PERFORMANCE ANALYSIS OF BLIND CHANNEL ESTIMATORS BASED ON NON-REDUNDANT PERIODIC MODULATION PRECODERS

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ABSTRACT

Periodic modulation precoders allow blind identification of SISO channels from output second-order cyclic statistics, irrespective of the location of channel zeros, color of additive stationary noise, or channel order overestimation errors. In this paper the performance of blind channel estimators relying on periodic precoders is investigated. Criteria for optimal designs of periodic modulation precoders are also presented.

1. INTRODUCTION

By inducing cyclostationarity (CS) at the transmitter, it has been shown that blind identification of FIR single-input single-output communication channels is possible only from the output second-order statistics and without using any assumption on the channel zeros, color of additive noise, and channel order overestimation errors [10], [5], [2], [8].

In the present work, we consider the performance analysis of blind channel estimators when CS is induced at the transmitter by means of a periodic modulation precoder. Consider the simplified baseband discrete-time channel shown in Fig. 1, where the zero-mean independently and identically distributed (i.i.d.) input stream $s(n)$ is modulated by the deterministic and periodic sequence $p(n)$ with period P, to obtain $w(n) = p(n)s(n)$, with $p(n) = p(n+P)\ \forall\ n$. Sequence $w(n)$ propagates through the unknown channel $h(n)$, whose output $x(n)$ is corrupted by the stationary noise $v(n)$ assumed to be uncorrelated with the inaccessible input $s(n)$. The input-output relation is described by

$$y(n):=x(n)+v(n)=\sum_{l=0}^{L} h(n-l)w(l)+v(n)\ , \quad (1)$$

where L denotes the order of the discrete-time channel h. The time-varying correlation at time n and lag τ of sequence $w(n)$ is defined by $c_{ww}(n;\tau) := Ew(n+\tau)\ w^*(n)$, and satisfies $c_{ww}(n;\tau) = |p(n)|^2\ \sigma_s^2\ \delta(\tau) = c_{ww}(n+lP;\tau), \forall\ l,\tau \in \mathbf{Z}$; $*$ stands for conjugation, $|\cdot|$ denotes absolute value, and $\sigma_s^2 := E|s(n)|^2 = 1$ (source power normalized to unity). The output time-varying correlation $c_{yy}(n;\tau)$ is given by

$$c_{yy}(n;\tau) = \sigma_s^2 \sum_{m=-\infty}^{\infty} |p(n-m)|^2 h(m+\tau)h^*(m) + c_{vv}(\tau). \quad (2)$$

It follows from (2) that $c_{yy}(n;\tau)$ is also a periodic function in n since $c_{yy}(n;\tau) = c_{yy}(n+P;\tau)$, for any n, τ. Being periodic, $c_{yy}(n;\tau)$ accepts a Fourier Series expansion over the set of complex exponentials with harmonic cycles, and with the set of cycles defined as: $A_{yy}^c := \{2\pi k/P, \quad k = 0,\ldots,P-1\}$; i.e., $c_{yy}(n;\tau)$ and its Fourier coefficients $C_{11y}(k;\tau)$, called cyclic correlations, are related by the discrete Fourier Series:

$$C_{11y}(k;\tau) = \frac{1}{P}\sum_{n=0}^{P-1} c_{yy}(n;\tau)e^{-j2\pi kn/P}. \quad (3)$$

We consider only non-zero cycles $k \neq 0$ such that the contribution of the additive stationary noise is cancelled out. Thus, we have

$$C_{11y}(k;\tau) = \sigma_s^2 P_2(k) \sum_{m=-\infty}^{\infty} h(m+\tau)h^*(m)e^{-j2\pi km/P} \quad (4)$$

$$P_2(k) := \frac{1}{P}\sum_{n=0}^{P-1} |p(n)|^2 e^{-j2\pi kn/P}\ . \quad (5)$$

The Fourier transform of the cyclic correlation $C_{11y}(k;\tau)$, for a fixed cycle k, is called the cyclic spectrum and is given by (c.f. (4))

$$S_{11y}(k;e^{j2\pi f}) = \sigma_s^2 P_2(k) H(e^{j2\pi f}) H^*(e^{j2\pi (f-\frac{k}{P})}), \quad (6)$$

where $H(e^{j2\pi f}) := \sum_{n=0}^{L} h(n)e^{-j2n\pi f}$.

