BRANCH-HOPPED WAVELET PACKET DIVISION MULTIPLEXING

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ABSTRACT

Wavelet Packet Division Multiplexing (WPDM) is a high-capacity, flexible and robust orthogonal multiplexing technique in which wavelet packet basis functions are chosen as the coding waveforms. By analogy with frequency-hopped communication schemes, incorporation of time variation into the WPDM scheme offers the potential for further performance improvements, especially in frequency-selective fading channels. We consider a 'Branch-Hopped' WPDM scheme which employs an efficient modular switched transmultiplexer structure to induce the time variation. We determine classes of 'slow' and 'fast' hopping schemes analogous to their frequency-hopped counterparts, and evaluate several switching strategies for the transmultiplexer. For a given switching strategy we then design the filters within the transmultiplexer modules to provide further robustness to frequency-selective fading channels.

1. INTRODUCTION

Wavelet Packet Division Multiplexing (WPDM) [1] is an emerging orthogonal multiplexing technique in which the the properties of wavelet packet basis functions [2] are used for multiplexing. In contrast to the conventional time division (TDM), frequency division (FDM) and code division (CDM) multiplexing schemes, the waveforms used to represent the data symbols of each user overlap in *both* time and frequency. The fact that the waveforms are of finite duration and overlap in time and frequency provides a substantial increase in capacity over TDM and FDM [1] and robustness to certain adverse channel environments [1, 3], whilst their close relationships with multi-rate filter banks (transmultiplexers) provide particularly simple transmitter and receiver structures [1].

Orthogonal multiplexing schemes are sensitive to the effects of frequency-selective channels. Whilst the frequency overlapping of the WPDM waveforms provides some robustness to these effects, the commonly used wavelet packet basis functions are still 'localized' in frequency, and hence are susceptible to the perturbation of frequency-selective channels. In frequency-hopped communication schemes, this susceptibility can be reduced by 'hopping' the carrier frequency between several frequencies in a pattern which is known by the receiver. In the present paper, we consider an efficient modular switched transmultiplexer structure for implementing an analogous hopping scheme for WPDM, which we will call *Branch-Hopped WPDM* (BH-WPDM). We determine classes of 'slow' and 'fast' hopping schemes with performance advantages analogous to those of slow and (coherently combined) fast frequency hopping, and evaluate several switching strategies for the transmultiplexer. For a given switching strategy we then design the filters within the transmultiplexer modules to provide further robustness to frequency-selective fading channels.

2. WAVELET PACKET DIVISION MULTIPLEXING

We begin with a brief review of the WPDM scheme [1]. (See [4] and references therein for some related work.) Let g_0 be a unit-energy real causal FIR filter of length L which is orthogonal to its even translates; i.e., $\sum_n g_0[n]g_0[n-2m] = \delta[m]$, where $\delta[m]$ is the Kronecker delta, and let g_1 be the (conjugate) quadrature mirror filter, $g_1[n] = (-1)^n g_0[L-1-n]$. If g_0 satisfies some mild technical conditions [2], we can use an iterative algorithm to find the function $\phi_{01}(t) = \sqrt{2}\sum_n g_0[n]\phi_{01}(2t-nT_0)$, for a given T_0. Subsequently, we define the family of functions $\phi_{\ell m}$, $\ell \geq 0$, $1 \leq m \leq 2^\ell$ in the following (binary) tree-structured manner:

$$\phi_{\ell+1,2m-1}(t) = \sum_n g_0[n]\phi_{\ell m}(t-nT_\ell), \tag{1a}$$

$$\phi_{\ell+1,2m}(t) = \sum_n g_1[n]\phi_{\ell m}(t-nT_\ell), \tag{1b}$$

where $T_\ell = 2^\ell T_0$. For any given tree structure, the functions at the *terminals* of the tree form a *wavelet packet* [2]. They have a finite duration, $(L-1)T_\ell$, and are self- and mutually-orthogonal at integer multiples of dyadic intervals, and hence they are a natural choice for multiplexing applications. More precisely, if T denotes the set of terminal index pairs, then for $(\ell,m),(\lambda,\mu) \in T$

$$\langle \phi_{\ell m}(t-nT_\ell), \phi_{\lambda\mu}(t-kT_\lambda)\rangle = \delta[\ell-\lambda]\,\delta[m-\mu]\,\delta[n-k]. \tag{2}$$

