ON SOURCES COVARIANCE MATRIX SINGULARITIES AND HIGH-RESOLUTION ACTIVE WIDEBAND SOURCE LOCALIZATION

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ABSTRACT

High resolution eigenstructure-based techniques for signal source localization are known to be ineffective when the source covariance matrix is not of full rank. We present here two techniques to circumvent this problem in the context of wideband active source localization. An extension is made to show how eigenstructure methods can be applied even when there is only one snapshot available to estimate the wideband spectral matrices.

1. INTRODUCTION

For many years, a considerable amount of attention has focused on eigenstructure methods for source localization. In spite of their very interesting asymptotic properties, this class of high-resolution techniques is difficult to apply with success when the underlying source covariance matrix happens to be singular.

This problem of rank deficiency occurs in particular when the sources are fully correlated (coherent sources). This is even worse for active methods, this means when the received signal is composed of time-delayed and amplitude-weighted replicas of a single signal: the emitted one.

Some specific "de-correlating" techniques have been proposed in the context of narrowband passive array processing [3], [5], [6]. The extension of such methods to active wideband arrays is not so easy to handle. The structure of the matrices involved in wideband modeling are indeed very specific and imply various combinations of the information obtained at different frequency bins for different sensors. Though, even if the properties are not really the same as in the narrowband model, some efficient wideband smoothing methods can be derived from the narrowband ones [2].

Another important limitation of eigenstructure methods is that they require good estimates of the spectral matrices and consequently numerous snapshots.

Our aim in this paper is to underline that the problems of sources coherence and of source localization using a single snapshot are very similar. They are in fact both strongly linked with some matrices rank deficiencies. After presenting two wideband methods capable of increasing the rank of the source covariance matrix, we show how to perform wideband eigenstructure-based source localization with a single snapshot. We also present some simulation results to illustrate the performance of wideband active localization with a single snapshot.

2. PROBLEM STATEMENT

Consider a uniform linear array composed of M equispaced sensors with P planewaves impinging on it. At frequency ν_f the Fourier transform of the signal received on the m^{th} sensor can be expressed as:

$$X_{m,f} = \sum_{p=1}^{P} c_p S_f \exp[-j\nu_f(\Psi_p + (m-1)\Phi_p)] + N_{m,f} \quad (1)$$

where c_p is the complex attenuation from the location of the p^{th} source to the array, S_f and $N_{m,f}$ are respectively the Fourier transforms at frequency ν_f of the emitted signal and of the noise received on the m^{th} sensor, and with :

$$\Psi_p = 2\pi T_p \qquad \Phi_p = 2\pi \frac{D\sin(\theta_p)}{C} \quad (2)$$

T_p is the time-delay between the p^{th} source and the first sensor ($m = 1$), D is the spacing between two adjacent sensors, C is the speed of the wavefronts, and θ_p is the direction of arrival of the p^{th} wavefront.

For the whole array, the information coming from the F different frequency bins of the band can be taken into account by combining the $M.F$ elements $X_{m,f}$ given in (1) to form a wideband vector as follows [1]:

$$\mathbf{X} = [X_{1,1}, X_{2,1}, \cdots, X_{M,1}, X_{1,2}, \cdots, X_{M,F}]^T \quad (3)$$

If the noise is not correlated with the signal sources, the wideband spectral matrix can be written as:

$$\mathbf{\Gamma_X} = E[\mathbf{X}.\mathbf{X}^H] = \mathbf{A}.\mathbf{\Gamma_C}.\mathbf{A}^H + \mathbf{\Gamma_N} = \mathbf{\Gamma_Y} + \mathbf{\Gamma_N} \quad (4)$$

$\mathbf{\Gamma_C}$ is the source covariance matrix of dimension $P \times P$. $\mathbf{\Gamma_Y}$ is the wideband spectral matrix of the received signal without noise. $\mathbf{\Gamma_N}$ is the spectral matrix of the noise. With the model introduced in (3), the spectral matrices $\mathbf{\Gamma_Y}$ and $\mathbf{\Gamma_N}$ are composed of F^2 matrix blocs (respectively $\mathbf{\Gamma}_{\mathbf{Y}_{f,g}}$ and $\mathbf{\Gamma}_{\mathbf{N}_{f,g}}$) whose structure are similar to narrowband spectral matrices of dimension $M \times M$. $\mathbf{A}$ is a $MF \times P$ matrix whose columns are the P wideband steering vectors $\mathbf{a}_p$. $\mathbf{A}$ can be seen as the juxtaposition of F matrix blocs corresponding each to the narrowband steering matrix $\mathbf{A}_f$ obtained at frequency ν_f. It is to be noted that each element of the vector $\mathbf{a}_p$ depends on both temporal and spatial parameters of the p^{th} source and takes the form:

$$a_{m,f,p} = S_f \exp[-j\nu_f(\Psi_p + (m-1)\Phi_p)] \qquad (5)$$

Thus $\mathbf{\Gamma_X}$ carries information about the time-delay and the direction of arrival of the sources. The active wideband model introduced above enables high-resolution techniques to perform spatio-temporal localization of the sources. For example, a MUSIC-like estimator can be defined as [1]:

$$MSCL(\theta, T) = \frac{1}{1 - \sum_{p=1}^{P} |\mathbf{a}^H(\theta, T)\mathbf{V}_p|^2} \qquad (6)$$

