FAST BLIND IDENTIFICATION OF FIR COMMUNICATIONS CHANNELS

Claudio Becchetti, Gaetano Scarano and Giovanni Jacovitti

Dip. INFOCOM, Università di Roma "La Sapienza"
Via Eudossiana 18, I-00184 Roma, Italy Fax: +39.6.4873.300, Tel: +39.6.4458.5500
E-Mail: **cbecc, gaetano, gjacov@infocom.ing.uniroma1.it**

ABSTRACT

This contribution describes a fast frequency domain approach for blind channel identification which does not rely on the statistic of the symbols. The proposed approach is based on the so–called "intraspectral relations" of DFT's of PAM fractionally sampled signals. The use of DFT's is allowed under certain conditions commonly encountered in data communication systems. The intraspectral relations are equivalent to analogous relations introduced in the time domain in [1, 2, 3]. From the intraspectral relations, asymptotically efficient solutions are derived which turn out to be either more accurate or less expensive in term of complexity w.r.t. the time domain counterparts. Simulation results are provided to assess the validity of the proposed approach in comparison with the Rao Cramer bound and with other approaches from the literature.

1. INTRODUCTION

Blind identification and equalization of data communication channels are interesting as they do not require training sequences thus saving channel bandwidth. Older methods were based on statistical assumptions about the transmitted symbol sequence. In recent years new methods based on spectral redundancy have been investigated. In these methods channel estimation was connected to the multichannel nature of the oversampled received sequence. In those related algorithms, the unknown channel response is the solution of a Least Square system obtained by imposing the "Cross Relations" among the different temporal subchannels [1] (CR method). These relations assure, under some conditions,an exact estimation of the channel for infinite symbol sequences and/or for no additive noise conditions. More robust estimates can be achieved by the subspace (SS) blind identification introduced in [2] which is derived from MUSIC -like approaches. In this method, the observed data space is decomposed in the channel and noise subspaces. The solution is represented by the orthogonal vectors to the noise subspace.

A widely known property of linear systems is that if the noise affects only the vector of known terms and if the perturbation is white Gaussian then the Least Square estimation is equivalent to the Maximum likelihood estimation. This condition is achieved by weighing the linear system with a proper matrix. The Two Steps Maximum Likelihood method (TSML) [3] is based on these concepts. The algorithm consists of computing a first channel estimate thorough CR or SS methods using it to weight a new CR system. Another class of methods invoke the cyclostationarity of the fractionally sampled signals. The so called "cyclic spectra" are estimated in order to extract phase channel information. In [4, 5], blind channel identification methods based on cyclic spectra are proposed. In [6], adaptive and optimal solution are further presented. In this contribution, we present a deterministic method based on signal representation in the discrete time Fourier domain. The CR relations are converted into "intra spectral" DFT relations valid for data burst spaced by time guards. This condition is often satisfied especially in mobile communication. We show that these "intra-spectral relations" project data on the eigenspace of the Fourier basis and we introduce consistent and two steps asymptotically efficient solutions. Among the benefits, this eigendecomposition allows for a considerable reduction of the computational complexity of the method with respect to the time domain solutions.

2. THE SPECTRAL REDUNDANCY

The proposed method relies on the spectral properties of the Pulse Amplitude Modulated (PAM) signals. A typical PAM signal is defined as:

$$a(t) = \sum_{n=-\infty}^{\infty} a_n \cdot \delta(t - nT) \quad (1)$$

where a_n is a complex sequence of symbols, $\delta(t)$ is the Dirac distribution and T is the symbol period. The Continuous Time Fourier Transform (CTFT) $A(f) = \mathcal{F}\{a(t)\}$ is characterised by a $1/T$ periodicity. In fact we have

$$A(f) = \sum_{n=-\infty}^{\infty} a_n \cdot e^{-j2\pi fT} = A\left(f + \frac{1}{T}\right) \quad (2)$$

The spectral redundancy can be used to derive "intraspectral relations" among the received signal $y(t)$ and the channel response $h(t)$ regardless the actual symbol sequence a_n . Considering the (noiseless) received signal directly in the frequency domain we have:

$$Y(f) = A(f) \cdot H(f) \tag{3}$$

$$Y\left(f - \frac{1}{T}\right) = A(f) \cdot H\left(f - \frac{1}{T}\right) \tag{4}$$

where $X(f)$ and $H(f)$ are the CTFT of $x(t)$ and $h(t)$, respectively, and all shaping filters have been included in the channel $h(t)$.

