BLIND MULTI-USER MMSE DETECTION OF CDMA SIGNALS

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ABSTRACT

The recovery of code-division multiple access (CDMA) information signals in a frequency selective fading channel is a problem of great theoretical and practical interest. This paper addresses the estimation of an optimal (within the class of linear detectors) multi-user CDMA receiver. A novel approach is introduced that enables the estimation of the minimum mean-square error (MMSE) detector in a blind setting. The MMSE detector is obtained through a double subspace projection that exploits the subspace structure associated with both the code of the desired user and the estimated signal subspace of the covariance matrix for the observed signals. The technique allows for interference rejection without requiring the knowledge of the codes for the interferers.

1. INTRODUCTION

Direct sequence code-division multiple access (DS-CDMA) systems are currently gaining a significant share of the wireless telecommunications market. In a DS-CDMA system, all the users share the same temporal and spectral resources. Source separation is made possible by spreading each user's information bit stream by a unique orthogonal (or quasi-orthogonal) code prior to the transmission.

There exist two main approaches to the separation of the CDMA signals at the receiver. A *single-user* receiver basically estimates the signal of a desired user by modeling the interfering users as noise. In contrast, a *multi-user* detector includes all the users in the signal model.

A popular single-user receiver is the RAKE combiner. The RAKE correlates the received signal with the code of the desired user, at time instants corresponding to the time-delays introduced by the multipath channel, and the outputs are combined to maximize the signal-to-noise ratio (SNR) at the receiver. Hence, the RAKE receiver performs a matched filtering operation, both with respect to the channel and the code of the user. As such, due to imperfect correlation properties of the spreading codes, the RAKE receiver is sensitive to multipath and near-far problems.

Significant improvement can be obtained with a multi-user receiver. Such a detector typically requires the knowledge of the codes and channels of all the users. The channels are in general estimated with the use of training sequences or pilot symbols, which however reduces the spectral efficiency. Blind approaches, that estimate the channel without the use of training, have been proposed as a way to alleviate this problem and are therefore of great interest.

The blind multi-user CDMA detection problem is similar in principle to the recently studied blind multiple-input multiple output (MIMO) equalization problem [3, 6] for TDMA. However, a significant advantage of blind CDMA detection over the conventional MIMO problem is the availability of a rich code structure which can be exploited to overcome the robustness and convergence problems observed with blind separation methods.

The current literature on blind CDMA mainly focuses on the channel estimation problem because it is somewhat straightforward to apply the subspace-based ideas developed for the usual MIMO case to this context [1, 4]. In parallel, the incorporation of blind approaches in the design of the conventional single user RAKE receiver is also gaining attention since it offers a simple (yet suboptimal) alternative to the multi-user methods [2, 7].

The problem of directly estimating a multi-user detector for CDMA signals in unknown channels is perhaps more challenging. In [5] a minimum output energy technique is used which results in a linear receiver being close the MMSE receiver at high SNR.

In this paper we address the blind estimation of the *true* MMSE receiver. The approach is based on the exploitation of a set of linear conditions that take advantage of 1) the subspace structure associated with the code of the desired user, and 2) the estimated "signal subspace" of the covariance matrix for the observed signals. We show that these conditions in general fully characterize the desired solution. A practical algorithm, based on a two-fold projection, is introduced and evaluated by means of some numerical experiments.

2. NOTATIONS AND ASSUMPTIONS

We assume that a total number of Q CDMA signals impinge on an array of $M \geq 1$ sensors. The qth user transmits a symbol stream $s^q(k)$ at rate $1/T_s$ (where T_s is the symbol duration). The symbols are assumed to be unit variance zero mean independent and identically distributed (iid) random variables. To each user is associated a code sequence $\{c^q(p)\}_{p=0}^{P-1}$ of length P that spreads the symbols. The resulting (chip) sequences are denoted by $d^q(k)$ and have rate $1/T_c$, with $T_c = T_s/P$. The received (continuous

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time) base band signal at the mth antenna can be written

$$x_m(t) = \sum_{q=1}^{Q}\sum_{l=0}^{L} h_m^q(t - lT_s)s^q(l) + \varepsilon_m(t) \tag{1}$$

where $h_m^q(t)$ is the global channel from the qth user's to the mth antenna, and hence includes equipment filtering, propagation effects, time delay, and also the code sequence of the user. The degree of the channel is denoted by L and is assumed to be the same for all users ($L = 1$ is a typical value for current CDMA rates). The term $\varepsilon_m(t)$ is an additive noise which may also contain any residual interference.