The blind channel estimators based on the second-order statistics can be categorized in two classes: 1) the linear (or subspace) approaches that estimate directly the channel from a linear system of equations [1], [9], [2], [8], and 2) the nonlinear approaches based on the minimization of a certain nonlinear cost function [6], [8]. The advantages of one class with respect to the other are well-known [6]. For the periodic modulation framework, it has been shown that the linear approaches guarantee identifiability when the period is

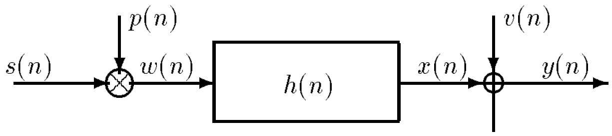

Figure 1: Periodic Modulation Precoder

chosen to satisfy $P \geq L+1$, while nonlinear correlation matching requires only $P > [L/2]$ [2], [8]. The main disadvantages of the nonlinear approach lie in its high computational complexity and local convergence problems. However, since nonlinear correlation matchings offer the lowest asymptotic covariance matrix, they are useful in practice since their performance serves as a benchmark for the linear approaches. Next, we study the asymptotic performance of these algorithms. Due to the reduced amount of space, the proofs of all results are omitted, and we refer the reader to [3].

2. BLIND CHANNEL ESTIMATION AND PERFORMANCE ANALYSIS

We consider first the signal subspace formulation of the linear approach.

2.1. Signal Subspace Approach

By collecting all the second-order cyclic spectral information into the vector

$$\mathbf{s}(e^{j2\pi f}) := \begin{bmatrix} S^*_{11y}(P-1; e^{j2\pi f}) \\ S^*_{11y}(P-2; e^{j2\pi f}) \\ \vdots \\ S^*_{11y}(1; e^{j2\pi f}) \end{bmatrix}, \tag{7}$$

it is easily seen that $\mathbf{s}(e^{j2\pi f})$ can be factorized as (see (6))

$$\mathbf{s}(e^{j2\pi f}) = \sigma_s^2 \mathbf{h}(e^{j2\pi f}) H^*(e^{j2\pi f}) \tag{8}$$

where

$$\mathbf{h}(e^{j2\pi f}) := \begin{bmatrix} P_2^*(1) H(e^{j2\pi(f-\frac{1}{P})}) \\ P_2^*(2) H(e^{j2\pi(f-\frac{2}{P})}) \\ \vdots \\ P_2^*(P-1) H(e^{j2\pi(f-\frac{P-1}{P})}) \end{bmatrix}. \tag{9}$$

For a $(P-1)$-dimensional vector $\mathbf{a} = [a_1, ..., a_{P-1}]^T$ define the $(P-1)(P-2)/2 \times (P-1)$ matrix

$$\mathcal{F}(\mathbf{a}) = \left[\begin{array}{ccccc} -a_2 & a_1 & & & 0 \\ \vdots & 0 & \ddots & & \\ -a_{P-1} & 0... & 0... & a_1 & 0 \\ \hline 0 & -a_3 & a_2 & & \\ & \vdots & & \ddots & \\ & -a_{P-1} & & & a_2 \\ \hline \vdots & & & \vdots & \vdots \end{array}\right].$$

The channel estimation algorithm we propose relies on the orthogonality between the rows of $\mathcal{F}(\mathbf{h}(e^{j2\pi f})^T)$ and the columns of $\mathbf{h}(e^{j2\pi f})$. We have the result:

Theorem 1: *The subspace equation*

$$\mathcal{F}\left(\mathbf{x}^T(e^{j2\pi f})\right) \mathbf{s}(e^{j2\pi f}) = 0, \tag{10}$$

over the set of $(P-1)$-dimensional polynomial vectors having the structure (9) has a unique solution (within a constant) $\mathbf{x}(e^{j2\pi f}) = \mathbf{h}(e^{j2\pi f})$.

