A NEW TEST OF STATIONARITY AND ITS APPLICATION TO TELETRAFFIC DATA

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ABSTRACT

In this contribution we generalize the test of sphericity as a test of stationarity for time-series. The sphericity statistics is in our case a measure of distance between the empirical correlations calculated on two contiguous segments of the same process. We prove that under the hypothesis of stationarity the logarithm of the sphericity converges in distribution to a quadratic form in a multidimensional gaussian random variable with a convergence rate that is equal to the length of the observation window. We then derive a test of proportionality of the correlations of the process on the two segments. This new test of stationarity is applied to test if the traffic measured on today's broadband networks is stationary. The results that we obtain are connected to many previous works according to which the traffic generated by modern high-speed networks is a stationary and long-range dependent process.

1. A NEW TEST OF STATIONARITY

1.1. Introduction

In [1] Mauchly introduces the sphericity statistics and he demonstrates how this statistics can be used to test whether two gaussian random vectors have the same covariance matrix. In [2] and [3] Mokkadem generalizes the sphericity statistics to the case of time series. He proves that this statistics is an entropy ratio. He then uses the statistics to test whether two processes have proportional spectra and to test whether a process has a given spectrum (with the particular case of the test of whiteness). Our idea is to use the same statistics to test if a process is stationary. The basic idea is to compare the empirical correlations, up to a fixed lag, calculated on two segments of the same process. We prove that under the null hypothesis of stationarity the logarithm of the sphericity multiplied by the length of the observation window converges on a quadratic form in a multidimensional normal variable. We then derive a test of adequacy of the sequence of empirical correlations. The stationarity is rejected if the empirical sphericity is in the distant quantiles of the asymptotic distribution. The generalization of the test of sphericity as a test of stationarity is to our best knowledge original.

1.2. The sphericity statistics

In [1] Mauchly defines the sphericity of two N-dimensional random variables X and Y as $S = \frac{(det(\Sigma_X \Sigma_Y^{-1}))^{1/N}}{1/N Tr(\Sigma_X \Sigma_Y^{-1})}$ where Σ_X and Σ_Y denote the covariance matrices of X and Y. Mokkadem generalizes this definition for two independent random processes $\{X_t\}$ and $\{Y_t\}$ by defining $S = \frac{(det(\Sigma_X \Sigma_Y^{-1}))^{1/N}}{1/N Tr(\Sigma_X \Sigma_Y^{-1})}$ where Σ_X and Σ_Y denote the Toeplitz matrices of the correlations of the two processes up to a fixed lag (N-1). One can demonstrate that when the truncation order N tends to infinity then the sphericity tends to the ratio of $\exp(\int_A \log \phi_X(\omega)\phi_Y^{-1}(\omega)d\omega)$ and of $\int_A \phi_X(\omega)\phi_Y^{-1}(\omega)d\omega$ ($A =]-\pi, +\pi]$). This quantity is always inferior or equal to one and the equality is fulfilled iff $\{X_t\}$ and $\{Y_t\}$ have proportional spectra. The basic idea in any version of the test of sphericity is that the hypothesis of proportionality can be rejected if the empirical sphericity is significantly lower than one. The originality of our work consists in the generalization of the test of sphericity to the comparison of the empirical spectra calculated on two neighbour segments of the same time series.

Denote by $\{X_t\}_{t=1}^T$ a finite length observation of a discrete time stochastic process $\{X_t\}$. The observation may be partitioned into two subsets within each of which the process is assumed to be a second-order stationary and regular process. $1 < \tau_1 < T$ represents the presumed change point. Taking $\tau_0 = 0$ and $\tau_2 = T$, the length of the nth epoch is $T_n = \tau_n - \tau_{n-1}$. Although T_n is finite, it is convenient to regard the n-th epoch as a realization of an infinitely long process $\{X_t^n\}$. The sphericity statistics is then defined as $S = \frac{(\det(R_1 R_2^{-1}))^{1/N}}{1/N Tr(R_1 R_2^{-1})}$ where R_i represents the Toeplitz matrix whose elements are the correlations $(\rho_i(k))_{0 \le k \le N-1}$ up to lag (N-1) of $\{X_t^i\}$ (define $R_i = (\rho_i(|k-l|))_{0\le k \le N-1, 0 \le l \le N-1}$).