In the WPDM scheme, the (binary) message data at the (ℓ,m)th terminal, $d_{\ell m}[n]$, are waveform coded by pulse amplitude modulation (PAM) of the function at that terminal. Hence the WPDM composite signal is

$$s(t) = \sum_{(\ell,m)\in T}\sum_n d_{\ell m}[n]\phi_{\ell m}(t-nT_\ell).$$

(Note that the terminals on different levels have different symbol rates, $1/T_\ell$.) Due to the orthogonality relationship in Eq. (2) the data can be extracted from the transmitted signal without inter-symbol interference or crosstalk by using a simple matched filter receiver for each terminal. By exploiting the structure in Eq. (1) we obtain an alternative transmitter structure using a tree-structured multi-rate filter bank and a single PAM modulator (Fig. 1)

$$s(t) = \sum_k \sigma_{01}[k]\phi_{01}(t-kT_0), \tag{3}$$

where

$$\sigma_{01}[k] = \sum_{(\ell,m)\in T} \sum_n f_{\ell m}[k-2^{\ell}n] d_{\ell m}[n], \tag{4}$$

and $f_{\ell m}$ is the *equivalent filter* from the (ℓ,m)th terminal to the root node, which can be found recursively using Eq. (1) and $f_{\ell m}[k] = \langle \phi_{\ell m}(t), \phi_{01}(t-kT_0)\rangle$. The orthogonality properties of the equivalent filters, $\langle f_{\ell m}[k-2^{\ell}n], f_{\lambda\mu}[k-2^{\lambda}i]\rangle = \delta[\ell-\lambda]\delta[m-\mu]\delta[n-i]$, for $(\ell,m),(\lambda,\mu) \in T$, confirm that an alternative receiver structure consisting of a single matched filter and a tree-structured multirate filter bank is available (Fig. 1). By substituting Eq. (4) into Eq. (3) we can view the WPDM scheme as a member of a class of generalized orthogonal CDM schemes in which the 'codes' are the equivalent filters $f_{\ell m}$ and the 'chip' waveform is ϕ_{01}. This generalized class includes the conventional orthogonal CDM schemes (i.e., Walsh-Hadamard schemes), but extends those schemes to allow for real-valued orthogonal codes which overlap in time, and orthogonal chip waveforms which have a duration longer than the chip interval.

3. BRANCH-HOPPED WPDM

The BH-WPDM scheme is based on a modular switched transmultiplexer structure in which a two-input two-output memoryless switching unit is attached to the input of each 'merge' module at the transmitter and to the output of each corresponding 'split' module at the receiver, as illustrated in Fig. 1. If we toggle the state of each switch at the transmitter in a pattern which is known at the receiver, we 'hop' the branches of the tree-structured filter banks. 'Long' or 'short' intervals between switch state changes lead to schemes with analogies to the slow and fast frequency-hopped schemes, respectively. Of course, the BH-WPDM hopping schemes require synchronization of the switches, but that is no more arduous than the synchronization of frequency-hopped schemes.

If the *terminals* of a BH-WPDM scheme are those nodes which are terminals when all the switches are in the parallel state, then the BH-WPDM composite signal can be written as (Fig. 1)

$$\tilde{s}(t) = \sum_k \nu_{01}[k]\phi_{01}(t-kT_0), \tag{5}$$

where ν_{01} is obtained from the 'switched' filter bank,

$$\nu_{01}[k] = \sum_{(\ell,m)\in T} \sum_n \tilde{f}_{\ell m;n}[k] d_{\ell m}[n], \tag{6}$$

and $\tilde{f}_{\ell m;n}$ is the equivalent filter from the (ℓ,m)th terminal to the root node 'seen' by a unit sample at instant n at the (ℓ,m)th terminal. If we define $x_{\lambda\mu}[i]$ to be the state of the switch at the (λ,μ)th node at the ith instant, with zero representing a 'parallel' connection and one representing a 'cross' connection then $\tilde{f}_{\ell m;n}$ can be found recursively as

