# ORTHOGONAL SUBSPACE PROJECTION FILTERING FOR STEREO IMAGE COMPRESSION

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## Abstract

*This paper presents a 2–D filtering scheme for stereo image compression using orthogonal subspace projection. To provide more candidate blocks for input data, the support region for input data is extended in the reference image. In addition, edge blocks are added to the candidate input blocks in order to provide better compensation ability for edges and boundaries of objects. The best blocks for input data are selected one by one in order of importance to reconstruct the desired block using the Gram–Schmidt orthogonalization algorithm. Simulation results exhibit excellent performance of the proposed scheme when compared to those of the standard block–matching and least-squares(LS)-based 2–D filtering schemes.*

## 1. INTRODUCTION

Disparity estimation aims at finding the position errors, or the binocular parallaxes, between the points or blocks corresponding to the right and left images of a stereo image pair. This is very similar in nature to the motion estimation used to detect the object displacement in temporal image sequences. Consequently, many algorithms that have been used for motion estimation can also be applied to the disparity estimation problem. The estimation of disparity vectors, however, needs greater accuracy compared with the estimation of motion vectors in practical applications, since human eyes recognize still objects sharper than moving ones and have higher resolution with 3–D images than 2–D images. The error in estimating disparity vectors can greatly affect the quality of the reconstructed 3–D images. In addition, the stereo image compression involves mismatching problems between the left and right images due to reflectivity/illumination differences, deformation of objects, occlusion and noise that have to be compensated for in order to provide good quality reconstructed images at the receiver.

Block–matching method is widely used for the disparity estimation. However, this method suffers from limitations such as blocking artifacts on the reconstructed images and poor compensation capability for the mismatched areas. Recently, a least–squares (LS) based 2–D filtering scheme for disparity estimation is proposed [1] to provide more accurate estimates of the disparity vector and better compensation capability for stereo image compression. To minimize the number of filter coefficients needed for reconstruction, a reduced order filtering scheme is also introduced [1] to recursively allocate variable filter order depending on the quality of each reconstructed block. The reconstructed right images generated based upon the estimated disparity vectors and some principal filter coefficients exhibited good quality without most of the mismatching problems of the block–matching method. Nonetheless, the LS–based 2–D filtering scheme requires intensive computation to estimate disparity vectors, since the 2–D filter has to be applied for all the blocks inside searching region. In addition, the performance is limited by the fact that the filter input blocks are confined only to those blocks inside the support region, which is determined by the filter size.

The orthogonal subspace projection filtering has found applications in hyperspectral image classification [2] and also for stereo image compression as a post processing after the disparity estimation [3]. The input data in this scheme consisted of some edge/tilt blocks and the block at which the disparity is estimated. This simple subspace is not adequate to compensate for the mismatching problems.

In this paper, an enhanced 2–D filtering scheme using the orthogonal subspace projection is introduced to select the most effective input blocks one by one in order of importance for reconstruction from the extended candidate input blocks. In nature, this scheme provides a very efficient algorithm to be able to allocate variable number of input blocks depending on the characteristics of the blocks. This scheme greatly improves the compensation ability while reducing the encoding overhead and computational time.

## 2. ORTHOGONAL SUBSPACE PROJECTION SCHEME

As shown in Figure 1, the right image is considered as the desired image and the left image as the input image to this system. The right image is partitioned into blocks of size $K \times L$ and each block is used as the desired block needed to be reconstructed at the receiver. In the left image, a searching region is chosen for this block to estimate the disparity vector. A support region is also defined with in this searching region to provide the input data to the filter. Note that the blocks in the right im-

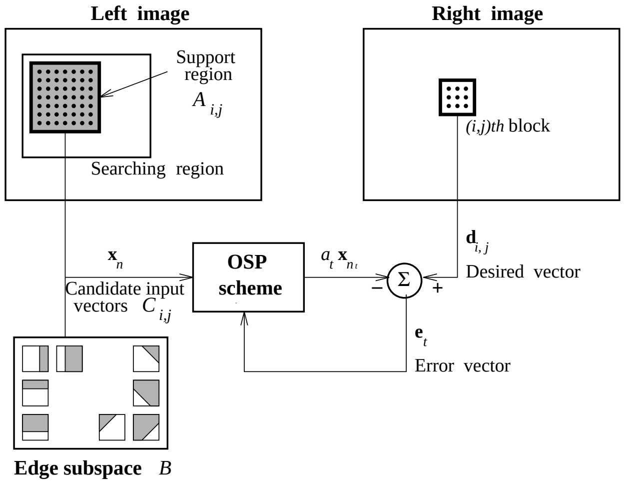



Figure 1: The orthogonal subspace projection (OSP) scheme for stereo image compression.