$\{X_t\}$ is observed on a finite window of length T. The tests will thus be based on the empirical counterpart $\hat{S}$ of S, the true correlations $\rho_i(k)$ being replaced by the biased estimators $\hat{\rho}_i(k) = \frac{1}{T_i}\sum_{\tau_{i-1}+1}^{\tau_i - k} X_t X_{t+k} - \hat{\mu}^2$ where $\hat{\mu} = \frac{1}{T_i}\sum_{\tau_{i-1}+1}^{\tau_i} X_t$. The reason why we choose the biased estimators is that it guarantees the positivity of the correlation matrices $\hat{R}_1$ and $\hat{R}_2$. If moreover $\{X_t^n\}, n = 1, 2$ are two regular processes then the empirical correlation matrices are definite with probability one. It is then possible to consider the logarithm of the sphericity $d = \log S$ and its empirical counterpart $\hat{d} = \log \hat{S}$.

1.3. A Central Limit Theorem for the sphericity statistics

The objective of this section is to derive a Central Limit Theorem (CLT) for $\hat{d} = \log(\hat{S})$.

Assumption 1 *The correlations of $\{X_t^1\}$ and of $\{X_t^2\}$ are proportional up to lag (N-1)*

$$\exists \alpha > 0 / \forall 0 \le k \le N-1,\ \rho_2(k) = \alpha \rho_1(k)$$

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Assumption 2 *$\{X_t\}$ is α-mixing with an α-mixing coefficient α_n that verifies $\sum_{n=0}^{+\infty} \alpha_n^{\delta/(2+\delta)} < +\infty$ and with $\mathbb{E}(|X_t|^{2+\delta}) < +\infty$.*

The Assumption 2 is verified by many usual processes and in particular by Markov processes, these processes being geometrically α-mixing (see for example [4]). It must be noticed that ARMA processes as well as Hidden Markov Models are particular cases of Markov chains and that the same mixing properties can be obtained by marginalization. For a survey about mixing processes and about the CLT for these processes we refer the reader to [5] and the references therein.

Theorem 1 *If Asssumption 2 is true then $\{X_t\}$ verifies a CLT*

$$\sqrt{T}\left(\frac{1}{T}\sum_{t=1}^{T} X_t - \mu\right) \sim \mathcal{AN}(O, \sigma^2)$$

where $\mu = \mathbb{E}(X_1)$ and $\sigma^2 = \sum_{\mathbb{Z}} \mathbb{E}((X_t - \mu)(X_{t+k} - \mu))$

Remark that the logarithm of the sphericity $\hat{d}$ is a C^∞ function of the vector $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ where $\hat{\rho}_i = (\hat{\rho}_i(0), \cdots, \hat{\rho}_i(N-1))$. To prove a CLT for $\hat{d}$ we first prove that $\hat{\rho}$ verifies a CLT and we conclude by making a Taylor development of Φ at point $\rho = (\rho_1, \alpha\rho_1)$. To prove a CLT for $\hat{\rho}$ we need the additional technical assumption

Assumption 3 *$T_i \overset{T\to\infty}{\to} +\infty$ and $T_i/T \overset{T\to+\infty}{\to} c_i > 0$.*

Theorem 2 *If the Assumptions 1, 2 and 3 are true then*

$$\sqrt{T}(\hat{\rho} - \rho) \sim \mathcal{AN}(0, \Gamma) \text{with} \Gamma = \begin{pmatrix} c_1\Gamma & 0 \\ 0 & c_2\Gamma \end{pmatrix}$$

Γ denoting the $N \times N$ matrix whose entry (k,l) is equal to $\sum_{\mathbb{Z}} M_4(k, \tau, \tau+l)$ where $M_4(k,l,m)$ denotes the fourth centered moment of lag (k,l,m) of $\{X_t\}$ that is to say $M_4(k,l,m) = E((X_t-\mu)(X_{t+k}-\mu)(X_{t+l}-\mu)(X_{t+m}-\mu))$.

