A SEMI-BLIND APPROACH TO STRUCTURED CHANNEL EQUALIZATION *

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ABSTRACT

This paper describes a direct equalization approach for channels with some underlying structure. A semi-blind approach is taken here where a small amount of training symbols is available. A family of MMSE equalizers is obtained that includes some prior information about the channel structure. The channel structure assumed in this paper is that the channel vector lies approximately in the subspace of a matrix associated with the samples of the transmit pulse shape. Blind identifiability issues of the structured equalizer are also addressed. Numerical results using experimental indoor channel data indicate that these structured equalizers can achieve bit error rates that are significantly lower than traditional non-blind MMSE equalizers.

1 INTRODUCTION

Data communication through severe multipath radio channel causes inter-symbol interference (ISI) that requires the use of an equalization device to compensate for the channel effects. Conventional systems require the periodic sending of data bursts that contain both training data, and regular information data. The training part may be exploited in order to acquire the taps of a linear equalizer. The equalizer is then used on the received information signal in order to recover the transmitted symbols, until a new data burst is transmitted at which the equalization process is repeated. Conventional training-based equalization methods can suffer from significant estimation errors due to the limited number of training symbols. There is considerable interest in constructing improved estimators that can cope with limited amount of training. Typically, such improved equalizers should resort to extra levels of available information:

• **Semi-blind estimators**: Blind equalization methods estimate the equalizer on the basis of the non-training part [1, 2]. Hence, the regular data provides useful information too. However, pure blind methods are unable to exploit the knowledge of the training symbols. In contrast, "semi-blind" estimators can combine the advantages of blind and training-based techniques [3, 4].

• **Estimators based on prior information**: The fact that the unknown channel contains known transmit and receive pulse shaping filters offers an extra source of information for the receiver design. Specifically, the vector of channel coefficients lies approximately in a subspace associated with the pulse shaping filter convolution matrix. Previous work has been done to exploit such channel structure to improve the performance of blind [5, 6, 7] as well as non-blind channel estimation techniques [8]. However, there appears to be very little work on designing direct equalizers that take into account the underlying channel structure. It is believed that the estimation of the channel equalizer can benefit from the channel structure information.

In this paper, we construct direct equalizers that are able to exploit information provided by the training symbols, the information data, and the underlying channel structure altogether. The equalizers are derived from a minimum mean square error (MMSE) cost function that contains a cost that relates to the finite-sample mean-square error over the training part of the data and a second cost component that makes use of the channel structure information. This latter cost component is derived from projection equations obtained from the asymptotic MMSE solution. We show that, under certain conditions, the projection equations alone fully determine a family of channel equalizers.

2 DATA MODEL

Consider the digital communication system in Figure 1, where $g(t)$ is the impulse response of the transmit filter[1], $c(t)$ is the physical propagation channel, $v(t)$ is the additive noise, P is the oversampling factor with respect to the baud rate T, $\mathbf{w}$ represents a discrete time equalizer, and $x(t)$ is the received signal. The received signal is

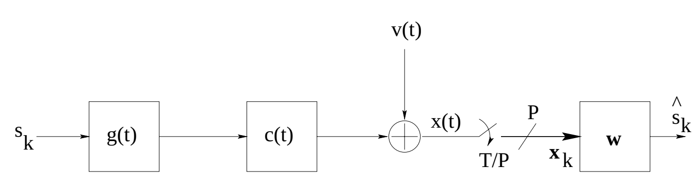

Figure 1: Block diagram of channel and equalizer.

$$x(t) = \sum_n s_n h(t - nT) + v(t) \qquad (1)$$

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[1]Without loss of generality, the effects of a receive filter can also be included into $g(t)$

where $\{s_n\}$ denotes the data symbols. The overall channel can be written as

$$h(t) = \int_0^{T_c} g(t-\tau)c(\tau)d\tau \qquad (2)$$

where we assumed $c(t)$ is of finite duration T_c, and causal. For all practical purposes, $g(t)$ can also be assumed to have finite duration, that is, $g(t) = 0 \ \forall t \notin [0, T_g]$. By approximating the convolution integral (2) with a finite Riemann sum with ν ($\nu > 1$) terms, we can re-write (2) as