By construction, $h_m^q(t)$ is the convolution between the code sequence $c^q(p)$ and the physical channel seen by the chip sequence, denoted by $g_m^q(t)$, and can thus be expressed as

$$h_m^q(t) = \sum_{p=0}^{P-1} c^q(p)g_m^q(t - pT_c) \tag{2}$$

The signal (1) received at antenna m is sampled at P times the symbol rate. Therefore, per symbol, P samples can be stacked in a column vector as

$$\mathbf{x}_m(k) = (x_m(kT_s), \ldots, x_m(kT_s + (P-1)T_c))^T \tag{3}$$

Stacking the vectors (3) for all the antennas yields the MP dimensional vector

$$\mathbf{x}(k) = (\mathbf{x}_1^T(k), \mathbf{x}_2^T(k), \ldots, \mathbf{x}_M^T(k))^T \tag{4}$$

In view of (1) we can write

$$\mathbf{x}(k) = \sum_{q=1}^{Q} \mathbf{H}^q\mathbf{s}^q(k) + \boldsymbol{\varepsilon}(k) \tag{5}$$

where the $MP \times (L+1)$-dimensional matrix $\mathbf{H}^q$ is the channel matrix for user q, its $((m-1)P+p+1)$th row (with $1 \le m \le M$ and $0 \le p \le P-1$) being given by

$$(h_m^q(pT_c), h_m^q(pT_c + T_s), \ldots, h_m^q(pT_c + LT_s)) \tag{6}$$

In (5), $\mathbf{s}^q(k) \triangleq (s^q(k), s^q(k-1), \ldots, s^q(k-L))^T$ is the symbol vector and $\boldsymbol{\varepsilon}(k)$ contains the noise vectors for all the M antennas and is defined in the same manner as in (3)–(4). For later purposes the following MNP-dimensional augmented output vector is introduced:

$$\mathbf{X}(k) = (\mathbf{x}^T(k), \mathbf{x}^T(k-1), \ldots, \mathbf{x}^T(k-N+1))^T \tag{7}$$

where N is called the smoothing lag or the memory of the equalizer. The vector $\mathbf{X}(k)$ can, similarly to $\mathbf{x}(k)$, be written in matrix form. We obtain the following linear model:

$$\mathbf{X}(k) = \sum_{q=1}^{Q}\begin{pmatrix} \boxed{\mathbf{H}^q} & & 0 \\ & \boxed{\mathbf{H}^q} & \\ & \ddots & \\ 0 & & \boxed{\mathbf{H}^q}\end{pmatrix}\begin{pmatrix} s^q(k) \\ s^q(k\text{-}1) \\ \vdots\end{pmatrix} + \begin{pmatrix}\boldsymbol{\varepsilon}(k) \\ \boldsymbol{\varepsilon}(k\text{-}1) \\ \vdots\end{pmatrix} \tag{8}$$

$$\triangleq \sum_{q=1}^{Q} \boldsymbol{\mathcal{H}}^q\mathbf{S}^q(k) + \mathbf{e}(k) \tag{9}$$

Note that the global channel matrix $\boldsymbol{\mathcal{H}} \triangleq (\boldsymbol{\mathcal{H}}^1, \ldots, \boldsymbol{\mathcal{H}}^Q)$ has dimension $MNP \times Q(L+N)$. We will assume $MP \ge Q$ so that $\boldsymbol{\mathcal{H}}$ can be made tall (i.e. it has more rows than columns) for a sufficiently large N. This makes (9) a low-rank model which is a property that we will later exploit. We further assume that the conditions under which $\boldsymbol{\mathcal{H}}$ has full column rank are satisfied [3].

3. BLIND MULTI-USER DETECTION

The problem of linear multi-user detection can be stated as follows: given the measured data $\mathbf{X}(k)$, find a vector $\mathbf{w}$ with MNP entries that satisfies (or approximates):

$$\mathbf{w}^*\boldsymbol{\mathcal{H}}^q = (0, \ldots, 0, 1, 0, \ldots, 0), \tag{10}$$

$$\mathbf{w}^*\boldsymbol{\mathcal{H}}^u = (0, \ldots, 0),\ u \ne q \tag{11}$$

where q now denotes the index of the desired source and where "*" denotes complex conjugate transpose. Note that, while (10) corresponds to channel equalization, (11) allows for the source separation. The position of the "1" element in (10) indicates the delay of the receiver. In the presence of noise, the best (in the MMSE sense) linear receiver with delay δ, for user q is given by:

$$\mathbf{w} = \arg\min E|\mathbf{w}^*\mathbf{X}(k) - s^q(k-\delta)|^2 \tag{12}$$