It is easily demonstrated that if $\{X_t\}$ verifies the Assumption 2 then the process $\{Y_t\} = \{(X_t^2, X_tX_{t+1}, \cdots, X_tX_{t+N-1})\}$ verifies the Assumption 2 as well and that a CLT can consequently be established for $\{Y_t\}$. Supposing that the Assumption 3 is true we easily get by application of the Theorem 1 a CLT for the vector of unbiased estimators $\bar{\rho}_i = (\bar{\rho}_i(0), \cdots, \bar{\rho}_i(N-1))$ where $\bar{\rho}_i(k) = \frac{1}{T_i}\sum_{\tau_{i-1}+1}^{\tau_i} X_tX_{t+k}$: $\sqrt{T}(\bar{\rho}_i - \rho_i) \sim \mathcal{AN}(0, c_i\Gamma)$. What is more $\sqrt{T}(\bar{\rho}_i - \hat{\rho}_i)$ converges in probability to zero when T tends to infinity. $\sqrt{T}(\bar{\rho}_i - \rho_i)$ and $\sqrt{T}(\hat{\rho}_i - \rho_i)$ are consequently equally distributed and $\sqrt{T}(\hat{\rho}_i - \rho_i) \sim \mathcal{AN}(0, c_i\Gamma)$.

We are now going to prove that $\sqrt{T}(\hat{\rho}_1 - \rho_1)$ and $\sqrt{T}(\hat{\rho}_2 - \rho_2)$ are asymptotically independent; that will end the demonstration of the Theorem 2. The demonstration of this point is a bit more technical. In this contribution we mimic the approach of Epps [6]. Let us introduce a sequence of integers (q_t) that converges to infinity and that verifies $q_t = o(t)$ as t tends to infinity. The basic idea is to omit the first q_T terms in the biased estimators $\hat{\rho}_1$ and $\hat{\rho}_2$. Denote by $\tilde{\rho}_i = (\tilde{\rho}_i(0), \cdots, \tilde{\rho}_i(N-1)), i = 1, 2$ the new estimators obtained: $\tilde{\rho}_i(k) = \frac{1}{T_i}\sum_{t=\tau_{i-1}+q_T+1}^{\tau_i}(X_t - \tilde{\mu}_i)(X_{t+k} - \tilde{\mu}_i)$ where $\tilde{\mu}_i = \frac{1}{T_i}\sum_{t=\tau_{i-1}+q_T+1}^{\tau_i} X_t$. The assumptions on q_T guarantee that $\tilde{\rho}_1$ and $\tilde{\rho}_2$ are mutually asymptotically independent and that if $\tilde{\rho}$ denotes $\tilde{\rho} = (\tilde{\rho}_1, \tilde{\rho}_2)$ then $\sqrt{T}(\tilde{\rho} - \rho)$ and $\sqrt{T}(\bar{\rho} - \rho)$ are asymptotically identically distributed.

As we have mentioned previously $\hat{d}$ is a C^∞ function of $\hat{\rho}$. $\hat{d} = \Phi(\hat{\rho})$ where $\Phi(\hat{\rho}) = \log \frac{(\det(\hat{R}_1\hat{R}_2^{-1}))^{1/N}}{1/NTr(\hat{R}_1\hat{R}_2^{-1})}$. In this expression $\hat{R}_i$ denotes the Toeplitz matrix whose elements are the $(\hat{\rho}_i(k))$. This permits the derivation of an asymptotic distribution result for $\hat{d}$. The demonstration of this result is based on a Taylor development of Φ at the point $\rho = (\rho_1, \rho_2)$. Under the null hypothesis of stationarity a development of order 2 is needed whereas under the alternative hypothesis a development of order 1 suffices. The difference comes from the fact that under the null hypothesis the term of order 1 in the Taylor development of Φ is equal to zero since the function Φ is maximum at point $\rho = (\rho_1, \alpha\rho_1)$. This results in a difference in the rate of convergence and also in the asymptotic distribution under the two alternative hypotheses.