$$h(t) \approx \Delta_c \sum_{l=0}^{\nu-1} g(t - l\Delta_c)\, c(l\Delta_c) = \mathbf{g}^T(t)\,\mathbf{c} \qquad (3)$$

where Δ_c is the discretization interval and is given by $\Delta_c = T_c/\nu$, $\mathbf{c} = \Delta_c[c(0), c(\Delta_c), \cdots, c((\nu-1)\Delta_c)]^T$, and $\mathbf{g}(t) = [g(t), g(t-\Delta_c), \cdots, g(t-(\nu-1)\Delta_c)]^T$.

Let $\mathbf{x}_k$ be the oversampled signal vector at the kth symbol interval, i.e., $\mathbf{x}_k = [x(kT), x(kT+T/P), \cdots, x(kT+(P-1)T/P)]^T$, and L be the total channel length ($L = \lceil T_g + T_c/T \rceil$). The data model is then:

$$\mathbf{x}_k = \mathbf{H}\mathbf{s}_k + \mathbf{v}_k \qquad (4)$$

where $\mathbf{s}_k = [s_k, s_{k-1}, \cdots, s_{k-L+1}]^T$, $\mathbf{v}_k = [v(kT), \cdots v(kT + (P-1)T/P)]^T$, and $\mathbf{H}$ is the $P \times L$ channel matrix. In addition, we can use (3) to establish [8]

$$\mathbf{h} = \text{vec}(\mathbf{H}) \approx \mathbf{G}\mathbf{c} \qquad (5)$$

where the $PL \times \nu$ pulse shaping matrix $\mathbf{G}$ is given by $\mathbf{G} = [\mathbf{g}_{00}, \mathbf{g}_{10}, \cdots, \mathbf{g}_{(P-1)0}, \mathbf{g}_{01}, \cdots, \mathbf{g}_{(P-1)(L-1)}]^T$, and where $\mathbf{g}_{jk} = [g(kT+jT/P), \cdots, g(kT+jT/P-(\nu-1)\Delta_c)]^T$.

The relation (5) indicates that the total vector channel lies approximately in the subspace spanned by the columns of $\mathbf{G}$. The approximation error can be made arbitrary small for "smooth" $c(t)$ by increasing ν.

Let the number of symbol spaced taps for the equalizer vector $\mathbf{w}$ be M (so $\mathbf{w}$ is $MP \times 1$) and let $\mathbf{y}_k = [\mathbf{x}_k^T, \mathbf{x}_{k-1}^T, \cdots, \mathbf{x}_{k-M+1}^T]^T$. Then we can write

$$\mathbf{y}_k = \boldsymbol{\mathcal{H}}\tilde{\mathbf{s}}_k + \mathbf{z}_k \qquad (6)$$

where $\tilde{\mathbf{s}}_k = [s_k, s_{k-1}, \cdots, s_{k-L-M+2}]^T$ and $\mathbf{z}_k = [\mathbf{v}_k^T, \mathbf{v}_{k-1}^T, \cdots, \mathbf{v}_{k-M+1}^T]^T$. The $MP \times (L+M-1)$ matrix $\boldsymbol{\mathcal{H}}$ is block-Toeplitz and is generated from the $P \times L$ channel matrix $\mathbf{H}$.

The goal of this paper is to exploit the structure (5) and construct direct equalizers such that $\mathbf{w}^H\mathbf{y}_k$ is an estimate of the data.

3 MMSE EQUALIZERS

We start by considering the conventional minimum mean square error (MMSE) equalizer based on a training sequence. We then generalize the MMSE criterion to include a penalty term that imposes a structured constraint on the equalizer.