Eliminating $A(f)$, we obtain the following homogeneous equations:

$$Y\left(f - \frac{1}{T}\right) \cdot H(f) - Y(f) \cdot H\left(f - \frac{1}{T}\right) = 0 \tag{5}$$

This equality turns out to be the CR condition for the case of signal pair extracted from PAM signal. Its discrete time version has been also outlined in [5]

3. CHANNEL IDENTIFIABILITY

The relation (5) enlightens a simple general condition for channel identifiability. Constant complex factors multipling $H(f)$ are not identifiable since they can be dropped out in (5). More generally, if $H(f)$ can be expressed as $H(f) = H(f) \cdot H"(f)$ where $H"(f)$ shows a $1/T$ periodicity, *i.e.* $H"(f) = H"(f + 1/T)$, then $H"(f)$ can be dropped out from (5) and it cannot be identified by the intraspectral relations. Those periodical frequency responses do correspond to time responses of the kind $h"(t) = \sum \alpha_k \delta(t - kT)$. Therefore our identifiability condition for PAM signals is the following: *Channel factors originating from echoes at integer multiples of the symbol period T are not identifiable*. This condition generalizes the identifiability conditions previously reported in literature for channels represented by polynomial models (see [7]). In fact, if we assume a FIR model for the channel, the previous identifiability condition are equivalent to the well-known condition on the system function of the channel which must not have zeroes uniformly spaced on a circle or equivalently common zeroes between subchannels [7].

4. A NEW BLIND INDENTIFICATION METHOD

Let us consider the fractionally sampling of the received signal $y(n) = y(nT/2)$. In this case, $y(n)$ still embodies the spectral redundancy provided that the roll-off of the shaping filters is greater then zero. In fact the fractionally sampling periodizes the spectrum at distances $2/T$ thus preserving the band in which redundancy is present. We then introduce the Discrete Fourier Transforms (DFT) of the fractionally sampled sequences when N data samples are considered:

$$Y(k) = \sum_{n=0}^{N-1} y(n) e^{-j\frac{2\pi}{N}nk} \quad ; \quad A(k) = \sum_{n=0}^{N-1} a_n e^{-j\frac{2\pi}{N}nk}$$

The periodicity of the spectrum $A(f)$ (2) still holds even for its DFT counterpart $A(k)$. In fact it is easily shown that

$$A(k) = A(k + N/2] \quad ; \qquad k = 0, \cdots, N/4 - 1 \tag{6}$$

On the contrary, the CTFT products in (3) and (4) are equivalent to DFT products only if we can embed the underlying linear convolution into a properly defined circular convolution. This occurs when an L order FIR filter $h(n)$ is considered and a_n is suitably zero–padded from $N-L-1$ and $N-1$. In this case, the intraspectral relations (5) holds also for DFT's:

$$Y(k) \cdot H(k + N/2) - Y(k + N/2) \cdot H(k) = 0 \tag{7}$$
$$k = 0, \cdots, N/4 - 1$$

These conditions are not rarely satisfied in digital data communication. For example, in mobile communication, TDMA systems require time guards between subsequent data bursts in order to guarantee transmission into the proper time slot bounds. These time guards must be long at least the time required by the signal to be transmitted to the maximum intra–cell distance which can be approximately considered as the maximum time extension of the channel. Moreover, other zeroed symbols are generally appended on the head and the tail of a burst of data for synchronization purposes.

Let us proceed with the identification algorithm and let us collect the coefficients of the channel impulse response in the L-vector $\mathbf{h} = [h(0) \cdots h(L-1)]^{\mathrm{T}}$. Now, the useful equations of (5) are obtained considering frequency bins k which correspond to non vanishing spectra. Taking into account the selectivity of the shaping filters and denoting by ρ their roll–off parameter, we can collect all the useful equations in (5) by introducing the matrices $\mathbf{Y}$ and $\widetilde{\mathbf{Y}}$ whose elements are given by:

$$\mathbf{Y}_{pq} = Y(p) \cdot \delta_{pq} \quad ; \quad \widetilde{\mathbf{Y}}_{pq} = Y\left(p + \frac{N}{2}\right) \cdot \delta_{pq} \tag{8}$$

