DIFFUSION OF THE ATTRACTOR OF FRACTAL CODING FOR EDGE RESTORATION°

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ABSTRACT

Diffusion of the attractor or reconstructed image of the fractal code provides us a technique to restore edge information. Because of coding error associated with the fractal mappings, edges are degraded at high compression ratios. Partitioning compensates for the degradation, but lowers the compression ratios significantly and does not insure the retention of significant edges. The diffusion technique uses the image gradient to control the rate and direction of diffusion. This allows for smoothing in flat (low intensity transitions) regions and sharpening in edge (high intensity transitions) regions. The usage of the image gradient in this method insures the retention of significant edges. The diffusion technique presented in this paper lessens the degree of degradation of edges from fractal coding at a lower bit rate cost than partitioning at small block sizes.

1. INTRODUCTION

Fractal coding compresses digital images by relating areas of local self similarity at different scales in the images [6]. In an image, numerous smaller regions look like larger regions in the image; for example, a small cloud pretty much looks like a large cloud. Taking advantage of this correlation, the fractal code pairs up similar regions creating a list of fractal mappings. The attractor of this code is an approximation of the original image. The accuracy of the approximation depends on the degree of similarity between the larger to smaller mapping.

In order to increase the degree of similarity, the first automatic fractal coder proposed by Jacquin [3] uses quadtree splitting to partition the image into two block sizes. The quadtree splitting method divides a large square or block into four equal sized smaller squares. In smooth areas of the image, using the larger block size gives higher compression ratios. In areas of edges, the smaller block size increases the accuracy of the fractal mapping. This is still the most popular method for partitioning the image; although there exist more elaborate methods [2][5].

Compression ratio in a region decreases by a factor of four when that region needs to be partitioned to maintain a level of accuracy. In order to reconstruct that region, the fractal code requires four entries instead of one. Assuming each line in the fractal code requires 29 bits [2], covering an 8x8 pixel image block with 8 bpp (bits per pixel) results in a compression ratio of 17.65 (8*8*8/29) or bpp rate of 0.45 (29/8/8). Splitting this block into four 4 x 4 pixel blocks causes the compression ratio to be reduced to 4.4 (4*4*8/29) or a bpp rate of 1.8 (29/4/4).

Partitioning increases the probability of restoring edges in the attractor; however, it does not insure the retention of edges. Degradation of edges and loss of edge information occur in the attractor due to the limitation of fractal code to completely describe the similarities in the image with contractive affine transformations. In addition, the iterative nature of spatially reducing larger regions to smaller regions in the attractor for image reconstruction tends to "flatten" or "blur" the image even more [5].

Modeling edge degradation as diffusion allows us to develop a technique to sharpen and restore edges in the attractor. In recent years, diffusion techniques [1][8][9] have been used for image segmentation, edge detection, and image enhancement. Because the image gradient is a good estimate of the edges, choosing a diffusion coefficient as a function of the image gradient results in smoothing of flat regions and sharpening of edge regions. The diffusion coefficient can be selected to incur backward diffusion at certain location to sharpen edges and restore edges.

Our diffusion technique takes advantage of this backward diffusion to restore edges as well as sharpen edges in attractor image. We selected the diffusion coefficient to be a function of both the gradient of the original image and the gradient attractor. The advantage gained for this selection allows us to restore lost edge information. Section 2 of this paper discusses the diffusion technique and its implementation. Section 3 gives the experimental results of our implementation and Section 4 summarizes our approach and outlines future work.

2. DIFFUSION TECHNIQUE

2.1 Diffusion Model

Diffusion technique evolved from modeling the intensity of the image *f* as an anisotropic diffusion [9] in the form of

$$\frac{df}{dt} = \nabla \bullet (k \nabla f(x, y, t)) \qquad (1)$$

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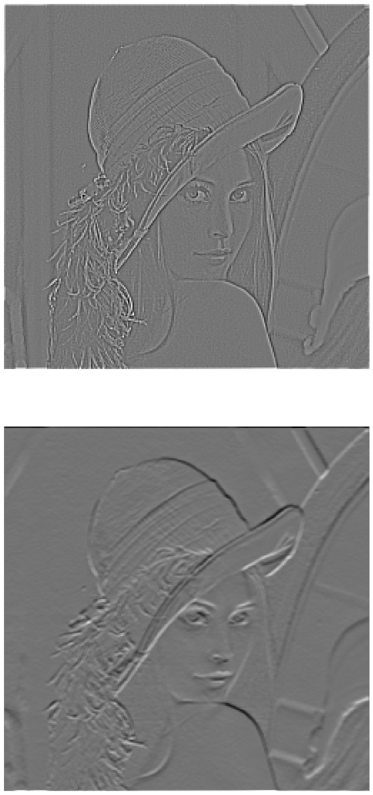

Figure 1. Comparison of the Laplacian image (top) to the gradient image (bottom).

where $\nabla\bullet$ and ∇ define the divergence and gradient operators. Letting the diffusion constant *k* vary, we can controlled the direction and rate of the diffusion spatially. In order to increase the backward diffusion for edge sharpening and restoration, we extended equation (1) to

$$\frac{df}{dt} = \nabla \bullet (g(x,y,t)\nabla f(x,y,t)) \qquad (2)$$

where *f* and *g* are scalar functions representing the intensity of the original image and the approximated image respectively. Using Green's theorems [4], we expanded equation (2) to

$$\frac{df}{dt} = g(x,y,t)\nabla^2 f(x,y,t) + \nabla(g(x,y,t)\nabla f(x,y,t)) \qquad (3)$$

where∇^2 defines the Laplacian operator. Use of equation (3) allowed edges defined by the Laplacian of the original image to be restored using backward diffusion.