Theorem 3 *Suppose that the technical Assumptions 2 and 3 hold. Then under the Assumption 1 of stationarity*

$$2T\hat{d} \overset{T\to\infty}{\to}_{(d)} Z^H \nabla^2\Phi(\rho) Z \text{with} Z \sim \mathcal{N}(0, \Gamma)$$

and under the alternative assumption

$$\sqrt{T}(\hat{d} - \Phi(\rho)) \sim \mathcal{AN}(0, \nabla\Phi^H(\rho)\Gamma\nabla\Phi(\rho))$$

where $\nabla\Phi(\rho)$ and $\nabla^2\Phi(\rho)$ denote respectively the gradient and the Hessian of Φ taken at point ρ.

We are now going to give the main points that are needed to establish the expression of $\nabla\Phi$ and of $\nabla^2\Phi$. The only technical points that are needed are the expression of the derivative of the determinant and of the inverse of a matrix M. These expressions are $\Delta\det(M) = \det(M)Tr(M^{-1}\Delta M)$ and $\Delta M^{-1} = -M^{-1}\Delta M M^{-1}$. The Hessian matrix $\nabla^2\Phi(\rho)$ depends on ρ in a more complicated manner. Rather than deriving the analytical expression of $\nabla^2\Phi(\rho)$ we suggest to simulate the Hessian matrix by small variations on $\nabla\Phi$.

1.4. A new test of stationarity for time series

The test of stationarity that we propose relies on the first part of Theorem 3. Under the Assumption 1 and under the additional technical Assumptions 2 and 3 the statistics $2T\hat{d}$ is asymptotically distributed as a quadratic form of a normally distributed vector. Denote by A and B the square roots of Γ and $\nabla^2\Phi(\rho)$, $\Gamma = AA^H = A^HA$ and $\nabla^2\Phi(\rho) = BB^H = B^HB$, and denote by $(\lambda_k)_{1\le k\le N}$ the eigenvalues of $A^HB(A^HB)^H$. Then the first part of Theorem 3 can be formulated in the following manner. If the Assumptions 1, 2 and 3 are true then

$$2T\hat{d} \overset{T\to\infty}{\to}_{(d)} \sum_{k=1}^{N} \lambda_k Y_k^2 \text{where} Y \sim \mathcal{N}(0, I_N) \quad (1)$$

The Assumption 1 is then rejected if the obtained value $2T\hat{d}$ is in the distant quantiles of the asymptotic distribution. The repartition function of the asymptotic distribution is computed by Monte-Carlo simulations. One simulates many iid random variables distributed as $\sum_k \lambda_k Y_k^2$; the null hypothesis of stationarity is rejected if the proportion of cases where this random variable is greater than $2T\hat{d}$ is inferior to α, α being the false alarm probability that is accepted.

Remark that the asymptotic covariance matrix Γ is unknown. Theorems 2 and 3 still hold if the asymptotic variance is replaced by a consistent estimator. $\hat{\rho}$ is a consistent estimator of ρ. This result can be seen as a consequence of Theorem 2 by invoking the second Borel-Cantelli's theorem. Consequently $\nabla\Phi(\hat{\rho})$ and $\nabla^2\Phi(\hat{\rho})$ are consistent estimators of $\nabla\Phi(\rho)$ and of $\nabla^2\Phi(\rho)$. It remains to find a consistent estimator of Γ. Γ is equal to the spectral density function of $\{Y_t = (X_t^2, \cdots, X_t X_{t+N-1})\}$ taken at point $\omega = 0$; one can therefore use the smoothed periodogram at the frequency $\omega = 0$

$$\hat{\Gamma} = \frac{1}{T} \sum_{0}^{+m_T} w(k) \Re((\sum_{1}^{T} (Y_t - \hat{\rho}) e^{j \frac{k+1}{T} t})^H (\sum_{1}^{T} (Y_t - \hat{\rho}) e^{j \frac{k+1}{T} t}))$$

with $m_T = \sqrt{T}$ and $w(k) = 1\!\!1_{k=0} + \frac{2}{2m_T+1} 1\!\!1_{1 \le k \le m_T}$.

