by the circulant $L \times L$ matrix $\mathbf{H}$

$$\mathbf{H} = \begin{bmatrix} h_0 & 0 & \cdots & 0 & h_\nu & h_{\nu-1} & \cdots & h_1 \\ h_1 & h_0 & \cdots & 0 & 0 & h_\nu & \cdots & h_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & h_\nu & h_{\nu-1} & \cdots & h_1 & h_0 \end{bmatrix} \tag{12}$$

Denoting the DFT of $[h_0, \ldots h_\nu, 0, \ldots, 0]'$ by $[H_0, \ldots, H_{L-1}]'$ we obtain

$$\mathbf{H} = F^H \text{diag}(H_0, \ldots, H_{L-1}) F \tag{13}$$

where F is the DFT matrix with elements

$$f_{k+1,n+1} = \frac{1}{\sqrt{L}} \exp\{-j2\pi \frac{kn}{L-1}\} \tag{14}$$

for $k, n = 0, \ldots L-1$. By choosing $\Psi = \Phi = F^H$, we obtain

$$\begin{aligned} \mathbf{z} &= F\mathbf{y} = F(\mathbf{Hv} + \mathbf{w}) = F\mathbf{H}F^H\mathbf{u} + F\mathbf{w} \\ &= FF^H \text{diag}(H_0, \ldots, H_{L-1}) FF^H \mathbf{u} + F\mathbf{w} \\ &= \text{diag}(H_0, \ldots, H_{L-1}) \mathbf{u} + \mathbf{W} \end{aligned} \tag{15}$$

where $\mathbf{W} = [W_0, \ldots, W_{L-1}]'$ is the DFT of $\mathbf{w}$. The MMSE receiver uses the DFT $\mathbf{z}$ of the received block $\mathbf{y}$. Notice that $\mathbf{v}$ is the inverse DFT of $\mathbf{u}$. Since the DFT is a unitary transform, the components of $\mathbf{W}$ are uncorrelated, with variances equal to the power in w_k. Notice that above described use of the circulant matrix $\mathbf{H}$ is just an equivalent representation. In reality we have a linear convolution instead of a circular convolution and we precede the channel input $\mathbf{v}$ by a preamble of size equal to the channel memory (as in the more general case of the vector channels)

$$v_{-i} = v_{L-i}, \text{ for } 1 \leq i \leq \nu \tag{16}$$

The use of this preamble makes the linear convolution equal to the circular convolution and, thus, eliminates the interblock interference. The receiver computes the metrics $p(\mathbf{z}|\mathbf{u} = \mathbf{s}_i)$ for $i = 1, \ldots N$ to be used in (1). The components of the input $\mathbf{u}$ are just scaled by the square roots of the eigenvalues of the $\mathbf{HH}^H$ matrix in the first case, and by the DFT coefficients of the channel impulse response in the second case.

As the block size L increases, the DMT and vector channel asymptotically become equivalent, since the limit eigenvectors of $\mathbf{HH}^H$ when $L \to \infty$ are complex exponentials, and the limiting eigenvalue distribution is the magnitude square of the DTFT of the channel impulse response. The efficiency of these procedures increases, as the ratio L/ν increases.

4. JOINT SOURCE CHANNEL CODING FOR VECTOR CHANNELS

Since the vector channel model decomposes the ISI channel to L independent channels, we can apply the technique for joint source channel coding for vector Gaussian channels developed in [5]. In principle, we could assume that $p = L$, and transmit a signal s_i^l on the subchannel l. The design problem is to find the optimal codebook vectors, encoding regions and modulation signal points in the $2L$-dimensional signal set. However, to reduce complexity, instead of designing $2L$-dimensional VQ and signal set, we design a number of VQ's and signal sets for groups of p subchannels, where p divides L. We map each VQ output to the corresponding signal point transmitted on a group of p subchannels, not necessarily adjacent. We order the subchannels according to the magnitudes of the DFT coefficients in the DMT approach or singular values in the SVD based vector channel approach; we then group $p = 2m$ subchannels together by choosing m subchannels from the top and m from the bottom of the list, and then continue in the same fashion.

As a result of this procedure we design L/p VQ's and signal sets ($2p$-dimensional). For each VQ design we use (3), (4) and (5) with $P_c(j|i)$ computed for the DMC approximation of the corresponding group of subchannels. The total distortion is obtained by averaging the distortion over all groups of subchannels. The receiver must be able to properly group the subchannels in order to use centroids of the VQ designed for this group of subchannels in (1). Notice that in (1) vectors $\mathbf{u}_n$ and $\mathbf{z}_n$ are $2p$-dimensional, where p is a factor of L. Also, $\mathbf{x}_n$, $\mathbf{c}_j$ and $\hat{\mathbf{c}}_j$ in (3), (4) and (5) are $2p$-dimensional.