$$\tilde{f}_{\ell+1,2m-1;n}[k] = \sum_i \big(g_0[i-2n](1-x_{\ell m}[n]) + g_1[i-2n]x_{\ell m}[n]\big)\,\tilde{f}_{\ell m;i}[k],$$
$$\tilde{f}_{\ell+1,2m;n}[k] = \sum_i \big(g_0[i-2n]x_{\ell m}[n] + g_1[i-2n](1-x_{\ell m}[n])\big)\,\tilde{f}_{\ell m;i}[k],$$

with $\tilde{f}_{01;n}[k] = \delta[k-n]$. (If all the switches are in the parallel state then $\tilde{f}_{\ell m;n}[k] = f_{\ell m}[k-2^{\ell}n]$.) The equivalent filters retain the orthogonality properties of those in the WPDM scheme, $\langle \tilde{f}_{\ell m;n}[k], \tilde{f}_{\lambda\mu;i}[k]\rangle = \delta[\ell-\lambda]\delta[m-\mu]\delta[n-i]$, for $(\ell,m),(\lambda,\mu) \in T$, and hence the receiver consists of a single matched filter and a 'switched' filter bank. The properties of $\tilde{f}_{\ell m;n}$ ensure that (with white binary data) the BH-WPDM composite signal, $\tilde{s}$, has the same cyclic spectra as that of the static WPDM scheme. Hence BH-WPDM retains the capacity advantages of WPDM. What the BH-WPDM scheme does change is the way in which the spectra of $\tilde{s}$ are allocated to the user at each terminal. The BH-WPDM composite signal can be viewed as a collection of PAM signals with index-dependent (orthogonal) waveforms, $\tilde{s}(t) = \sum_{(\ell,m)\in T}\sum_n d_{\ell m}[n]p_{\ell m;n}(t)$, where $p_{\ell m;n}(t) = \sum_k \tilde{f}_{\ell m;n}[k]\phi_{01}(t-kT_0)$, inherits the orthogonality properties of $\tilde{f}_{\ell m;n}$. By substituting Eq. (6) into Eq. (5) it can also be viewed as a 'code-hopped' extension to the class of generalized orthogonal CDM schemes in Section 2 in which the orthogonal codes allocated to the data symbols at a given terminal may vary from symbol to symbol, as illustrated in Fig. 2.

4. SWITCHING STRATEGIES AND FILTER DESIGN FOR BH-WPDM

Once the underlying tree structure of a BH-WPDM scheme has been chosen, its performance depends on the switching strategy and the filter g_0. In this section we evaluate several switching strategies for a given g_0, and then for a given switching strategy we design a g_0 to provide further robustness to frequency selective channels.

The term *switching strategy for the* (λ,μ)*th node* will refer to the mechanism which determines the switch state sequence, $x_{\lambda\mu}[i]$. An *overall switching strategy* determines a switching strategy for each node. A central observation in our analysis is that $\tilde{f}_{\ell m;n}$ depends on $x_{\lambda\mu}[i]$ at the nodes which are 'ancestors' of the (ℓ,m)th terminal, for $i \in [2^{\ell-\lambda-1}n, 2^{\ell-\lambda-1}n + L_{\ell}^{\lambda+1} - 1]$, where $L_{\ell}^{\lambda} = (2^{\ell-\lambda}-1)(L-1)+1$. We will refer to these states as the *collection of contributing switch states for* $\tilde{f}_{\ell m;n}$. For each terminal, the overall switching strategy creates a set of distinct equivalent filters and hops $\tilde{f}_{\ell m;n}$ amongst (shifted versions of) that set, where we say that two filters are *distinct* if they are not shifted versions of each other. If each subset is well chosen, improved performance in certain classes of channels may be obtained. Due to the structure of the BH-WPDM schemes, the set of distinct equivalent filters for a given terminal is not necessarily distinct from the set for another terminal at that level. Indeed, the performance averaging offered by such an overlap is often desirable. A measure of the degree of overlapping is the ratio of the number of distinct equivalent filters at the (ℓ,m)th terminal, denoted by $W_{\ell m}$, and the number of distinct equivalent filters for the whole scheme, denoted by W.