3.1 Non-blind MMSE equalizer

In this equalizer, the oversampled channel output is weighted and summed to produce the desired output. The $MP \times 1$ equalizer weight vector $\mathbf{w}$ is chosen to minimize the expected squared error, that is,

$$\mathbf{w}(D) = \arg\min_{\mathbf{w}} E\left|\mathbf{w}^H\mathbf{y}_k - s_{k-D}\right|^2$$

where D is a user-chosen integer delay. The optimal weight vector is given by the Wiener solution

$$\mathbf{w}(D) = \mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{r}_{\mathbf{y}s}(D) \qquad (7)$$

where $\mathbf{R}_{\mathbf{yy}}$ is the covariance matrix of $\mathbf{y}_k$ and $\mathbf{r}_{\mathbf{y}s}(D)$ is the cross-correlation vector between $\mathbf{y}_k$ and s_{k-D}.

In practice, we compute the finite sample estimates of $\mathbf{R}_{\mathbf{yy}}$ and $\mathbf{r}_{\mathbf{y}s}(D)$ using the received samples during the training period. The optimal weight vector computed from (7) during the training period is then used for the entire time slot if the channel does not vary significantly over the slot period. In addition, the optimal D could be found by optimizing the mean square error with respect to D.

3.2 Structured MMSE equalizers

The non-blind MMSE equalizer does not incorporate any prior information of the channel structure. However, if we assume independent symbols, then using (6), it is easily shown that

$$\mathbf{r}_{\mathbf{y}s}(D) = \boldsymbol{\mathcal{H}}\mathbf{e}_{D+1}\ , \qquad (8)$$

where $\mathbf{e}_i$ is the $(L+M-1)\times 1$ unit vector with a one in the ith position and zeros elsewhere. We make the following two assumptions: (1) $M \geq L$, and (2) $M-1 \geq D \geq L-1$. The first assumption ensures that at least one of the columns of $\boldsymbol{\mathcal{H}}$ contains all the columns of $\mathbf{H}$. The second assumption means that $\mathbf{r}_{\mathbf{y}s}(D)$ contains all the elements of $\mathbf{H}$, which translates to choosing an equalizer delay where most of the signal energy is concentrated. With these assumptions and using (8) and (5), it can be shown that

$$\mathbf{r}_{\mathbf{y}s}(D) = \mathbf{G}_D\mathbf{c}\ , \qquad (9)$$

where

$$\mathbf{G}_D = \begin{bmatrix} \mathbf{0}_{(D-L+1)P\times\nu} \\ \mathbf{J}\mathbf{G} \\ \mathbf{0}_{(M-D-1)P\times\nu} \end{bmatrix},\ \mathbf{J} = \begin{bmatrix} & & \mathbf{I}_P \\ & \iddots & \\ \mathbf{I}_P & & \end{bmatrix}_{PL\times PL}. \qquad (10)$$

Let $\mathbf{U}_D$ be a basis for the null space of $\mathbf{G}_D^H$. Then it follows from (7) and (9) that the MMSE equalizer also satisfies

$$\mathbf{U}_D^H\mathbf{R}_{\mathbf{yy}}\mathbf{w}(D) = \mathbf{0}\ . \qquad (11)$$

The relation (11) suggests that the MMSE equalizer lies in the nullspace of $\mathbf{U}_D^H\mathbf{R}_{\mathbf{yy}}$. This relation is the key to designing a new family of equalizers.

Since (5) is an approximate relation, (11) is also approximately true. However, this structural constraint on the equalizer allows robust performance against noise as shown in the simulations. This motivates us to define a new cost function for designing equalizers. The new cost function includes (11) as a penalty term and is given by

$$\mathbf{w}(D) = \arg\min_{\mathbf{w}} \sum_{k=1}^{N_t}\left|\mathbf{w}^H\mathbf{y}_k - s_{k-D}\right|^2 + \alpha\left|\mathbf{U}_D^H\mathbf{R}_{\mathbf{yy}}\mathbf{w}\right|^2\ , \qquad (12)$$

where N_t is the number of training symbols, $\mathbf{R_{yy}}$ is the sample covariance matrix of $\mathbf{y}_k$ taken over all the data, and α is a non-negative penalty parameter that weighs the relative importance of the structural constraint as compared to the non-blind MMSE solution. The solution to (12) is

$$\mathbf{w}(D) = \left(\hat{\mathbf{R}}_{\mathbf{yy}} + \alpha \mathbf{R_{yy}} \mathbf{U}_D \mathbf{U}_D^H \mathbf{R_{yy}}\right)^{-1} \hat{\mathbf{r}}_{\mathbf{y}s}(D) \ , \qquad (13)$$

where $\hat{\mathbf{R}}_{\mathbf{yy}}$ and $\hat{\mathbf{r}}_{\mathbf{y}s}(D)$ indicate estimates based on the training sequence alone. Depending on the value of α, different equalizers can be obtained. For example, for $\alpha = 0$, the non-blind MMSE equalizer is obtained. For positive values of α, a family of structured equalizers is obtained.