(where E denotes the expectation operator). The detector $\mathbf{w}$ found from (12) satisfies the classical equation

$$\mathbf{R}\mathbf{w} = \mathbf{r}^q \tag{13}$$

where $\mathbf{R} = E(\mathbf{X}(k)\mathbf{X}^*(k))$ denotes the covariance matrix and $\mathbf{r}^q = E(\mathbf{X}(k)s^{q*}(k-\delta))$ denotes the inter-correlation vector. In what follows we introduce a subspace approach that leads to the blind identification of $\mathbf{w}$.

3.1. Code subspace properties

Under the iid symbols hypothesis, it is found from (8) that $\mathbf{r}^q$ coincides with the $(\delta+1)$th column of $\boldsymbol{\mathcal{H}}^q$. If the detector delay δ is chosen appropriately (namely $L \le \delta \le N-1$ assuming $L \le N-1$), the vector $\mathbf{r}^q$ contains all the $(L+1)MP$ channel coefficients for user q, as well as $(N-L-1)MP$ zeros. The point here is that, for each antenna m, there exist a simple selection-permutation matrix $\mathbf{T}_m$ that selects the $(L+1)P$ channel coefficients in $\mathbf{r}^q$ associated with this antenna, together with $(N-L-1)P$ of the zeros, and puts the selected entries in a chronologically ordered vector so that

$$\mathbf{T}_m\mathbf{r}^q = (h_m^q(0), \ldots, h_m^q((LP+P-1)T_c), 0, \ldots, 0)^T$$

For notational convenience, in what follows we will assume $\delta = L$, but the final result holds true for any admissible value of the delay. From (2) we can now write

$$\mathbf{T}_m\mathbf{r}^q = \begin{pmatrix} c^q(0) & & 0 \\ \vdots & \ddots & \\ c^q(P-1) & & c^q(0) \\ & \ddots & \vdots \\ 0 & & c^q(P-1) \\ \vdots & & \\ 0 & & 0\end{pmatrix}\begin{pmatrix} g_m^q(0) \\ g_m^q(T_c) \\ \vdots \\ g_m^q(LPT_c)\end{pmatrix}$$

$$\triangleq \mathbf{C}^q\mathbf{g}_m^q \tag{14}$$

The code matrix $\mathbf{C}^q$ is tall, of dimension $PN \times LP + 1$ and is known a priori. In contrast, $\mathbf{g}_m^q$ is unknown and contains a certain number of non-zero entries. Note however that the knowledge of this number is not necessary for the proposed method. Introduce $\mathbf{U}^q$, an orthonormal basis for the orthogonal complement of $\mathbf{C}^q$. Hence, $\mathbf{U}^q$ is of size $PN \times (P(N-L)-1)$. Then, from (13) and (14), it holds

$$\mathbf{U}^{q\,*}\mathbf{T}_m\mathbf{R}\mathbf{w} = 0 \tag{15}$$

Of course, (15) is valid for $m = 1, \ldots, M$.

3.2. Signal subspace properties

Let us assume the additive noise is white with variance σ^2. The covariance matrix of the received signal is then given by

$$\mathbf{R} = \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^* + \sigma^2\mathbf{I} \tag{16}$$

where $\mathbf{I}$ is the identity matrix of size MNP. Hence, $\boldsymbol{\mathcal{H}}$ spans a "signal" subspace of dimension $Q(N+L)$. The "noise" subspace is its orthogonal complement and can be described by the $MNP \times (MNP - Q(N+L))$ matrix, denoted by $\mathbf{E}_n$, containing the noise eigenvectors of $\mathbf{R}$.

A very useful result is the fact the desired MMSE detector *always* lies in the signal subspace, as can be shown as follows. From (13) and (16) we have

$$(\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^* + \sigma^2\mathbf{I})\mathbf{w} = \mathbf{r}^q$$

which implies

$$\mathbf{w} = (\mathbf{r}^q - \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^*\mathbf{w})/\sigma^2 \tag{17}$$

In view of (17), $\mathbf{w}$ belongs to $\mathrm{span}(\boldsymbol{\mathcal{H}})$ (recall that $\mathbf{r}^q$ is by construction in the signal subspace) and it follows that

$$\mathbf{E}_n^*\mathbf{w} = 0 \tag{18}$$

3.3. Algorithm

Based on the projection equations in (15) and (18) $\mathbf{w}$ can be written as the solution to

$$\begin{pmatrix} \mathbf{U}^{q\,*}\mathbf{T}_1\mathbf{R} \\ \vdots \\ \mathbf{U}^{q\,*}\mathbf{T}_M\mathbf{R} \\ \mathbf{E}_n^* \end{pmatrix} \mathbf{w} = 0 \tag{19}$$