In order to classify the BH-WPDM schemes, we observe that if the collection of contributing switch states for $\tilde{f}_{\ell m;n}$ contains only one state of each switch, then $\tilde{f}_{\ell m;n}$ is simply a shifted version of a WPDM equivalent filter at the ℓth level. In that case, n will be said to lie in a *non-transient zone* for that terminal. Otherwise, $\tilde{f}_{\ell m;n}$ depends on both filters in the 'merge' unit at at least one node, and n is said to lie in a *transient zone*. Since g_0 and g_1 tend to be 'low-pass' and 'high-pass', respectively, an equivalent filter in a transient zone tends to have a broader frequency response (as a fraction of the bandwidth of the whole multiplexing scheme) than the corresponding WPDM filters.

If the overall switching strategy is such that the the duration of the transient zones are short with respect to the non-transient zones, then the set $\{\tilde{f}_{\ell m;n}[k]\}_{n\in\mathbb{Z}}$ is dominated by shifted versions

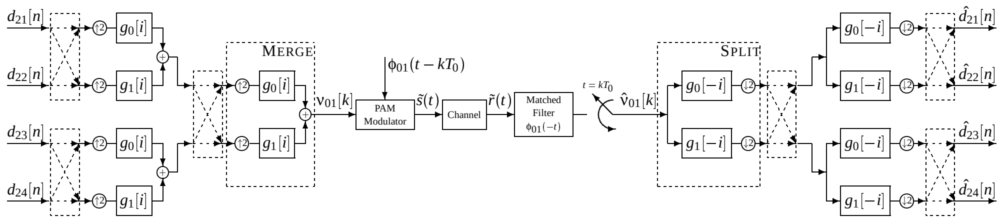

Figure 1: A four-user BH-WPDM scheme. The unmarked dashed boxes represent ‘switching’ units which provide either a ‘parallel’ or a ‘cross’ connection at each instant. If the switches remain in the ‘parallel’ state we have a WPDM scheme.

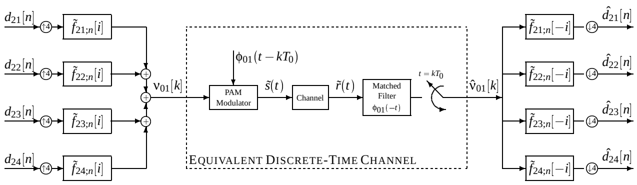

Figure 2: An equivalent model for Fig. 1.

of $f_{\ell m'}[k]$, $1 \leq m' \leq 2^{\ell}$, and the scheme will be said to be a *slow BH-WPDM scheme*. If each of these dominant equivalent filters is used for an equal proportion of bits, then in a slowly varying channel the performance at the (ℓ,m)th terminal will be an average of the performance of each terminal in the WPDM scheme which has the same g_0 and all its terminals at the ℓth level, an averaging process analogous to that of slow frequency hopping. If the duration of the transient zones is substantial, the scheme will be said to be a *fast BH-WPDM scheme*. Whilst the slow BH-WPDM schemes hop $\tilde{f}_{\ell m;n}$ amongst the equivalent filters provided by a static WPDM scheme, the fast BH-WPDM schemes hop $\tilde{f}_{\ell m;n}$ amongst a set of distinct equivalent filters of potentially broader frequency response. (The number and nature of the distinct equivalent filters is determined by the overall switching strategy.) In a slowly-varying channel, the performance at the (ℓ,m)th terminal in a fast scheme will be an average of the performance obtained from each individual equivalent filter $\tilde{f}_{\ell m;n}$, an averaging process with analogies to (coherently combined) fast frequency hopping. We illustrate the performance of the slow and fast BH-WPDM schemes in the following example. The switching strategies for each node are taken from the class of $(a\!:\!b)$ *switching strategies*, in which the switch toggles at each instant for a instants and then remains in its final state for b instants, and the pattern is repeated. For example, in a $(3\!:\!1)$ strategy $x_{\lambda\mu}[i]$ is periodic with one period given by the sequence $0,1,0,1,1,0,1,0$.