age move block–wise, *i.e.*, no overlapping while the blocks within the searching region in the left image move pixel–wise, *i.e.*, overlapped. The subspace, $C_{i,j}$, which is defined to provide input data for the $(i,j)$th desired (right) image block, consists of signal subspace, $A_{i,j}$, inside the support region and the edge subspace, $B$, as shown in Figure 1. The signal subspace, $A_{i,j}$, includes $M_1N_1$ blocks of the support region, $A_{i,j} = [\ \mathbf{x}_1 \ \ \mathbf{x}_2 \ \ \mathbf{x}_3 \ \ \cdots\cdots \ \ \mathbf{x}_{M_1N_1}\ ]$, where each block of size $K \times L$ inside the support region is reshaped into column vector, $\mathbf{x}_n$, of size $KL \times 1$. The edge subspace, $B$, on the other hand, includes $MN - M_1N_1$ different shape edge blocks, $B = [\ \mathbf{x}_{M_1N_1+1} \ \ \mathbf{x}_{M_1N_1+2} \ \ \cdots\cdots \ \ \mathbf{x}_{MN}\ ]$, where each edge block of size $K \times L$ is reshaped into column vector, $\mathbf{x}_n$, of size $KL \times 1$. The $(i,j)$th desired block of size $K \times L$ is also reshaped into column vector, $\mathbf{d}_{i,j}$, of size $KL \times 1$.

In this scheme, the blocks for input data are selected from both the extended support region comparing to the 2–D filtering scheme, *i.e.*, $A_{i,j}$, and the edge subspace, $B$. The number of blocks can be determined optimally by examining the quality of reconstructed block at each stage, since the blocks for input data are selected one by one in order of importance. It is neither necessary to calculate all the orthogonal basis for the candidate input blocks in $C_{i,j}$ nor it is necessary to transmit all the projection coefficients as in the subspace projection technique scheme [3]. Note that in this scheme, the disparity vector is estimated using the block–matching method.

To select the best block for the input data at each stage which provides the greatest reduction in the mean squared error (MSE) between the desired block and the approximated one, the orthogonal subspace projection method using Gram-Schmidt orthogonalization procedure [4] is employed. The first projection coefficient, $a_1$, is obtained by

$$a_1 = \max_{\mathbf{x}_n \in C_{i,j}} \left| \frac{\mathbf{d}_{i,j}^T \mathbf{x}_n}{\mathbf{x}_n^T \mathbf{x}_n} \right|. \tag{1}$$

The vector corresponding to $a_1$ is the first basis vector defined by $\mathbf{x}_{n_1}$. Then, the error vector is calculated using

$$\mathbf{e}_1 = \mathbf{d}_{i,j} - a_1 \mathbf{x}_{n_1}. \tag{2}$$

The remaining vectors in the subspace $C_{i,j}$ are orthogonalized to the first basis vector, $\mathbf{x}_{n_1}$, *i.e.*,

$$\mathbf{x}_n^{(1)} = \mathbf{x}_n - \frac{\mathbf{x}_n^T \mathbf{x}_{n_1}}{\mathbf{x}_{n_1}^T \mathbf{x}_{n_1}} \mathbf{x}_{n_1}, \quad \mathbf{x}_n^{(1)} \in C_{i,j}^{(1)},\ \forall n \neq n_1, \tag{3}$$

where $\mathbf{x}_n^{(1)}$ contains the information not included in $\mathbf{x}_{n_1}$, *i.e.*, the innovation part. This can be used to select the next most effective basis vector to approximate the error signal, $\mathbf{e}_1$.

The $t$th projection coefficient is calculated based on the vectors in $C_{i,j}^{(t-1)}$, *i.e.*,

$$a_t = \max_{\mathbf{x}_n^{(t-1)} \in C_{i,j}^{(t-1)}} \left| \frac{\mathbf{e}_{t-1}^T \mathbf{x}_n^{(t-1)}}{\mathbf{x}_n^{(t-1)^T} \mathbf{x}_n^{(t-1)}} \right| \tag{4}$$

and the vector corresponding to $a_t$ is the $t$th basis vector defined by $\mathbf{x}_{n_t}$. Then, the error vector is defined by

$$\mathbf{e}_t = \mathbf{e}_{t-1} - a_t \mathbf{x}_{n_t} \tag{5}$$

The remaining vectors in $C_{i,j}^{(t-1)}$ are orthogonalized to the newly chosen basis vector, $\mathbf{x}_{n_t}$, *i.e.*,

$$x_n^{(t)} = \mathbf{x}_n^{(t-1)} - \frac{\mathbf{x}_n^{(t-1)^T} \mathbf{x}_{n_t}}{\mathbf{x}_{n_t}{}^T \mathbf{x}_{n_t}} \mathbf{x}_{n_t}, \quad \mathbf{x}_n^{(t)} \in C_{i,j}^{(t)},\ \forall n \neq n_t. \tag{6}$$

This procedure continues iteratively until the quality threshold of the reconstructed block is reached. The desired block is then reconstructed using

$$\hat{\mathbf{d}}_{i,j} = \sum_{t=1}^{T} a_t \mathbf{x}_{n_t} \tag{7}$$

where $T$ is variable depending on the characteristics of each desired block and its corresponding subspace.