4 BLIND IDENTIFIABILITY

The case of $\alpha \to \infty$ suggests an approach for obtaining blind structured equalizers. We consider the issue of identifying $\mathbf{w}$ based on the subspace constraint (11). We consider the case when $\alpha \to \infty$ under a noise-free assumption. The following result addresses the blind identifiability of equalizers $\mathbf{w}$ based on (11):

Proposition 4.1 *Suppose that $MP - \nu > L + M - 1$ and $[\widetilde{\boldsymbol{\mathcal{H}}}, \mathbf{G}_D]$ has full column rank, where $\widetilde{\boldsymbol{\mathcal{H}}}$ is the matrix $\boldsymbol{\mathcal{H}}$ with the $(D+1)$th column removed, then (i) the rank of $\mathbf{Q} \stackrel{\triangle}{=} \mathbf{U}_D^H \mathbf{R_{yy}}$ is given by $M + L - 2$, and (ii) all solutions to (11) are channel equalizers:*

$$\mathbf{w}^H(D)\, \boldsymbol{\mathcal{H}} = \gamma \mathbf{e}_{D+1}^H \qquad (14)$$

where γ is an arbitrary scalar.

• **Proof:** The proof is sketched as follows. In the noise free case, we have $\mathbf{R_{yy}} = \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^H$, hence:

$$\mathbf{Q} = \mathbf{U}_D^H \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^H$$

Since $\boldsymbol{\mathcal{H}}$ has $M+L-1$ columns and $\mathbf{U}_D^H$ has $MP-\nu$ rows, the rank of $\mathbf{Q}$ is at most $M+L-1$. In fact the rank drops by one because the $(D+1)$th column of $\boldsymbol{\mathcal{H}}$ is by construction in the null space of $\mathbf{U}_D^H$. Then the rank is at most $L+M-2$. Suppose that $\exists$ some $\boldsymbol{\eta} \neq \mathbf{0}$ s.t. $\mathbf{U}_D^H \widetilde{\boldsymbol{\mathcal{H}}}\, \eta = \mathbf{0}$, then $\widetilde{\boldsymbol{\mathcal{H}}} \boldsymbol{\eta} = \mathbf{G}_D \boldsymbol{\beta}$ for some non-zero $\boldsymbol{\beta}$. Under the hypothesis that $[\widetilde{\boldsymbol{\mathcal{H}}}, \mathbf{G}_D]$ has full column rank, it follows by contradiction that the rank of $\mathbf{Q}$ is exactly $L+M-2$. Result (ii) comes from the following: all solutions of (14) must satisfy (11). The solutions of (14) span a subspace of dimension $MP - M - L + 2$. From (i), the solutions of (11) span a subspace of similar dimension. Therefore, (11) determines all the channel equalizers. ■

Note that, in the presence of noise, the solutions of (11) become approximate equalizers. This analysis above shows that the new cost function (12) can be interpreted as a mixture between non-blind and blind schemes, with the "blindness" of the equalizer emphasized by the parameter α.

5 SIMULATION RESULTS

To investigate the performance of the structured MMSE equalizers, we consider a digital communication example where the true channel $c(t)$ is obtained from experimental channel impulse response (CIR) data measured at 2.4 GHz in an indoor environment [9]. Figure 2 shows the CIRs for two channels: line-of-sight (LOS) and non-LOS. The LOS CIR has a smaller delay spread than the non-LOS. The time slot length used is 300 symbols and a training preamble of 25 symbols is assumed. The transmit filter frequency response is a raised cosine with 35% roll-off factor where the impulse response is truncated to 6 symbol durations. An oversampling factor of two is used and $\nu = 4$.