$$p, q = (1-\rho)\frac{N}{4}, \cdots \frac{N}{4} - 1 \tag{9}$$

where $\delta_{pq} = 1$ with $p = q$ and 0 otherwise. Now, (5) is rewritten in following compact form:

$$\left(\mathbf{Y} \cdot \widetilde{\mathbf{W}} - \widetilde{\mathbf{Y}} \cdot \mathbf{W}\right) \cdot \mathbf{h} = \mathbf{0} \tag{10}$$

where the matrices $\mathbf{W}$ and $\widetilde{\mathbf{W}}$ are (partial) DFT matrices defined as

$$\mathbf{W}_{pl} = e^{-j\frac{2\pi}{N}pl} \quad ; \quad \widetilde{\mathbf{W}}_{pl} = e^{-j\frac{2\pi}{N}\left(p+\frac{N}{2}\right)l}$$
$$p = (1-\rho)\frac{N}{4}, \cdots \frac{N}{4} - 1, \quad l = 0, \cdots, L-1$$

Noted that in (10), the received data $Y(k)$ are present in the form of diagonal matrix. This will reduce the computational load of the weighed solutions required in presence of noise.

Least Square solutions may be employed in order to solve (10) in presence of noise. We introduce the DFT of the noise corrupted received signal $Z(k) = Y(k) + V(k)$ where $V(k)$is the DFT of a Gaussian zero mean i.i.d. noise $v(n)$ with variance σ_v^2. Defining the diagonal matrices $\mathbf{Z}$ and $\mathbf{V}$, corresponding to $Z(k)$ and $V(k)$ as in (9), the intra-spectral relations are modified as follows:

$$\left(\mathbf{Z} \cdot \widetilde{\mathbf{W}} - \widetilde{\mathbf{Z}} \cdot \mathbf{W}\right) \cdot \mathbf{h} = \mathbf{r} \tag{11}$$

where the residue vector $\mathbf{r}$ is given by $\mathbf{r} = \widetilde{\mathbf{V}} \cdot \mathbf{H}_f - \mathbf{V} \cdot \widetilde{\mathbf{H}}_f$ being

$$\mathbf{H}_f = \left[H\left((1-\rho)\frac{N}{4}\right) \cdots H\left(\frac{N}{4} - 1\right)\right]^{\mathrm{T}}$$
$$\widetilde{\mathbf{H}}_f = \left[H\left((1-\rho)\frac{N}{4} + \frac{N}{2}\right) \cdots H\left(\frac{3N}{4} - 1\right)\right]^{\mathrm{T}}$$

A simple solution of (11) can be obtained by a constrained ordinary least square solution:

$$\underset{\mathbf{h}}{\mathrm{Min}} \left\| \left(\mathbf{Z} \cdot \widetilde{\mathbf{W}} - \widetilde{\mathbf{Z}} \cdot \mathbf{W}\right) \cdot \mathbf{h} \right\|^2 \tag{12}$$

with the constraint $|\mathbf{h}|| = 1$. This solution which will referred to as "FCR" is easily proved to be statistically unbiased and consistent.

The Minimum Variance Unbiased (MVU) solution makes explictely use of the (square root of the) covariance matrix of the residue $\mathbf{r}$, say $\mathbf{R}_{\mathrm{r}} = \mathbf{P}\mathbf{P}^{\mathrm{H}}$. The covariance matrix $\mathbf{R}_{\mathrm{r}}$ is a diagonal matrix having nonzero entries

$$(\mathbf{R}_{\mathrm{r}})_{kk} = \sigma_r^2(k) = \sigma_v^2 \cdot \left(|H(k)|^2 + |H(k + N/2)|^2\right)$$

and each equation in (11) is optimally weighed by $\sigma_r(k)$. Unfortunately, the covariance $\mathbf{R}_{\mathrm{r}}$ depends on the actual (unknown) channel response. Then, a two steps solution can be employed whereas the first steps must obtain a consistent estimation of the channel $\mathbf{h}$ using (11) and an estimation of the optimal weights σ_r is accomplished using (4). In the second steps an optimally weighed version of (11) is solved for $\mathbf{h}$.