The extraction of the channel vector $\mathbf{h} := [h(0) \ldots h(L)]^T$ is performed by re-writing the orthogonality condition in the time domain. The Laurent expansion of $\mathbf{s}(z)$ can be written $\mathbf{s}(z) = \sum_{\tau=-N}^{N} \mathbf{s}(\tau) z^{-\tau}$, with $\mathbf{s}(\tau) := [C_{11y}(P-1; -\tau), ..., C_{11y}(1; -\tau)]^T$ and $N \geq 2L$. Define $\mathbf{s} := [\mathbf{s}(N)^T ... \mathbf{s}(-N)^T]^T$. Associate to an arbitrary Mth degree polynomial $F(z)$ the $(N+1) \times (M+N+1)$ Toeplitz matrix $\mathcal{T}_N(F)$ with the first column and row $[F(0)\ 0 \ldots 0]^T$ and $[F(0) \ldots F(M)\ 0 \ldots 0]$, respectively. Eq. (10) can be re-written in the forms

$$\begin{array}{rcl} \mathcal{T}_{2N-L}\left(\mathcal{F}(\mathbf{x}^T(e^{j2\pi f}))\right) \mathbf{s} & = & \mathbf{0}, \\ \mathcal{D}(\mathbf{s}) \mathcal{X} & = & \mathbf{0}, \end{array} \tag{11}$$

where the matrix $\mathcal{D}(\mathbf{s})$ is defined as

$$\mathcal{D}(\mathbf{s}) := \begin{bmatrix} \mathcal{F}(\mathbf{s}^T(N)) & \ldots & \ldots & \mathcal{F}(\mathbf{s}^T(N-L)) \\ \vdots & & & \vdots \\ \mathcal{F}(\mathbf{s}^T(-N+L)) & \ldots & \ldots & \mathcal{F}(\mathbf{s}^T(-N)) \end{bmatrix},$$

and $\mathcal{X} := [\mathcal{X}(0)^T, ..., \mathcal{X}(L)^T]^T$. According to (9), we have $\mathcal{X} = \mathcal{P}\mathbf{x}$, where $\mathbf{x} := [x(0), ..., x(L)]^T$ and $\mathcal{P}$ is the block-diagonal matrix whose (k,k) block is:

$$[\mathcal{P}]_{k,k} = [P_2^*(1) e^{i2\pi k/P}, ..., P_2^*(P-1) e^{i2\pi k(P-1)/P}]^T.$$

The following identifiability result holds:

Theorem 2: *If $P \geq L$, the quadratic form*

$$\mathbf{Q} = \mathcal{P}^* \mathcal{D}(\mathbf{s})^* \mathcal{D}(\mathbf{s}) \mathcal{P} \tag{12}$$

admits a kernel of dimension 1 generated by $\mathbf{h}$.

Given the observations $\{y(n)\}_{n=0}^{T-1}$, the cyclic correlation at cycle k and lag m can be estimated using:

$$\hat{C}_{11y}^{(T)}(k; m) := \frac{1}{T} \sum_{\ell=0}^{T-1} y(\ell+m) y^*(\ell)\, e^{-j2\pi k\ell/P}. \tag{13}$$