Figure 3 shows typical equalizer outputs for the non-LOS channel data for a single time slot with 300 QPSK symbols. The various parameters are: SNR = 10 dB, $\alpha = 0.2$, and $M = L+2$. We observe that the structured equalizer output has less variance than the non-blind MMSE equalizer.

Figures 4 and 5 show the improvement in BER of the structured equalizers as compared to the non-blind MMSE equalizer for BPSK data for low to moderate SNRs for the LOS and non-LOS channel data respectively. We observe that for low to moderate SNR, the simulated values of α gave almost the same BER performance and they are superior to the non-blind ($\alpha = 0$) equalizer. At high SNR, we see that the BERs of the structured equalizers appear to saturate. This is suggested by the fact that structured equalizers are biased since they rely on the approximate relation in (5). At a target BER of 10^{-2} we get SNR gains of 5 dB for both the LOS and non-LOS channel data. If we assume that the true channel vector satisfies (5) exactly, then the structured MMSE equalizers still outperform the non-blind MMSE equalizer for higher SNRs. This is shown in Figure 6.

6 CONCLUDING REMARKS

We have developed direct semi-blind structured MMSE equalizers that can outperform traditional training-based MMSE equalizers in noise-limited environments. The optimal choice of α for given values of SNR, training length, and number of equalizer taps requires further investigation. This paper also proposes a novel blind equalization criterion as a by product. Many extensions of the proposed equalizers are possible. For example, other channel information such as known directions of arrival and delays of multipath in a specular channel could be used to obtain other structured equalizers. The extension to multiple antennas for designing space-time structured MMSE equalizers that can deal with co-channel interferences is straightforward. Another possible area to explore is to design structured adaptive equalizers to deal with time-varying channels.

References

[1] S. Haykin, Ed., *Blind Deconvolution*, Prentice-Hall, Englewood Cliffs, NJ, 1994.

[2] H. Liu, G. Xu, L. Tong and T. Kailath, "Recent developments in blind channel equalization: From cyclostationarity to subspaces," *Sig. Proc.*, vol. 50, no. 1-2, pp. 83–99, April 1996.

[3] A. Gorokhov and P. Loubaton, "Semi-blind second order identification of convolutive channels," in *Proc. Int.*

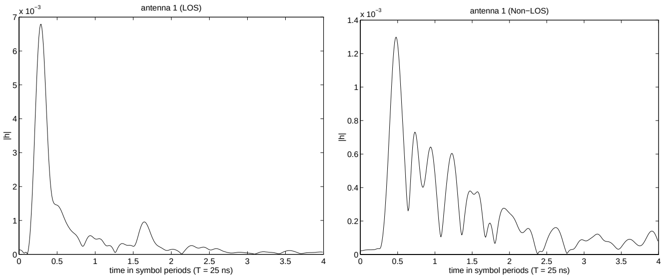

Figure 2: Indoor experimental CIR data: LOS (left) and non-LOS (right).

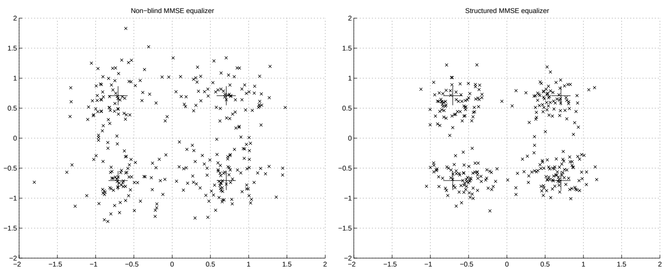

Figure 3: Scatter plots for the non-blind (left) and structured (right) MMSE equalizers.

Conf. Acoust., Speech and Sig. Proc., Munich, Germany, April 1997.

[4] E. De Carvalho and D. T. M. Slock, "Cramer-Rao bounds for semi-blind, blind and training sequence based channel estimation," in *IEEE Workshop on Sig. Proc. Advances for Wireless Commun.*, Paris, France, April 1997.