The Frequency domain Two Steps (FTS) solution is an asymptotically MVU solution provided that unbiasedness and consistency are guarantied for the solution obtained in the first step. Moreover, if the residue $\mathbf{r}$ is Gaussian, as in this case, the two steps solution is also an asymptotically Maximum Likelihood solution. A FTS solution is sensible to a poor initial estimation of $\sigma_r^2(k)$ achieved in the first step. Assuming i.i.d. with variance σ_a^2, another estimation of the optimal weigths can be obtained using directly the observations $Z(k)$:

$$\hat{\sigma}_r^2 = \sigma_a^2 \cdot \left(|Z(k)|^2 + \left|Z\left(k + \frac{N}{2}\right)\right|^2 - 2\sigma_v^2\right) \tag{13}$$

This first step solution obtained through the weighting (13) has been revelead to be more robust then the simple LS solution (12) with respect to channel conditions. For this reason the solution with estimated weigths in the frequency domain (EWF) has been used as the first step of FTS.

The eigendecomposition of the data on the Fourier basis allows for a considerable reduction of the computational cost. In essence, similar performances are achieved by FTS w.r.t. TSML at lower complexity. In addition, the FTS achieves better performances w.r.t. SS and CR methods at a comparable computational cost. In particular, the number of multiplications required by the TSML are $O(N^3)+2(W-L)O(L^2)+LN$ where W is the window length (see [2]), N is the number of observations, L is the channel order. FTS requires only $3P^2+N/2\log_2 N+5/2NP$. Then the requirement of $O(N^3)$ multiplications of TSML is reduced to $N/2\log_2(N)$ for the proposed method while obtaining similar results (see next section). SS and CR require L^2+LN, $(W-L)O(L^2)+LN$ multiplications respectively [8]. If we consider short length of data so that $\log(N)/2$ is comparable to L, the proposed FTS requires a similar or smaller number of multiplications while outperforming the CR and SS time domain methods. In order to improve accuracy, the (known) shaping filter must be separately accounted for in either the time and the proposed frequency method. In the time domain methods, this is obtained by a subchannel convolution between the shaping function and the observations [9]. If the length of the (truncated FIR) shaping filter is L_g, then an extra cost of $O(NL_g)$ must be considered for the time domain methods while frequency domain methods only require an $O(N)$ extra cost. The lower computational complexity for the FTS method with respect to TSML is due to the weighing operation that for FTS is fast. In fact, for each equation it consists only in dividing the two factors $Z(k)$ and $Z(k + N/2)$ for the standard deviation of the residue $\sigma_r(k)$ whereas in the time domain must be performed a full matrix matrix product after a pseudoinversion!

5. SIMULATION RESULTS

Tthe statistical performance of the three proposed frequency domain solutions (FCR, EWF, FTS) are reported. Then, performance of the FTS method are compared to time domain counterparts [1, 2, 3]. Comparison a +re performed *vs.* SNR, and *vs.* a channel parameter δ related to its identifiability (lack of disparity). Performance are also compared to the Cramer Rao bound (CRB) drawn from [3]. The channel impulse response is $\mathbf{h}=[1, 1, -2\cos\theta, -2\cos(\theta+\delta), 1, 1]^{\mathrm{T}}$ where θ is set to $\pi/10$. δ takes care of the lack of disparity. In fact, for $\delta = 0$ channel is not identifiable while $\delta = \pi$ is the best identifiability condition. SNR is computed according to $\mathrm{SNR}_{dB} = 20\log_{10}\left(\frac{\|\mathbf{h}\|\sigma_a}{\sqrt{2}\sigma_u}\right)$. Symbols are binary i.i.d. (+1,-1). Performance are drawn from $N_r = 100$ Montecarlo runs. The mean square error of the channel parameters is $\mathrm{MSEF}_{dB} = 20\log_{10}\left(\frac{1}{\|\mathbf{h}\|}\sqrt{\frac{1}{N_r}\sum_{i=1}^{N_r}\|\tilde{\mathbf{h}} - \mathbf{h}\|^2}\right)$.