2. APPLICATION TO REAL-LIFE TELETRAFFIC

2.1. The data under investigation

In this section we investigate real-life traffic measured on today's broadband networks. In the past few years high quality traffic measurements have become available and a large number of contributions have consecutively been devoted to the statistical study of these data ([7],[8],[9],[10]...). The authors have got to the point that the traffic is heavy tailed and long-range dependent no matter what kind of traffic (WWW, video, LAN, WAN ...) and what statistics (inter-arrival times or block packet counts) they considered. These contributions are very significant. The conclusions are indeed at variance with the traditional models of traffic such as the Poisson process or the Markov Modulated Poisson Process. It can be demonstrated that in the case of long-range dependence quality measures such as the overflow probability or the average packet delay are strongly underestimated by the traditional short-range dependent models. The challenge is then to propose realistic and yet analytically tractable models and to elaborate routing policies that take advantage of the auto-similarity of the traffic.

Our idea is that the traffic may not be a stationary and long-range dependent process but a short range dependent process that exhibits some kinds of non-stationarities. Teverovsky and Taqqu [11] have indeed demonstrated recently that some kinds of non stationarities (namely deterministic jumps or deterministic trends in the mean) can lead, if they are not detected, to the untrue conclusion that a time series is long-range dependent. In the above-mentioned contributions the authors investigate minutes or even hours of traffic, that is to say hundreds of thousands of packets; peculiarly these authors do not test if the measured traffic is stationary on these time-scales.

In what follows we expose the results of the application of the test of stationarity developed in this contribution to one of the data streams that are commonly studied in the litterature (LBL-PKT3, [9]). This data stream consists of the traffic measured at the gateway of Berkeley's university in 1994 during working hours. This trace was originally investigated by Paxson and Floyd [9]. The original trace lasts two hours and we take out seven minutes of traffic not long before 4pm. We study separately the data coming into the university and the data getting out of the university. The average time between two consecutive packets in one direction is equal to about $6\mathrm{ms}$ no matter which direction is considered. The traffic is composed of ftp, Network News, telnet, and mail data but we do not distinguish between the different protocols. We neither take into account the size of the packets. We only study the sequence of times between two consecutive packets in one direction.

2.2. Results of the tests

We use the test of sphericity to compare the first correlation coefficients of many neighbour segments of LBL-PKT3. The experience is replicated (i) separately for the packets entering the university and for the packets getting out of the university (ii) for different time-scales ranging from a few seconds to a few minutes (iii) for two truncation orders (N=5 and N=15) (iv) and for ten pairs of neighbour segments for each time scale, each truncation order and each stream. By a time-scale of, say, one minute, we mean that we compare two neighbour segments that last each thirty seconds; that permits to come to a decision concerning the stationarity of the first N correlation coefficients on the time-scale of one minute.

The results of our simulations are listed in Table 1. For all the pairs of neighbour segments we give the probability, under the hypothesis of stationarity, that a random variable distributed as $2T\hat{d}$ when T tends to infinity is inferior to the value $2T\hat{d}$ that we obtain in practice. If this probability is inferior to the false alarm probability $\alpha = 0.05$ then we decide that the segment is non stationary. The segments for which we decide the non-stationarity correspond to the values that are underlined in Table 1. If the traffic were stationary then for this false alarm probability 1 value out of 20 would be underlined.

2.3. Critical analysis of the results

The comparison of the N=15 first correlations coefficients and, to a lower extent, of the N=5 first correlation coefficients, reveals the presence of some non stationary segments. These non stationarities are very frequent on the time scales of a few minutes. The tests were conducted for the same pairs of neighbour segments for the truncation orders N=5 and N=15. It is clear on this example that the decision concerning the stationarity depends on the number of correlations that are taken into account. In some previous work [12] we have tested the stationarity of the marginal distribution for the same pairs of neighbour segments as in the present work. For the test on the marginal distribution the stationarity was rejected very violently for all the pairs of neighbour segments over a 13 seconds time-scale. It is thus difficult to give a limit under which the traffic is stationary and over which the traffic is non stationary. It depends of the quantities that are compared and of the part of the trace that is considered.