[5] S. V. Schell, D. L. Smith and S. Roy, "Blind channel identification using subchannel response matching," in *28th Annual Conf. of Info. Science and Systems*, Princeton, New Jersey, Mar. 1994, pp. 859–862.

[6] Z. Ding, "Multipath channel identification based on partial system information," *IEEE Trans. on Sig. Proc.*, vol. 45, no. 1, pp. 235–240, Jan. 1997.

[7] M. Cedervall, B. C. Ng and A. Paulraj, "Structured methods for blind multi-channel identification," in *Proc. 13th Int. Conf. Dig. Sig. Proc.*, Santorini, Greece, July 1997, pp. 387–390.

[8] B. C. Ng, M. Cedervall and A. Paulraj, "A structured channel estimator for maximum likelihood sequence detection," *IEEE Comm. Lett.*, vol. 1, no. 2, pp. 52–55, Mar. 1997.

[9] G. Janssen, P. Stigter and R. Prasad, "Wideband indoor channel measurements and BER analysis of frequency selective multipath channels at 2.4, 4.75 and 11.5 GHz," *IEEE Trans. Commun.*, vol. COM-44, no. 10, pp. 1272–1288, Oct. 1996.

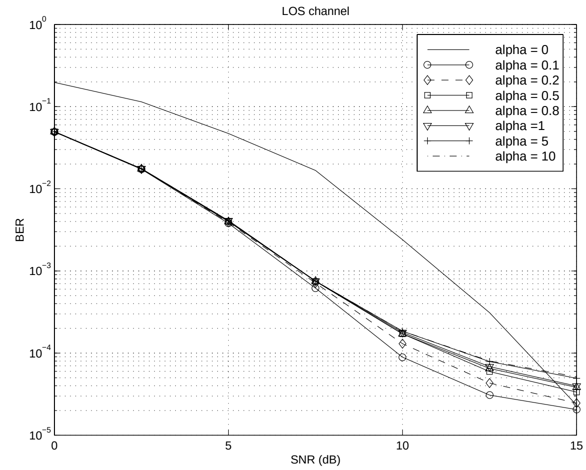

Figure 4: BER for the structured and non-blind MMSE equalizers for the LOS channel.

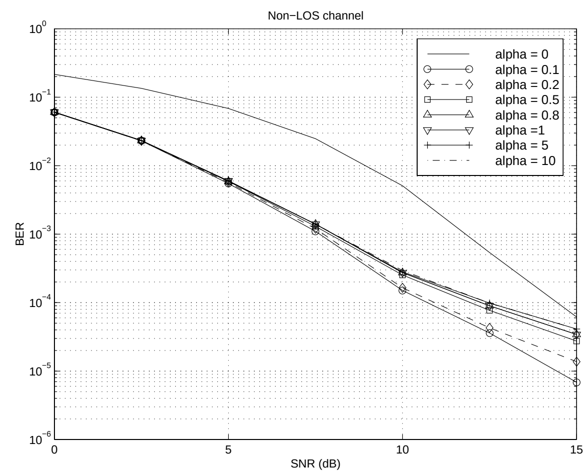

Figure 5: BER for the structured and non-blind MMSE equalizers for the non-LOS channel.

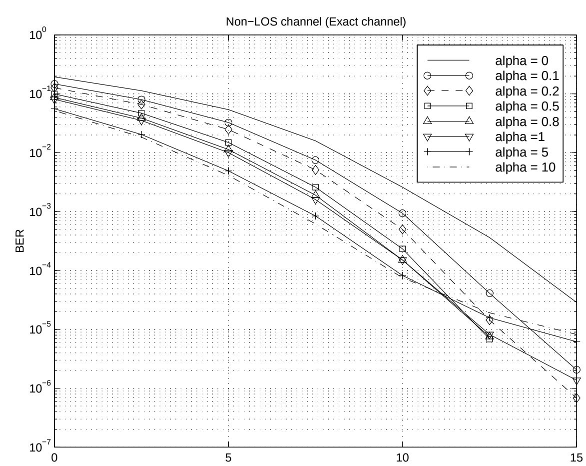

Figure 6: BER for the structured and non-blind MMSE equalizers with exact non-LOS channel taps.