In this expression, $\mathbf{V}_p$ represents the eigenvector associated with the p^{th} largest eigenvalue of $\mathbf{\Gamma_X}$ and $\mathbf{a}(\theta, T)$ represents the wideband steering vector of parameters (θ, T) (see (5)). This 2D-functional exhibits peaks for the true direction of arrival and time-delay (θ_p, T_p) corresponding to the P sources.

As in the narrowband case, any eigenstructure-based localization method derived from the wideband model given in (3) requires $\mathbf{\Gamma_C}$ to be of full rank. Under this assumption, the rank of $\mathbf{\Gamma_Y}$ is equal to the number of sources and the P largest eigenvalues of $\mathbf{\Gamma_Y}$ are associated with the eigenvectors spanning the signal subspace.

But, if the source covariance matrix is not of full rank, the eigendecomposition fails to obtain the signal subspace. In practice, this results in a dramatical loss of resolution of eigenstructure-based sources localization methods. The denomination “High Resolution” is no longer appropriate as, in general, the estimation accuracy is then similar to the one given by conventional beamformer techniques.

When the singularity of the source covariance matrix is due to the presence of some coherent (i.e. fully correlated) sources, an efficient “de-correlating” scheme known as “spatial-smoothing” can be applied. This method was first proposed in a passive narrowband context and we have recently shown how to extend it to active wideband model [2]. In the next section, the main properties of this method are summarized, leading us to introduce a spectral-smoothing technique.

3. WIDEBAND SMOOTHING TECHNIQUES

In the remainder of the document the P observed sources are divided into Q groups of P_q coherent sources each. This means that two sources are fully correlated if they belong to the same group q, and that they are partially correlated if they belong to two different groups $q1$ and $q2$. Thus, the presence of partially correlated sources is represented by singletons, i.e. groups with $P_q = 1$.

3.1. Wideband Spatial Smoothing

As in the narrowband case, the idea is to divide the array originally composed of M_0 sensors into K overlapping subarrays of size M, the k^{th} subarray containing the sensors $[k, k+1, \cdots, k+M]$. A set of K spectral matrices $\mathbf{\Gamma}^k_\mathbf{X} = E\left[\mathbf{X}^k.(\mathbf{X}^k)^H\right]$ is estimated from the K subarrays outputs and the smoothed spectral matrix is defined by:

$$\mathbf{\Gamma}^S_\mathbf{X} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{\Gamma}^k_\mathbf{X} \qquad (7)$$

Following the wideband vector notation introduced in (3), it can be shown that the F^2 matrix blocs $\mathbf{\Gamma}^S_{\mathbf{Y}_{f,g}}$ forming the spectral matrix $\mathbf{\Gamma}^S_\mathbf{Y}$ can be written as:

$$\mathbf{\Gamma}^S_{\mathbf{Y}_{f,g}} = \mathbf{A}_f \left(\frac{1}{K}\sum_{k=1}^{K}(\mathbf{B}_f)^{(k-1)}\mathbf{\Gamma_C}(\mathbf{B}^H_g)^{(k-1)}\right)\mathbf{A}^H_g \quad (8)$$

where $\mathbf{B}^k_f$ is a diagonal matrix whose P nonzero elements are:

$$B^k_{p,f} = \exp[-j\nu_f(k-1)\Phi_p] \qquad (9)$$

The main results provided by the study of the algebraic properties induced by (8) and (9) are [2]:

- Spatial smoothing mainly results in modifications of $\mathbf{\Gamma_C}$, and the decomposition given in (4) still holds with this smoothed source covariance matrix.

- The rank of the smoothed source covariance matrix only depends on the number K of subarrays and on the number Q of coherent sources groups. In particular, if $K \geq \frac{P}{Q}$ the smoothed source covariance matrix is at least of rank P.

These two points guaranty that the smoothed wideband spectral matrices have the same form as the ones obtained with partially correlated sources. Consequently spatial smoothing enables eigenstructure-based methods to successfully localize the sources.