One should remark that the Assumption 2 concerning the mixing properties of the process is not verified if the process is long-range dependent. This is a big problem since the test of sphericity developed in Section 1 is consequently not valid if the true process is long-range dependent. The test developed in this contribution does not enable us to decide between non-stationarities and long-range dependence. Contrary to many authors who decide in favor of the long range dependence hypothesis we give a greater place to the hypothesis of non stationarity. This solution has indeed many advantages. One can benefit from queuing results and from estimation procedures that already exist for these models. This is not the case when the measured traffic is modeled as a long-range dependent process. Our solution has proved to be fruitful. In some previous work [12] we have indeed proposed to model the LBL-PKT3 data stream as a locally stationary Hidden Markov Chain. This model has given us satisfying results in terms of overflow probability, of adequacy of the marginal distribution and of adequacy of the first correlation coefficients on the short time scales.

Traffic coming into Berkeley's university; N=5				
6sec.	25sec.	1min.	2min.	3min.
9.1e-01	3.7e-02	7.2e-01	1.9e-01	2.3e-02
4.7e-01	4.0e-01	4.1e-01	6.1e-02	5.7e-01
6.8e-01	9.9e-01	1.0e-01	2.3e-01	6.9e-01
8.0e-01	3.1e-01	2.1e-01	4.3e-01	6.8e-02
3.4e-02	6.3e-01	4.3e-01	5.1e-02	3.0e-03
3.0e-01	9.3e-01	2.8e-01	4.0e-02	5.0e-04
7.9e-01	4.5e-01	4.9e-01	1.0e-01	2.8e-03
6.7e-01	7.2e-01	8.3e-01	6.8e-01	4.3e-01
8.2e-01	5.6e-01	7.7e-02	1.8e-01	6.1e-01
7.0e-01	7.0e-02	6.9e-01	2.5e-01	1.7e-01

Traffic getting out of Berkeley's university; N=5				
6sec.	25sec.	1min.	2min.	3min.
8.7e-01	4.0e-01	8.9e-01	5.5e-01	1.6e-01
9.0e-01	7.0e-01	1.4e-01	3.9e-01	2.5e-01
7.8e-01	2.9e-01	4.3e-02	1.4e-01	3.4e-01
4.4e-01	9.3e-01	5.9e-01	6.6e-01	6.7e-01
5.4e-01	9.7e-01	9.2e-01	7.2e-02	4.8e-01
1	8.4e-01	8.9e-01	6.0e-01	4.2e-02
8.3e-01	5.7e-01	1.2e-01	7.0e-02	2.6e-02
2.2e-01	7.9e-01	3.3e-01	9.8e-01	1.1e-01
3.7e-01	3.3e-01	9.5e-02	2.3e-01	4.8e-01
4.2e-01	7.1e-01	1.9e-01	5.7e-01	8.6e-01

Traffic coming into Berkeley's university; N=15				
6sec.	25sec.	1min.	2min.	3min.
8.5e-01	1.1e-03	4.8e-01	2.4e-02	$\leq$1.0e-03
7.5e-02	1.8e-02	3.1e-02	8.0e-03	4.4e-01
6.8e-02	9.9e-01	5.0e-03	2.7e-02	3.4e-01
4.4e-01	6.9e-02	3.0e-03	3.8e-01	$\leq$1.0e-03
2.2e-02	8.2e-01	8.1e-02	$\leq$1.0e-03	$\leq$1.0e-03
1.9e-02	6.6e-01	1.9e-01	$\leq$1.0e-03	$\leq$1.0e-03
7.7e-01	8.8e-02	5.9e-02	1.9e-02	$\leq$1.0e-03
7.4e-01	1.3e-01	9.6e-01	4.0e-01	1.7e-01
5.2e-01	1.1e-01	$\leq$1.0e-03	2.0e-02	3.7e-01
1.6e-01	$\leq$1.0e-03	2.1e-01	1.3e-01	4.0e-03