The estimator $\hat{C}_{11y}^{(T)}(k; m)$ is mean-square sense (m.s.s.) consistent and asymptotically normal since $h(l)$ has finite memory and $w(n)$ has finite moments. The estimation of unknown channel vector $\mathbf{h}$ is obtained by minimizing the quadratic form:

$$\left\{ \begin{array}{ccc} \hat{\mathbf{h}} & = & \arg\min \mathbf{x}^* \hat{\mathbf{Q}} \mathbf{x} \\ \text{s. to} & & \mathbf{x}^{T*}\mathbf{x} = 1 \end{array} \right., \tag{14}$$

where the estimate $\hat{\mathbf{Q}}$ is obtained by replacing the unknown statistics with the consistent estimates (13). Define the vector $\mathbf{d}_k(e^{i2\pi f}) := [1\ e^{-i2\pi f} ... e^{-i2\pi f k}]^T$. We have the asymptotic result:

Theorem 3: *If the input is circularly distributed and the additive noise is white and Gaussian, then*

$$\lim_{T\to\infty} T E\left\{(\hat{\mathbf{h}} - \mathbf{h})(\hat{\mathbf{h}} - \mathbf{h})^{T*}\right\} = ,_1 + ,_2, \tag{15}$$

where for i=1,2, we have set

$$,_i = \mathbf{Q}^\dagger \mathcal{P}^* \mathcal{D}(\mathbf{s})^* \mathbf{\Sigma}_i \mathcal{D}(\mathbf{s}) \mathcal{P} \mathbf{Q}^\dagger, \tag{16}$$

with

$$\boldsymbol{\Sigma}_1 = \sum_{\ell_1=0}^{P-1} e^{i2\pi\frac{\ell_1 N}{P}} \int_0^1 \mathbf{d}_{2N-L}(e^{i2\pi f})$$

$$\times \mathbf{d}^*_{2N-L}(e^{i2\pi(f-\frac{\ell_1}{P})}) \otimes \mathbf{T}^{(\ell_1)}(e^{i2\pi f})df,$$

$$\mathbf{T}^{(\ell_1)} = \mathcal{F}\left(\mathbf{h}^T(e^{i2\pi f})\right)\mathbf{M}^{(\ell_1)}(e^{i2\pi f})\left(H(e^{i2\pi(f-\frac{\ell_1}{P})})\right.$$

$$\left.\times H^*(e^{i2\pi f}) + \delta_{\ell_1} c_{11v}(0,0)\right)\mathcal{F}^*\left(\mathbf{h}^T(e^{i2\pi(f-\frac{\ell_1}{P})})\right)$$

matrix $\mathbf{M}^{(\ell_1)}(e^{i2\pi f})$ *is* $(L \times L)$ *with*

$$[\mathbf{M}^{(\ell_1)}(e^{i2\pi f})]_{n_1,n_2} = H(e^{i2\pi(f-\frac{n_1}{P})})H^*(e^{i2\pi(f-\frac{\ell_1+n_2}{P})}) + \delta_{\ell_1+n_2-n_1} c_{11v}(0,0),$$

$$\boldsymbol{\Sigma}_2 = \int_0^1\int_0^1 \mathbf{d}_{2N-L}(e^{i2\pi\nu_1})\mathbf{d}^*_{2N-L}(e^{i2\pi\nu_2}) \otimes \mathbf{U}(e^{i2\pi\nu_1}, e^{i2\pi\nu_2})d\nu_1 d\nu_2,$$

$$\mathbf{U}(e^{i2\pi\nu_1}, e^{i2\pi\nu_2}) = \mathcal{F}\left(\mathbf{h}^T(e^{i2\pi\nu_1})\right)\boldsymbol{\Theta}(e^{i2\pi\nu_1}, e^{i2\pi\nu_2}) \times \mathcal{F}^*\left(\mathbf{h}^T(e^{i2\pi\nu_2})\right),$$

matrix $\boldsymbol{\Theta}(e^{i2\pi\nu_1}, e^{i2\pi\nu_2})$ *is also* $(L \times L)$ *with*