3.2. Spectral Smoothing

In the frequency domain, under certain conditions that will be specified below, it is possible to define a smoothing technique similar to the spatial smoothing. This mainly holds on the very particular structure of the wideband spectral matrices obtained with uniform linear arrays. An easy way to apprehend this property is to reorder the elements $X_{m,f}$ defined in (1) as follows:

$$\mathbf{X} = [X_{1,1}, X_{1,2}, \cdots, X_{1,F}, X_{2,1}, \cdots, X_{M,F}]^T \tag{10}$$

Using this new wideband vector, the spectral matrix remains defined by the equation (4). It contains exactly the same elements as the spectral matrix used in the previous sections, but they are differently arranged. This time, the matrices $\mathbf{\Gamma_Y}$ and $\mathbf{\Gamma_N}$ are composed of M^2 matrix blocs ($\mathbf{\Gamma}_{\mathbf{Y}_{m,n}}$ and $\mathbf{\Gamma}_{\mathbf{Y}_{m,n}}$) of dimension $F \times F$, each bloc corresponding to the interaction between the sensors m and n for all the frequency bins. The wideband steering matrix $\mathbf{A}$ can also be seen as the juxtaposition of M matrix blocs of dimension $F \times F$, each bloc corresponding to a single sensor wideband steering matrix $\mathbf{A}_m$.

At this stage, if the difference between two frequency bins is constant ($\Delta = \nu_{k+1} - \nu_k$) and if the signal received on the array has been whitened, a spectral smoothing technique can be applied. It simply consists in dividing the band originally composed of F_0 bins into K overlapping sub-bands of F bins each. The k^{th} sub-band contains the bins $[k, k+1, \cdots, k+F]$ and provides an estimate $\mathbf{\Gamma}_{\mathbf{X}}^k$ of the wideband spectral matrix. The spectral smoothed matrix is then defined as in (7) by averaging the K wideband spectral matrices obtained on the K sub-bands. A similar expression to (8) is obtained, as the M^2 matrix blocs $\mathbf{\Gamma}_{\mathbf{Y}_{m,n}}^F$ forming the spectral smoothed matrix $\mathbf{\Gamma}_{\mathbf{Y}}^F$ have the form:

$$\mathbf{\Gamma}_{\mathbf{Y}_{m,n}}^F = \mathbf{A}_m \left(\frac{1}{K} \sum_{k=1}^{K} (\mathbf{B}_m)^{(k-1)} \mathbf{\Gamma_C} (\mathbf{B}_n^H)^{(k-1)} \right) \mathbf{A}_n^H \tag{11}$$

where $\mathbf{B}_m^k$ is a diagonal matrix whose P nonzero elements are:

$$B_{p,m}^k = \exp[-j\Delta(k-1)(\Psi_p + (m-1)\Phi_p)] \tag{12}$$

As the equations (11) and (12) are similar to (8) and (9), the algebraic properties of the spectral smoothing are the same as the wideband spatial smoothing ones. Thus, the growth of the rank of the spectral smoothed source covariance matrix only depends on K and Q.

Spectral smoothing is of a particular interest when the total number of sensors M_0 of the array is small, as in such a case the wideband spatial smoothing is very difficult to apply. The two kinds of wideband smoothing techniques presented here can of course be jointly used, the rank of the smoothed matrices depends then on the number of subarrays and on the number of sub-bands.

4. SINGLE SNAPSHOT LOCALIZATION

Another major drawbacks of eigenstructure-base methods is that they require a considerable number of snapshots to have a good estimation of the spectral matrices. They yield very low accurate results when only one single snapshot is available. This problem is in fact very close to the problems of sources coherence. Once again it concerns a problem of rank deficiency.

A first quick analogy can be done to illustrate this fact. Consider a unique group of P coherent sources ($Q = 1$) with no noise. The spectral matrices of the received signal $\mathbf{\Gamma_X}$ is then of rank one and the wideband smoothing techniques can be applied to increase the rank of the matrices as shown in the previous section.

If there only is one snapshot available, instead of using estimates of the spectral matrices $\mathbf{\Gamma_X} = E[\mathbf{X}.\mathbf{X}^H]$, it is interesting to use the quantity defined by:

$$\tilde{\mathbf{\Gamma}}_{\mathbf{X}} = \mathbf{X}.\mathbf{X}^H \tag{13}$$

$\tilde{\mathbf{\Gamma}}_{\mathbf{X}}$ is also of rank one and has exactly the same structure as $\mathbf{\Gamma_X}$. Applying wideband smoothing techniques to $\tilde{\mathbf{\Gamma}}_{\mathbf{X}}$ enables then eigenstructure-based method to provide estimates of the sources locations with success.

In the presence of noise, such an analogy is more difficult to maintain. But we can still reasonably expect that increasing the ranks of the spectral matrices by applying wideband smoothing techniques will help to overcome the problems linked with the rank deficiency of the matrix $\tilde{\mathbf{\Gamma}}_{\mathbf{X}}$.