Example 1 Consider the four-user BH-WPDM scheme illustrated in Fig. 1, with $d_{2m}[n] = \pm 1$. The filter g_0 is chosen to be the standard Daubechies minimum-phase scaling filter [2] of length $L = 8$ (resulting in a short delay and a substantial capacity advantage over a conventional FDM scheme [1]). Four schemes are evaluated: the WPDM scheme, a slow BH-WPDM scheme, and two fast BH-WPDM schemes, with $(1\!:\!0)$ and $(3\!:\!1)$ strategies, respectively. In the fast schemes we employ the same switching strategy at each node, and in the slow scheme the switches at the level 1 nodes employ a $(1\!:\!\beta-1)$ strategy and the switch at the $(0,1)$ node employs a $(1\!:\!4\beta-1)$ strategy, with $\beta \gg L-1$. The ratio $W_{\ell m}/W$ for each scheme are given in Tab. 1. The frequency responses of some of the equivalent filters are provided in Fig. 3, from which

Scheme	W_{2m}/W	$\bar{D}$
WPDM	$1/4$	1.4165
Slow	$4/4$	1.4165
Fast $(1:0)$	$2/4$	0.3167
Fast $(3:1)$	$8/8$	0.9239

Table 1: The number of distinct waveforms at each terminal, W_{2m}, and for the scheme as a whole, W, and the overall deviation from frequency flatness, $\bar{D}$, for the schemes in Ex. 1. For the slow scheme we have neglected the equivalent filters in the (rare) transient zones.

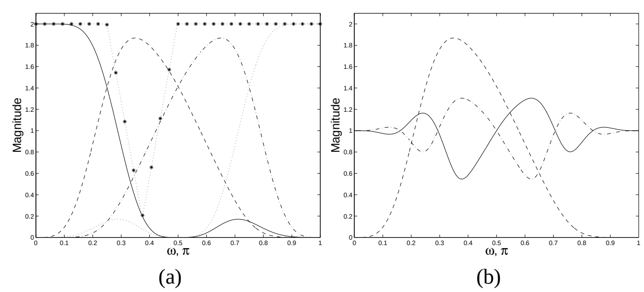

Figure 3: Frequency response of: (a) the equivalent filters for the WPDM scheme and the equivalent notch channel at a particular instant (dotted, with asterisks); (b) the equivalent filters for the $(2,2)$ terminal for the WPDM scheme (dashed) and the $(1\!:\!0)$ strategy (solid and dash-dot); for the schemes in Ex. 1.

the spectral broadening induced by the $(1\!:\!0)$ strategy is clear. The data from each user were transmitted over a slowly-varying notch channel, a snapshot of which is shown in Fig. 3(a), with an additive white Gaussian noise source at its output. The notch retained a constant shape and its position in frequency was varied slowly and uniformly between zero and π. The resulting overall bit error rate (BER) curves are plotted in Fig. 4. Although the slow BH-WPDM scheme tends to provide equal BERs for each terminal, it does so via time averaging. Hence the overall BER remains the same as that for WPDM. (See [5] for further details.) In contrast, the spectral broadening of the equivalent filters in the fast schemes provides a substantial overall BER improvement. □

The class of potential BH-WPDM switching strategies is large and diverse, which renders a general analysis of their performance intractable and performance evaluation via simulation computationally intense. As an alternative, we exploit the heuristic observation that the flatter the frequency response of the equivalent filter, the better the probability of error performance is likely to be in

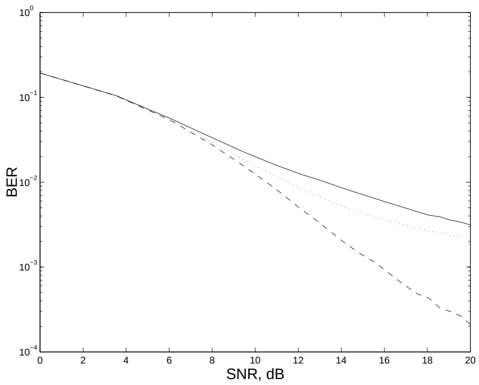

Figure 4: Simulated average BER against Signal-to-Noise Ratio (SNR) for the schemes in Ex. 1. Legend: WPDM and slow BH-WPDM (indistinguishable): solid; $(1:0)$ strategy: dashed; $(3:1)$ strategy: dotted.