$$[\boldsymbol{\Theta}(e^{i2\pi\nu_1}, e^{i2\pi\nu_2})]_{n_1,n_2} = c_{22s}(\mathbf{0})\theta(n_1,n_2)H(e^{i2\pi(\nu_1-\frac{n_1}{P})}) \times H^*(e^{i2\pi(\nu_2-\frac{n_2}{P})})H(e^{i2\pi\nu_2})H^*(e^{i2\pi\nu_1}),$$

$$\theta(n_1,n_2) = \kappa \sum_{\substack{j_1,j_2,j_3,j_4=0 \\ j_1-j_2+j_4-j_3 \\ \equiv n_1-n_2}}^{P-1} \lambda_{j_1}\lambda^*_{j_2}\lambda^*_{j_3}\lambda_{j_4},$$

κ *denotes the input kurtosis, and*

$$\lambda_k = \frac{1}{P}\sum_{n=0}^{P-1} p(n)e^{-i2\pi\frac{kn}{P}}.$$

2.2. Nonlinear Correlation Matching Estimator

It is easy to verify the following relations

$$C_{11y}(k;m) = e^{j2\pi km/P}\, C^*_{11y}(P-k;-m) \quad ,(17)$$

$$C_{11y}(k;m) = 0 \ \ \forall\, |m| > L \,. \quad (18)$$

The conjugate symmetry in the lags (17) and memory constraint (18) imply that all the non-redundant second-order statistical information about the complex-valued channel is contained at most in the set:

$$\mathcal{K} := \{C_{11y}(k;m); k = 1,2,\ldots,[P/2], m = -L,\ldots,L\}.$$

The cyclic correlations associated to the set $\mathcal{K}$ and their sample estimates are collected into the vectors

$$\mathbf{c} := [C_{11y}(1;-L),\ \ldots, C_{11y}([P/2];L)]^T \ ,$$

$$\hat{\mathbf{c}}^{(T)} := \left[\hat{C}^{(T)}_{11y}(1;-L),,\ \ldots, \hat{C}_{11y}([P/2];L)\right]^T.$$

The *normalized* asymptotic covariance of $\hat{\mathbf{c}}^{(T)}$ is defined as:

$$\boldsymbol{\Sigma} := \lim_{T\to\infty} T\mathrm{E}\{[\hat{\mathbf{c}}^{(T)} - \mathbf{c}][\hat{\mathbf{c}}^{(T)} - \mathbf{c}]^{T*}\} \ . \quad (19)$$

The nonlinear correlation matching estimator finds the channel estimate $\widehat{\mathbf{h}}$ such that the cyclic correlations $\mathbf{c}$ are closest to the observed correlations $\widehat{\mathbf{c}}^{(T)}$ in the least-squares sense. More precisely,

$$\widehat{\mathbf{h}} := \arg\min_{\mathbf{h}} J[\hat{\mathbf{c}}^{(T)};\mathbf{h}],\ J[\hat{\mathbf{c}}^{(T)};\mathbf{h}] := [\hat{\mathbf{c}}^{(T)} - \mathbf{c}]^{T*}[\hat{\mathbf{c}}^{(T)} - \mathbf{c}] \quad (20)$$

The following theorem establishes the consistency and the asymptotic performance of the cyclic correlation matching algorithm [7], [6].

Theorem 4: *The estimate* $\hat{\mathbf{h}}$ *obtained by minimizing* $J\ [\hat{\mathbf{c}}^{(T)};\mathbf{h}]$ *over a compact set converges in the mean-square sense to* $\mathbf{h}$ *provided that* $s(n)$ *has finite moments. The asymptotic covariance* $\mathbf{P}(\widehat{\mathbf{h}},\mathbf{h})$ *for the matching approach (20) is:*

$$\mathbf{P}(\widehat{\mathbf{h}},\mathbf{h}) = \mathbf{G}(\mathbf{h})\ \boldsymbol{\Sigma}\ \mathbf{G}^T(\mathbf{h}) \ , \quad (21)$$

where

$$\mathbf{G}(\mathbf{h}) := [\mathbf{F}^T(\mathbf{h})\ \mathbf{F}(\mathbf{h})]^{-1}\mathbf{F}^T(\mathbf{h}) \ ,$$

$$\mathbf{F}(\mathbf{h}) := [\nabla_{h(1)}\mathbf{c}\ldots\nabla_{h(L)}\mathbf{c}].$$

provided $\mathbf{F}(\mathbf{h})$ *is full column rank.*

3. SELECTION OF THE PRECODER

The following result holds [3]:

Theorem 5. *If in (6)* $P_2(k) = P_2(l)$, $\forall k,l = 0,\ldots, P-1$, *and* $v(n) = 0$, *then* $\boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \mathbf{0}$.

The constraint on $P_2(k)$'s translates into the fact that the periodic sequence takes $P-1$ values equal to zero within one period. From an identification viewpoint, such sequences are optimal. However, note that for such sequences, $P-1$ symbols every P consecutive symbols are lost. From an equalization viewpoint, less constraints have to be imposed on such sequences. One solution is to impose the equality only for $P_2(k)$, $k = 1,\ldots, P-1$, since in this case many terms in $\boldsymbol{\Sigma}_1$, $\boldsymbol{\Sigma}_2$ cancel out, and the equalization becomes well posed.

Since the dependence of cyclic correlation $C_{11y}(k;\tau)$ for a given cycle $k \neq 0$ is proportional to $P_2(k)$, it is natural to introduce as a measure of CS induced by the modulating sequence $p(n)$ the constants $P_2(k)$. High values for $P_2(k)$ imply a high degree of CS for the output sequence $y(n)$. In practice, the sample cyclic correlations are estimated from a finite number of samples. Thus, the residual noise due to noise/finite sample effects is present in our estimation approaches. Note that a small value of $P_2(k)$ (≈ 0) would imply that the cyclic correlations at cycle k assume small values. Thus, an additive noise of small power will highly affect the values of these cyclic correlations, and thus, the performance of the estimation algorithm drops significantly. In order to stress out the dependence on the periodic sequence $\mathbf{p}$, we will use the notation $P_{2,\mathbf{p}}(k)$ instead of $P_2(k)$. We pose the general question: do there exist periodic sequences $\{p(n)\}_{n=1}^{P}$ which under a power constraint, i.e., $\sum_{n=1}^{P}|p(n)|^2 = \alpha$, and the equalization constraint $\min_{n=1,\ldots,P}|p(n)| \geq \beta(>0)$, are such that $P_{2,p}(k) \geq P_{2,f}(k)$, for any $k = 1,\ldots,P-1$ and for any periodic sequence $\{f(n)\}_{n=1}^{P}$ which satisfies the power/equalization constraints? The answer is:

Theorem 6. *The optimal sequences under power and equalization constraints are given modulo a permutation and a phase-shift by*

$$\{|p(n)|\}_{n=1}^{P} = \{\beta,\ldots,\beta,\ \sqrt{\alpha-(P-1)\beta^2}\}\,. \qquad (22)$$

It follows that Theorem 7 corroborates the conclusions of Theorem 6.

4. SIMULATION EXPERIMENT

The asymptotic results have been compared with the simulation results for a real GSM channel of order $L = 5$ with coefficients $\mathbf{h} = [0.632-0.007i,\ 0.046+0.075i,\ -0.231-0.383i,\ -0.256-0.186i,\ 0.413+0.346i]^T$. Fig. 2 shows the simulation and theoretical results for two periodic modulating sequences $\mathbf{p}_1 = \{1,\ 1,\ 1,\ 1,\ 2\}/\sqrt{8}$ and $\mathbf{p}_2 = \{1,\ 1,\ \sqrt{2},\ \sqrt{2},\ \sqrt{2}\}/\sqrt{8}$. We note that both sequences satisfy the same power / equalization constraints. In Fig. 2, the normalized asymptotic covariance of the channel coefficients is plotted versus number of samples. The theoretical values are traces of the asymptotic covariance matrix and the simulation values are