Traffic getting out of Berkeley's university; N=15				
6sec.	25sec.	1min.	2min.	3min.
7.7e-01	4.1e-02	6.5e-01	2.5e-01	4.1e-02
1.1e-01	9.9e-02	6.0e-03	1.3e-01	8.6e-02
6.7e-02	8.9e-02	1.0e-03	5.0e-03	1.1e-01
8.8e-01	2.8e-01	9.1e-02	9.1e-02	4.7e-01
8.0e-02	3.7e-01	2.3e-01	6.1e-01	3.1e-01
4.1e-01	9.1e-01	4.1e-01	7.1e-01	7.1e-01
8.4e-01	5.8e-02	2.0e-03	1.0e-03	1.0e-03
2.4e-01	8.9e-01	1.1e-01	7.0e-01	7.0e-01
1.4e-01	5.5e-01	2.5e-02	4.6e-01	5.1e-02
3.8e-02	8.6e-02	$\leq$ 1.0e-03	1.9e-02	1.9e-02

Table 1: Tests of sphericity. Truncation orders N=5 and N=15.

3. CONCLUSION

In this contribution we have proposed a new test of stationarity for time series. We have established the asymptotic distribution under the null hypothesis of stationarity and under the alternative hypothesis of the normalized logarithm of the sphericity. The test rejects the null hypothesis if the obtained value is in the distant quantiles of the asymptotic distribution. We have then applied this new test to some teletraffic data. We have reached the conclusion that the assumptions of short range dependence and of stationarity are contradictory.And we have compared our findings with the work of many authors who affirm that the traffic measured on today's broadband networks is long range dependent.

4. REFERENCES

[1] A. Mauchly, "Significance test for sphericity of a normal n-variate distribution," *Ann. Math. Stat.*, vol. 11, pp. 204–209, 1940.

[2] A. Mokkadem and K. Drouiche, "A new test for time series," Prepublications d'Orsay.

[3] A. Mokkadem, "On some new tests for time series," *CRAS Paris, Series I*, vol. 318, pp. 755–758, 1994.

[4] S.P. Mein and R.L. Tweedie, *Markov chains and Stochastic Stability*, Communication and Control Engineering Series. Springer-Verlag, 1993.

[5] P. Doukhan, *Mixing: properties and examples*, Lecture Notes in Statistics. Springer-Verlag, 1994.

[6] T.W. Epps, "Testing that a gaussian process is stationary," *The Annals of Statistics*, vol. 16, no. 4, pp. 1667, dec 1988.

[7] W. Willinger, M. Taqqu, R. Sherman, and D.V. Wilson, "Self-similarity through high variability: Statistical analysis of ethernet lan traffic at the source level," in *SIGCOMM'95*, Cambridge, USA, jan 1995.

[8] W. Willinger, M. Taqqu, W.E. Leland, and D.V. Wilson, "Self-similarity in high-speed packet traffic: Analysis and modeling of ethernet traffic measurements," *Statistical Science*, vol. 10, no. 1, pp. 67–85, 1995.

[9] V. Paxson and S. Floyd, "Wide-area traffic: The failure of poisson modeling," in *SIGCOMM'94*, London, England, aug 1994, pp. 257–268.

[10] W. Willinger W.E. Leland, M.S. Taqqu and D.V. Wilson, "On the self-similar nature of ethernet traffic (extended version)," *IEEE ACM Transactions on Networking*, vol. 2, no. 1, pp. 1–14, feb 1994.

[11] V. Teverovski and M. Taqqu, "Testing for long-range dependence in the presence of shifting means or a slowly declining trends, using a variance type estimator," *J. of Time Series Analysis*, vol. 18, no. 3, pp. 279–304, 1997.

[12] S. Vaton, E. Moulines, H. Korezlioglu, and D. Kofman, "Statistical identification of lan traffic data," in *ATM'97*, Bradford, England, july 1997, pp. 15/1–10.