channels whose (unknown) frequency-selective effects are narrow with respect to the bandwidth of the whole multiplexing scheme. As a measure of flatness, we define the *deviation from frequency flatness* of the equivalent filter $\tilde{f}_{\ell m;n}$ to be $D_{\ell m;n} \geq 0$, where

$$D^2_{\ell m;n} = \frac{1}{\pi}\int_0^{\pi}\left(\left|\tilde{F}_{\ell m;n}(e^{j\omega})\right|^2 - 1\right)^2 d\omega,$$

and $\tilde{F}_{\ell m;n}(e^{j\omega})$ is the Discrete-Time Fourier Transform of $\tilde{f}_{\ell m;n}[k]$. An overall deviation from frequency flatness can be defined according to the particular tree structure in use. For example, for a BH-WPDM scheme with all its terminals at the ℓth level, an overall deviation from frequency flatness over an interval $\mathcal{N}$ containing N symbols can be defined via $\bar{D}^2 = 2^{-\ell}/N \sum_{m=1}^{2^\ell}\sum_{n\in\mathcal{N}} D^2_{\ell m;n}$. The deviations from frequency flatness are simple to compute and can be an effective guide towards 'good' switching strategies for narrow frequency-selective channels, as can be seen from Tab. 1 and Fig. 4.

The above discussion suggests that for a given switching strategy, greater robustness to narrow frequency-selective channels might be obtained by choosing a g_0 to further reduce the values of the deviations. In the following example we show that performance gains are indeed achievable in this manner.

Example 2 For the fast BH-WPDM scheme with the $(1:0)$ strategy a (unit-energy) filter g_0 of length $L = 8$ was designed to minimize the deviation, $D_{2m;n}$, of one particular equivalent filter, subject to the deviations, $D_{2\mu;i}$, of the three other equivalent filters being within one percent of $D_{2m;n}$, and to g_0 being self-orthogonal at even translations. In addition, to provide some robustness to time-selective effects, such as fading and impulsive noise [3], the penalty figures $V^2_{2\mu;i} = \sum_{k=4i}^{4i+L_2-1}\left(\tilde{f}_{2\mu;i}[k]^2 - 1/L_2\right)^2$, where $L_2 = 3L-2$, for each distinct equivalent filter were constrained to be within one percent of $V^2_{2m;n}$, and $V^2_{2m;n}$ was constrained to be of the order of the penalty figure for the corresponding WPDM filter from Ex. 1. (The design of g_0 can be decoupled from that of ϕ_{01} so no 'regularity' constraints [2] are required.) This procedure resulted in $\bar{D} = 0.2263$ and on comparison with the corresponding values from Ex. 1 in Tab. 1 we predict a performance gain if this 'optimal' g_0 is employed. The BER curves

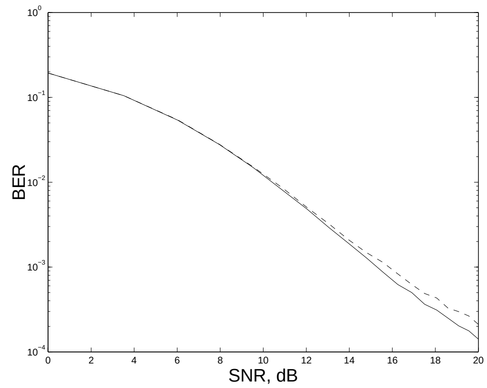

Figure 5: Simulated average BER against SNR for the fast scheme with the $(1:0)$ strategy using the 'optimal' g_0 (solid) from Ex. 2 and using the standard filter from Ex. 1 (dashed).

for transmission through the channel in Ex. 1 confirm that this predicted performance gain is in fact achieved. □

5. CONCLUSION

In this paper we have investigated the properties of the modular switched transmultiplexer which underlies the Branch-Hopped Wavelet Packet Division Multiplexing (BH-WPDM) scheme. We have shown that it can provide performance averaging analogous to that of frequency-hopped schemes whilst maintaining the capacity advantages of WPDM and the efficient structures for its implementation. We determined classes of slow and fast BH-WPDM schemes and isolated the deviation from frequency flatness as a convenient parameter in the evaluation of switching strategies for frequency-selective fading channels. Furthermore, we have shown that improved performance can be obtained by designing the filters in the transmultiplexer modules to reduce that deviation.

6. REFERENCES

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