$$RMSE = \frac{1}{||\mathbf{h}||}\sqrt{\frac{1}{MC}\sum_{m=1}^{MC}||\hat{\mathbf{h}}^{(m)}-\mathbf{h}||^2}\,,$$

where $\hat{\mathbf{h}}^{(m)}$ signifies the channel estimate in the mth Monte-Carlo run. The signal-to-noise ratio is fixed at $SNR = 10\ dB$, and $MC = 100$ Monte-Carlo simulations are performed. Note that the use of the optimal sequence $\mathbf{p}_1$ offers a much better performance than the suboptimal sequence $\mathbf{p}_2$. Note also the good agreement between simulation results and theoretical expressions. Performance of the subspace algorithm is improved by the nonlinear correlation matching algorithm only with suboptimal $\mathbf{p}(n)$'s.

5. REFERENCES

[1] K. Abed-Meraim, J.F. Cardoso, A. Gorokhov, P. Loubaton, and E.Moulines, "On subspace methods for blind identification of SIMO FIR systems," *IEEE Trans. on Signal Proc.*, Jan. 1997.

[2] A. Chevreuil and P. Loubaton, "Blind Second-Order Identification of FIR Channels: Forced Cyclostationarity and Structured Subspace Method," *IEEE Signal Processing Letters*, July 1997.

[3] A. Chevreuil, E. Serpedin, P. Loubaton, and G. B. Giannakis, "Performance Analysis of Blind Channel Estimators Based on Non-Redundant Periodic Modulation Precoders," *IEEE Trans. on Signal Processing*, (to be submitted).

[4] G. B. Giannakis, "Linear Cyclic Correlation Approaches for Blind Channel Identification of FIR Channels," *Proc. ASIMOLAR'95 Conf.*, Nov. 1995.

[5] G. B. Giannakis, "Filterbanks for Blind Channel Identification and Equalization," *IEEE Signal Processing Letters*, June 1997.

[6] G. B. Giannakis and S. D. Halford, "Asym. Optimal Blind FS Channel Estimation and Performance Analysis," *IEEE Trans. on Signal Proc.*, July 1997.

[7] B. Porat, *Digital Processing of Random Signals: Theory & Methods*, Prentice Hall, 1994.

[8] E. Serpedin and G. B. Giannakis, "Blind Channel Identification and Equalization with Modulation Induced Cyclostationarity," submitted to *IEEE Trans. on Signal Processing*, 1997, (also *Proc. CISS'97*, Baltimore, MD, March 1997).

[9] L. Tong, G. Xu, B. Hassibi, and T. Kailath, "Blind Channel Identification Based on Second-Order Statistics: A Frequency-Domain Approach," *IEEE Trans. on Inform. Theory*,Jan. 1995.

[10] M. K. Tsatsanis and G. B. Giannakis, "Transmitter Induced Cyclostationarity for Blind Ch. Equalization," *IEEE Trans. Signal Proc.*, July 1997.

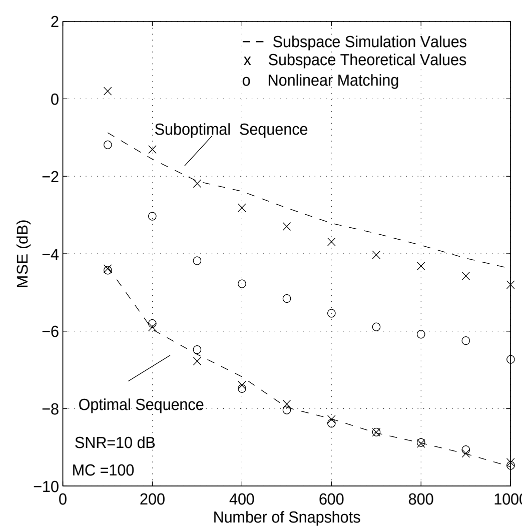

Figure 2: Simulation Results