To reduce complexity the signal points are obtained as Cartesian product of points in two-dimensional constellations. In particular, we consider PSK signals because each point in a PSK constellation has equal power. We optimize the power allocation to the component PSK's. Consider j-th group of subchannels, where $j = 1, \ldots, L/p$. Assuming that the power in the i-th component modulation is P_i, the total transmitted power on this group of subchannels is $P^j = \sum_{i=1}^{p} P_i$. The i-th constellation is given in terms of a fixed signal constellation S^* with signal points $s_k^* = \exp\{j2\pi k/M\}$, $k = 0, \ldots, M-1$ as:

$$s_k^i = s_k^* \frac{\sqrt{a_i}}{\sqrt{\sum_{i=1}^{p} a_i}} \sqrt{P^j} \tag{17}$$

where $\{a_i\}$ is a set of parameters that determine power allocation. The power in the i-th constellation is equal to $P_i = a_i P^j / \sum_{i=1}^{p} a_i$. We can also use modified parameters $a_i' = a_i / \sum_{i=1}^{p} a_i$. We apply the gradient descent method to obtain optimal values of a_i'. We assign equal power to each group of subchannels ($P^j = \text{const}$), and repeat the described procedure. The total transmitted power is $P_{av} = \sum_{j=1}^{L/p} P^j$. The design is performed for given SNR$= P_{av}/L\sigma^2$.

5. SIMULATION RESULTS

We use a first order Gauss-Markov source (autoregressive AR-1, with correlation coefficient 0.9). The channel is modeled as an FIR with 3 complex coefficients ($\nu = 2$). We use the vector channel and discrete multitone approaches, with block size $L = 16$. The singular values and singular vectors of the described channel matrix were computed and used for the vector channel approach. In the DMT approach, a FFT of the channel impulse response padded with zeros to obtain the block size $L = 16$ was performed, resulting in the subchannel complex gains. We fix the dimension of each VQ to $k = 4$, which with a source rate of 1bit/sample results in 16 codebook vectors. Setting $k = 2p$, we get $p = 2$ and $m = 1$. Thus, the number of subchannel groups, as well as codebooks and signal sets designed equals $L/p = 8$. The LBG algorithm was used for the initial VQ codebooks design. Our design is suboptimal, since we perform power allocation to the two component PSK modulations of each 4-dimensional modulation only, as opposed to the power allocation to all subchannels simultaneously. The simulation results are presented in Fig. 2 and compared to the OPTA (optimum performance theoretically attainable) limit and to our previous results [11] for the same channel. The results are slightly inferior to the results obtained

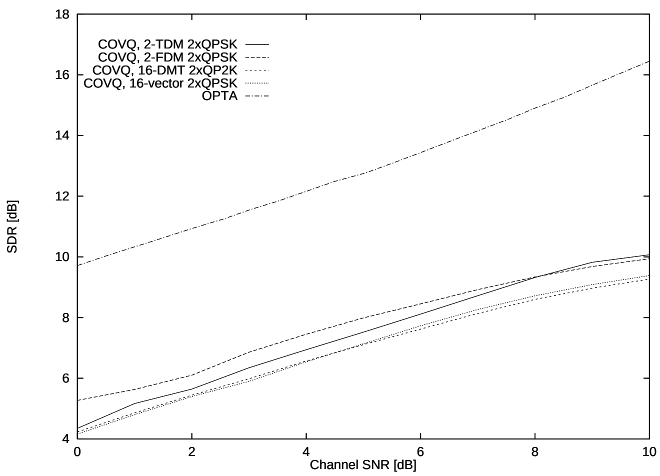

Figure 2: Reconstruction SDR in terms of channel SNR

using 2-TDM and 2-FDM (with a block MAP equalizer), because our vector channel implementation does not take the residual correlation between the VQ outputs into account. Both procedures avoid the use of an equalizer, but a large number of codebooks and signal sets is needed, as well as proper grouping of subchannels by the receiver. When using smaller blocks (small L), the efficiency of the procedure drops, due to lower L/ν. The complexity of the DMT approach is much lower than the SVD based approach due to FFT implementation, with comparable performance.

6. CONCLUSION

In this paper we proposed two procedures for joint source channel coding over channels with intersymbol interference, based upon vector channels and discrete multitone. Both procedures are parallel, and are applied to a block of data. Performance is somewhat inferior to previously described TDM and FDM approaches with a block MAP equalizer, since the residual correlation is not taken into account. Still, these procedures have lot of potential, due to lower implementation complexity, i.e. elimination of the need of computationally expensive equalization procedures. Especially attractive is the DMT procedure, due to the FFT implementation.

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