To conclude, it must be said that, when applied on a single snapshot, the wideband smoothing methods are another way of trying to estimate the spectral matrices. The problem addressed here is to construct a significant estimate of $\mathbf{\Gamma_X} = E[\mathbf{X}.\mathbf{X}^H]$. As there is only one snapshot, it is not possible to approximate the mathematical expectation operator by a time-averaging process.

The notion of coherence and/or correlation of the sources is now meaningless and other averaging methods have to be employed to estimate the spectral matrices [4]. In this context, the wideband smoothing schemes introduced above can be considered as an attempt to obtain these estimates. The classical assumption of signal ergodicity is then replaced by the assumptions of local stationarity in the space and frequency domains. Even if these two kind of averages involve necessarily finite sample data, they tend to provide matrices whose structure is close to the theoretical form of spectral matrices.

5. SIMULATION AND EXPERIMENTAL RESULTS

In the example presented in figure (1), there are 2 sources at location $(\theta_1 = -5deg, T_1 = 100ms)$ and $(\theta_2 = 10deg., T_2 =$

$125ms)$, $M = 6$, $F = 6$, $SNR = 20dB$. The number of subarrays is equal to the number of sub-bands and 3. We give the results: #1 obtained with the wideband active beamformer and #2 obtained with a wideband active MUSIC-like estimator applied on wideband smoothed spectral matrices. If the conventional 2D-localization technique

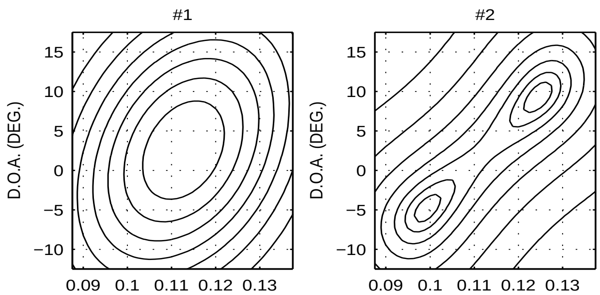

Figure 1: simulation of wideband localization with a single snapshot

is unable to localize the two sources, the high resolution properties of the MUSIC-like estimator are preserved and the two sources are correctly localized.

It is to be noted that we have also started to apply spectral smoothing on experimental data to localize wideband sources using one single snapshot and the results obtained are very encouraging. In the example given in figure (2), there are two predominant sources and several coherent echos. The array is composed of only $M = 3$ sensors, the number of frequency bins is $F_0 = 43$ and there was $K = 30$ sub-bands and no wideband spatial smoothing. We can see that the wideband active beamformer (#1) encounter some difficulties in localizing the weak sources. The MUSIC-like estimator (#2) applied on single snapshot estimates of the wideband smoothed spectral matrices localizes the other sources.

6. CONCLUSION

In this paper two kinds of problems encountered by eigenstructure-based source localization techniques were evoked. They are namely the presence of coherent sources and the possibility to perform wideband source localization using only a single snapshot. The both problems are caused by some rank deficiencies of the source covariance matrix. We have pointed out the strong similarities existing between these two problems and have proposed wideband smoothing techniques to circumvent them.

Even if the first results obtained in applying these methods to experimental data are convincing, it remains clear that some further work has to be done to determine with precision the influence of the various parameters. An interesting study would be, for example, to analyze the real impact of the pre-whitening stage.

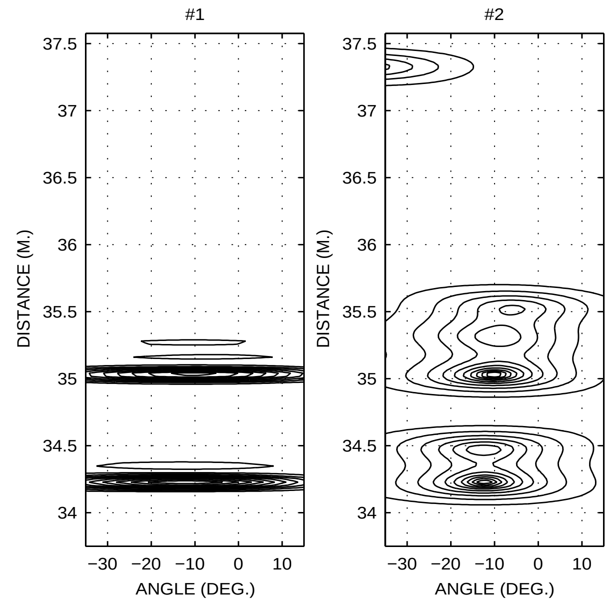

Figure 2: wideband localization with a single snapshot performed on experimental data

7. REFERENCES

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