## 3. SIMULATION RESULTS AND COMPARISON

The performance of the proposed scheme was compared to those of the standard block–matching method and the reduced order filtering scheme in [1] on a stereo image pair, "Chair", shown in Figures 3(a) and (b). The size of each image is $280 \times 320$ with 256 grey levels. This image pair is chosen for this testing, since it involves practical issues such as occlusion, reflectivity differences, and deformation of the objects. The block size was chosen to be $8 \times 8$. The searching region was defined as follows: left margin 48, right margin 8, upper margin 8, and lower margin 8. Since objects in the left image are shifted to the left side compared to the right image, more margin was given to the left side in the left image. To simplify the performance comparison of the disparity compensated reconstructed images, the original left image is used here instead of the encoded one.

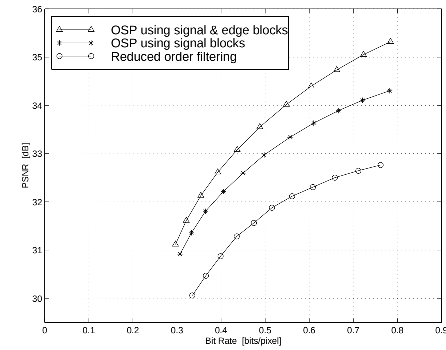
The orthogonal subspace projection scheme and the reduced order filtering scheme allocate variable number of input blocks to each desired block until to satisfy the predefined quality threshold of the reconstructed block. The maximum number of blocks for input data was set to 7. The performance is compared with respect to different threshold values. This leads to different bit-rate to satisfy the threshold condition. The reduced order filtering scheme [1] performs filtering on all the blocks inside the searching region. The support region of the reduced order filtering scheme contained 16 blocks, for a $4 \times 4$ possible order case, to find the most effective input blocks. If the support region becomes larger, the performance obviously degrades. This is due to the fact that the values of the filter weights are spreading over a larger support region, hence making it difficult to choose the best block by just using the lag of the largest weight in absolute value. In the orthogonal subspace projection scheme the subspace $C_{i,j}$ included 64 image blocks and 62 edge blocks. The best input blocks were selected one by one until the predefined threshold is satisfied. The disparity estimation is done using the standard block–matching method.

The coefficients were then encoded by Lloyd–Max quantizer and uniform coding method As shown in Figures 2(a) and (b), the performance of the proposed orthogonal subspace projection method is substantially better than that of the reduced order filtering scheme [1] in terms of both bit–rate to peak signal–to–noise ratio (PSNR) and the processing time. This is especially true for the case where both signal blocks and edge blocks were used as candidate input blocks.

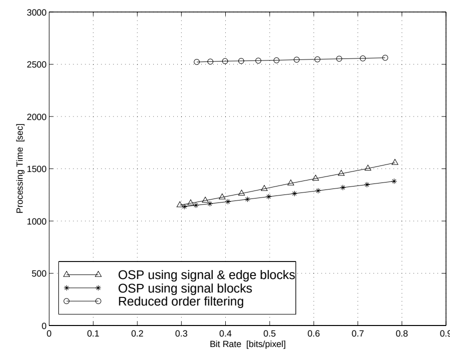
The reconstructed image using the standard block–matching method is shown in Figure 3(c). Clearly this image exhibits several mismatching problems: occlusion on the left side of the chair and the upper part of the top right object, blocking artifacts on the letter 'B' on the front container which appears as 'K', deformation of the objects as shown in some of the black squares in the checker board pattern, and reflectivity differences in the background on the top of the chair. The PSNR was measured to be 27.93 dB for this image. The reduced order filtering scheme compensates for most of the mismatching problems as shown in Figure 3(d) providing PSNR of 32.76 dB at 0.762 bpp. The reconstructed images using the proposed scheme for the case of only 64 signal blocks and the case of both 64 signal blocks and 62 edge blocks are shown in Figures 3(e) and (f), respectively. The PSNR for these images were found to be 34.30 and 35.32 dB at 0.782 and 0.784 bpp, respectively. The reconstructed images using the orthogonal subspace projection scheme are impressively much better than that of the reduced order filtering scheme both from the view points of visual quality as well as PSNR and bpp measures.

## 4. CONCLUSION

A new filtering scheme using the orthogonal subspace projection for stereo image compression was proposed. This scheme uses an extended support region to include more candidate blocks as well as the edge blocks to obtain the best candidate blocks for the reconstruction of the right image block. The scheme provides many desirable advantages in both implementation and performance over the other schemes for stereo image compression. Simulation results demonstrated excellent improvements compared to the LS–based 2–D filtering scheme for compensating for the effects of mismatching and improving the PSNR and bpp rates in stereo image compression applications.




(a) Bit–rate vs PSNR.




(b) Bit–rate vs processing time.

Figure 2: The performance comparison between the orthogonal subspace projection and the reduced order filtering schemes.

## 5. REFERENCES

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(a) Original left image.


(b) Original right image.

(c) Block–matching method (27.93 dB).

(d) Reduced order filtering scheme (32.76 dB, 0.762 bpp).

(e) OSP scheme using only signal blocks (34.30 dB, 0.782 bpp).

(f) OSP scheme using both signal and edge blocks (35.32 dB, 0.784 bpp).

Figure 3: Original stereo image pair, and reconstructed images using block–matching method, reduced order filtering scheme, and the orthogonal subspace projection scheme.