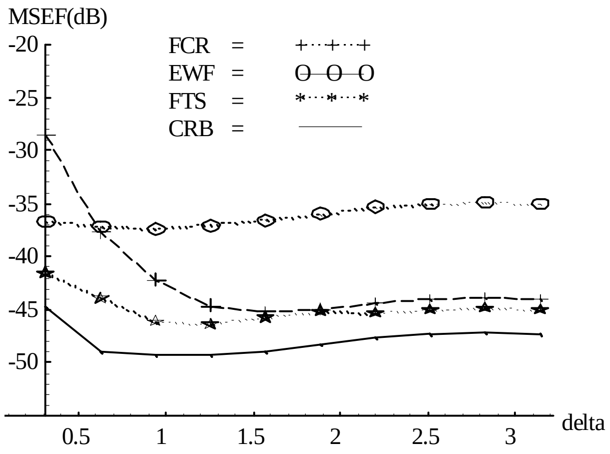

Figure 1: SNR = 45 dB, $\theta = \pi/10$, $N = 30$

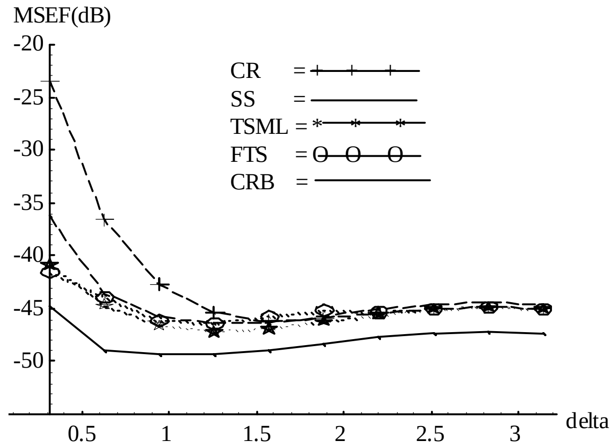

Figure 2: SNR = 45 dB, $\theta = \pi/10$, $N = 30$

In the first figure the performances of the three frequency domain solutions are reported (FCR, EWF, FTS). Experimental conditions are the same of [3]. In particular SNR = 45 dB, $N = 30$ symbols.

In the second figure the FTS method is compared to the time domain methods (CR, SS, TSML) in the same conditions of fig.1. For the SS method a window of W=6 is employed, and SS is used as the first steps of TSML method [3]). In the figure 3 the same test is reported for $\delta = \pi/10$ and the SNR parameter is variable. A linear dependency of the SNR parameter in the logarithmic scale can be assumed. This is confirmed by either the CRB either the theoretical performances of the methods. In fig.4 the performances of the methods FTS and TSML are reported when a raised cosine shaping function is employed.

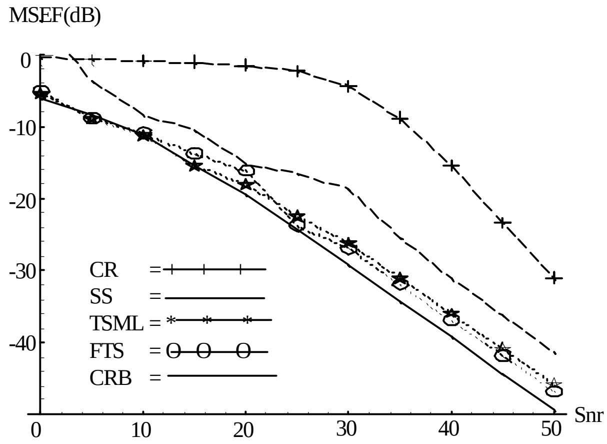

Figure 3: $\delta = \pi/10, \theta = \pi/10$, $N = 30$

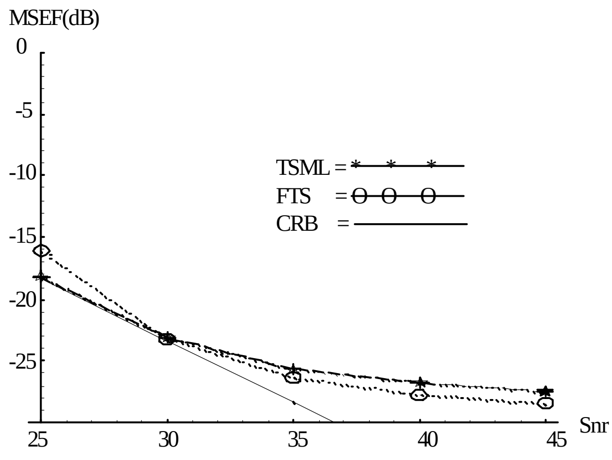

Figure 4: $\rho = 0.8, \delta = \pi/10, \theta = \pi/10$, $N = 64$

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