This provides us with a total number of $MNP - Q(L+N) + M(P(N-L)-1)$ linear equations to determine the MNP unknown entries of $\mathbf{w}$. Let us assume the following inequality to be satisfied

$$Q(L+N) \leq M(P(N-L)-1) \tag{20}$$

(note that for a given $MP > Q$, N can always be chosen to satisfy (20)). The generic rank of the matrix in (19) is $MNP-1$ (this fact was confirmed by numerical simulations over a wide range of channels). Under these conditions, the solution $\mathbf{w}$ to (19) is unique and is the MMSE equalizer. Of course, in the noise free case, $\mathbf{w}$ given by (19) is not unique since there exist several admissible solutions to (12). The study of channel conditions under which the rank drops compared to its generic value will be addressed in a forthcoming paper.

In order to determine $\mathbf{w}$, the projection equations (19) suggest the optimization of the following quadratic cost function

$$J_{MMSE}(\mathbf{w}) = \mathbf{w}^*\boldsymbol{\mathcal{A}}\mathbf{w} \tag{21}$$

with

$$\boldsymbol{\mathcal{A}} \triangleq \sum_{m=1}^{M} \mathbf{R}\mathbf{T}_m^*\mathbf{U}^q\mathbf{U}^{q\,*}\mathbf{T}_m\mathbf{R} + \alpha\mathbf{E}_n\mathbf{E}_n^* \tag{22}$$

where α is an arbitrary weight that can be chosen by the user. In order to avoid trivial solutions, some additional constraint must be added. In the particular case of a unit-norm constraint, $\mathbf{w}$ is found as the minimum eigenvector of $\boldsymbol{\mathcal{A}}$. Note that while $\mathbf{R}$ is estimated using sample averaging, $\mathbf{T}_m$ and $\mathbf{U}^q$ can be pre-computed.

4. NUMERICAL EXAMPLES

In this section, we validate the proposed algorithm with the means of a number of numerical experiments. In all the simulations, we use random channels and code sequences for all the users (note that orthogonality of the codes is not necessary). The channel order is taken to be $L = 1$ (a typical value for current CDMA systems) and we chose the memory of the equalizer to be $N = 2$. In all the experiments we use the weight $\alpha = 1$. The channels for all the users are normalized so we define the signal-to-noise ratio as

$$\text{SNR} = 10\log 1/\sigma^2 \tag{23}$$

and quantify the performance of our algorithm in terms of the mean-squared error (MSE) at the output of the receiver: MSE $= E|\hat{s}^q(k) - s^q(k)|^2$.

In Figure 1 we compare the proposed algorithm with the minimum output energy (MOE) method presented in [5]. MOE is formulated for the single antenna case only, so we have $M = 1$. As in [5], we assume a scenario with $Q = 10$ users and codes of length $P = 31$. We use 256 and 512 symbols in the estimation and assume perfect power control. We see that the proposed algorithm shows a significant performance improvement as compared to the MOE method, especially for low to medium-high SNRs. As the SNR increases, the performances of the two methods become similar, a result that is consistent with the fact that the MOE is shown to approach the MMSE solution for a high SNR.

In Figure 2 we study the robustness of the proposed method to the lack of power control (the near-far problem). We assume a scenario where the power (in dB) decreases linearly with user index, and there is a difference of 10dB in the power of the strongest and weakest of the 10 users (in this case, the SNR is defined with respect to the strongest user). We plot the MSE as a function of user index, and choose $M = 4$ and $P = 9$ to have a value of MP/Q comparable to the one in the previous example (512 symbols are used in the estimation). We see a very small increase in the MSE as the power of the users decrease. This shows that the method is robust to near-far situations, which is indeed a desired property of multi-user detectors as perfect power control can be difficult to realize in practice.

In Figure 3 we show that the performance of the proposed method approaches that of the asymptotic MMSE solution as the number of data points increases. We use the same multiple antenna scenario as above (but with perfect power control) and plot the MSE of one of the users as a function of SNR for different numbers of symbols together with the asymptotic curve.

5. CONCLUSIONS

In this paper we proposed a blind MMSE receiver for the detection of CDMA sequences. The algorithm exploits the code sequence of the desired user, through the subspace structure of the channel. It also exploits the fact that the MMSE detector always lies in the "signal" subspace of the channel output covariance matrix.