2.2 Implementation

In order to minimize the cost in terms of compression ratio and simplify the implementation we assumed the following:

Figure 2. Image of the residual code.

- Fractal coding degrades the edges in a manner similar to diffusion; therefore we can restore the attractor by iteratively applying the changes dictated by equation (3).
- For images, the gradient image is similar to the Laplacian image; therefore $\nabla f \approx \nabla^2 f$. Visually inspecting these images in Figure 1 shows this to be a reasonable assumption.

We implemented the discrete diffusion in the following manner:

$$g(x,y,t+1) =$$
$$g(x,y,t) + \alpha[g(x,y,t)\nabla^2 f(x,y,t) + \nabla g(x,y,t)\nabla^2 f(x,y,t)] \quad (4)$$

where α is the percent change in the cross correlation between the Laplacian of the attractor and the image. In equation (4), we reduced the additional information that needed to be sent with the fractal code to just the Laplacian of the original image.

Since the Laplacian of the attractor g also approximates the Laplacian of the image *f*, only the residual code *r* given by

$$r(x,y) = \begin{cases} 1; & if\ \nabla^2 f - \nabla^2 g > T \\ 0; & if\ \nabla^2 f - \nabla^2 g = 0 \\ -1; & if\ \nabla^2 f - \nabla^2 g < T \end{cases} \qquad (5)$$

where *T* is an error threshold set by the user to retain the edges required by the application was encoded into the fractal code. For example, the residual information for 'Lena' shown in figure 2 was calculated using *T* = 0.18. This nomenclature allowed us to reduce additional information and to code the information with a smaller number of bits.

3. EXPERIMENTAL RESULTS

Initial results using a 256x256 gray level version of 'Lena' validated the usage of equation (3) for image restoration.

Figure 3. Attractor of the fractal code.

An approximation of ‘Lena’ from a simple fixed 8x8 block fractal coder in Figure 3 showed image degradation in terms of blurring, some loss of edge information, and reconstruction artifacts. For a quantitative evaluation, two quality measures were used: peak signal to noise ratio (PSNR) and a measure of edge preservation (ξ).

PSNR is given by [[2]]

$$\mathrm{PSNR} = 20\log_{10}\frac{\mathrm{b}}{\mathrm{rsm}} \qquad (6),$$

where b is the largest possible intensity value (i.e., 255 for a gray scale image) and rms is the root mean square difference between the original image and the approximated image. The measure of edge preservation (ξ) is given by [8],

$$\xi = \frac{\sum_{x,y}\Delta v'\Delta u'}{\sum_{x,y}\Delta v'\Delta v'\sum \Delta u'\Delta u'} \qquad (7)$$

where Δv’ and Δu’ are sharpened (highpass filtered) versions of the original and approximated images normalized by their means. The measure of edge preservation should approach unity when the approximation is close to the original image.

The image in Figure 3 was restored at different threshold values resulting in the images in Figure 4. PSNR was 26.3 dB and ξ was 0.81 for the restored image using the full Laplacian image in comparison to 26.4 dB and 0.55. Because PSNR value only gives general indication of image quality and not edge preservation; the changes in PSNR values were not significant. There was a significant increase in ξ from the restoration of the edges.

The other two images had approximately the same ξ at around 72 with PSNR values around 26.2. However, we transferred less Laplacian information. From our experiments, we found that ninety percent of Laplacian information was zero or near zero for ‘Lena. Using a 4x4 block for comparison, the average number of bits required would be 0.9*4*4 +2*0.1*4*4 or 17.6 bits for sending this information. The bpp rate would be 1.1 (17.6/4/4) which is less than the bpp rate of 1.8 for the quadtree split.

Figure 4. Restored Images using all the Laplacian information (top), with T = $2\sigma^2$ (middle), and with T = σ^2 (bottom).

4. SUMMARY

To further improve our diffusion technique, we are working to integrate a diffusion process in the fractal transform to better describe similarities in the image. Identifying the edge regions that benefit most from this technique would allow us to further reduce the bpp rates. We are developing a direct spatial filter mask to simplify the complexity of the implementation.

Our work showed that the diffusion technique can enhance the quality of the reconstructed image by restoring lost edge information. This increased the edge correlation between the original image and the reconstructed image significantly. The bpp rate for using this method was lower than using the current quadtree partition method. This paper indicated that current diffusion techniques and fractal coding can be leverage to create a more robust method of image compression.

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