Numerical examples show that the proposed algorithm can outperform other existing methods, especially in cases when the SNR is low. The optimal choice of α for a given scenario deserves further attention. Also, the incorporation of pilot symbols in the criterion (21) is an interesting extension of this work.

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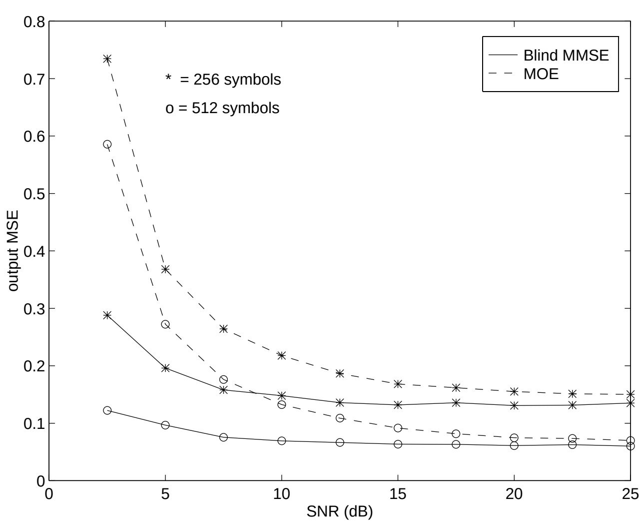

Figure 1: MSE as a function of SNR, for the proposed blind MMSE and the minimum output energy (MOE) methods ($Q = 10,\ P = 31,\ M = 1$). Perfect power control, 256 ("*") and 512 ("o") symbols are used in the estimation.

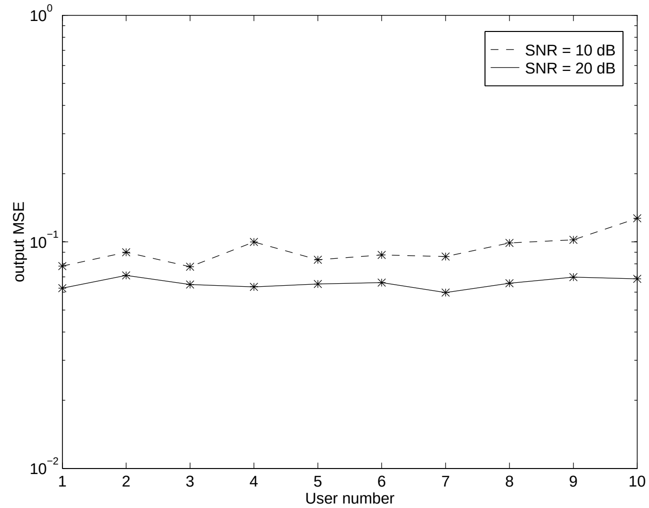

Figure 2: MSE as a function of user index for imperfect power control (user no. 1 has power 10dB above user no. 10). The SNR refers to the strongest user and 512 symbols are used in the estimation ($Q = 10,\ P = 9, M = 4$).

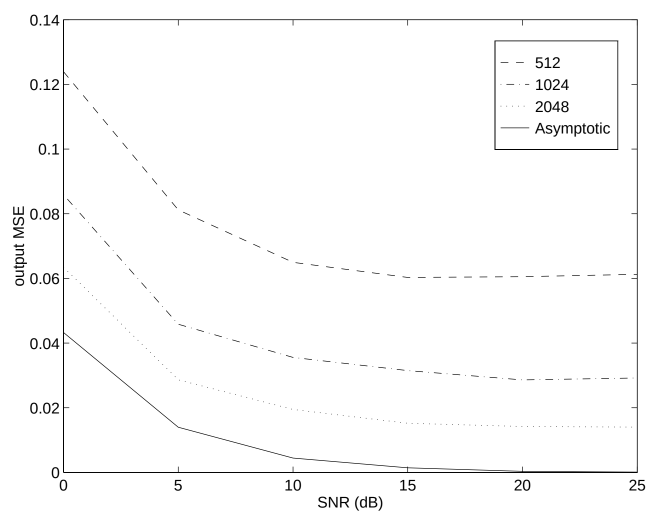

Figure 3: MSE as a function of SNR for different number of symbols (512, 1024 and 2048) used in the estimation, together with the MSE obtained using the true (asymptotic) covariance matrix. Perfect power control ($Q = 10,\ P = 9, M = 4$).