# RESOLUTION OF OVERLAPPING DOPPLER SHIFTED ECHOES

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**ABSTRACT**

This paper considers the problem of estimating the time delays and doppler shifts of a known waveform received via several distinct paths by an array of antennas. The general maximum likelihood estimator is presented, and is shown to require a $2d$-dimensional non-linear minimization, where $d$ is the number of received signal reflections. Two alternative solutions based on signal and noise subspace fitting are proposed, requiring only a $d$-dimensional minimization. In particular, we show how to decouple the required search into a two-step procedure, where the delays are estimated and the dopplers solved for explicitly. Initial conditions for the time delay search can be obtained by applying generalizations of the MUSIC and ESPRIT algorithms.

## 1. INTRODUCTION

The problem of using an antenna array to estimate the time delays and doppler shifts (or frequency offsets) of a known signal is important in two common applications. First, in active radar and sonar, a known waveform is transmitted and reflections from objects "illuminated" by the transmission are subsequently received. The received signals are often modeled as scaled, delayed, and doppler-shifted versions of the transmitted signal. Estimation of the signal amplitude, delay, and doppler shift provides information about the position and relative motion of the objects. The second application involves estimation of the parameters of a multipath communication channel in situations where the transmitter is rapidly moving or has an unknown frequency offset. For example, consider a situation where a remote mobile user transmits a known waveform (e.g., a training sequence) to a basestation for synchronization or equalization purposes. If the channel is frequency selective (non-zero delay spread), then the signal will be received with several distinct delays. In addition, due to the motion of the mobile and variations in the carrier frequency of the transmitter, the known signal can also be received with a small frequency offset. Estimation of the delays and frequency offsets, as well as the spatial signatures of the signal arrivals, is necessary in establishing a clean, inter-symbol and interference-free communication link. This paper presents a novel approach to solving the problems described above. The techniques presented are applicable in situations involving multiple antennas and, unlike classical methods, are asymptotically optimal at high SNR even when multiple overlapping copies of the signal are received. The frequency domain model used in [1, 2] for time-delay estimation is generalized to incorporate the presence of (small) frequency offsets. The resulting *signal manifold* in the frequency domain can be seen as a generalized version of the signal manifold of [1, 2], in much the same way that polarization [3, 4, 5] and local scattering [6] generalize the standard *array manifold* in direction of arrival (DOA) estimation. This observation motivates the development of subspace-based techniques similar to those in [4, 5, 6], which provide closed-form solutions for the linear parameters (in our case, the frequency/doppler offsets). The resulting algorithms require a search for the time delays, but it is seen that for small frequency offsets, the closed-form time-delay estimation techniques of [1, 2] provide excellent initial conditions.

This work was supported in part by the Swedish Institute, the Swedish National Board for Technical Development (NUTEK), the National Science Foundation under Grant MIP-9408154, and by the Office of Naval Research under grant N00014-96-1-0934.

## 2. MODELING

Suppose an $m$-element antenna array receives several scaled, time-delayed, and frequency/doppler-shifted copies of a known baseband signal, $s(t)$. The received signals could, for instance, be the echoes from a pulse transmitted by an active radar, or they could result from a training sequence sent over a multipath communication channel. In either case, we may model the output of the array for small frequency/doppler offsets as

$$\mathbf{x}(t) = \sum_{k=1}^{d} \mathbf{a}_k s(t-\tau_k) e^{j\omega_{D_k} t} + \mathbf{n}(t), \tag{1}$$

where $d$ represents the number of different multipath signals, and where the parameters $\tau_k, \omega_{D_k}$, and $\mathbf{a}_k$ are the time-delay, frequency offset, and spatial signature of the $k$:th arrival. The additive noise vector, $\mathbf{n}(t)$, is assumed to be a zero mean temporally and spatially white noise process with covariance $\sigma^2 \mathbf{I}$. The standard narrowband assumption is employed here. Note that, for the radar case, the frequency offset $\omega_{D_k}$ is a narrowband approximation to the stretching or shrinking of the frequency axis due to the doppler effect induced by the relative motion of the reflecting target.

The frequency domain representation of the array output in (1) is given by

$$\mathbf{x}(\omega) = \sum_{k=1}^{d} \mathbf{a}_k s(\omega - \omega_{D_k}) e^{-j\omega\tau_k} + \mathbf{n}(\omega), \tag{2}$$

where $\mathbf{x}(\omega)$, $s(\omega)$ and $\mathbf{n}(\omega)$ are the Fourier transforms of $\mathbf{x}(t)$, $s(t)$ and $\mathbf{n}(t)$, respectively. Under the assumption that the frequency/doppler offsets are "small", it is possible to simplify the dependence of (2) on the doppler frequencies by neglecting the higher order terms in the Taylor series expansion of $s(\omega - \omega_{D_k})$ :

$$\begin{bmatrix} s(\omega_1 - \omega_{D_k}) \\ \vdots \\ s(\omega_N - \omega_{D_k}) \end{bmatrix} \approx \begin{bmatrix} s(\omega_1) \\ \vdots \\ s(\omega_N) \end{bmatrix} - \omega_{D_k} \begin{bmatrix} d(\omega_1) \\ \vdots \\ d(\omega_N) \end{bmatrix}$$

$$\triangleq \mathbf{s} - \omega_{D_k} \mathbf{d} \tag{3}$$

where a total of $N$ snapshots are collected from the array, and

$$d(\omega_i) = \left. \frac{\partial s(\omega)}{\partial \omega} \right|_{\omega = \omega_i} .$$

Assuming that $\mathbf{x}(\omega)$ is an $m \times 1$ column vector, the data at frequencies $\omega_1, \ldots, \omega_N$ may be arranged in matrix form as

$$\begin{aligned} \mathbf{X} &\triangleq \begin{bmatrix} \mathbf{x}^T(\omega_1) \\ \vdots \\ \mathbf{x}^T(\omega_N) \end{bmatrix} \\ &= \left( \ \mathbf{S}\mathbf{V}(\boldsymbol{\tau}) - \mathbf{D}\mathbf{V}(\boldsymbol{\tau})\mathbf{\Phi}(\boldsymbol{\omega}) \ \right) \mathbf{A} + \mathbf{N} \\ &\triangleq \mathbf{Q}(\boldsymbol{\tau}, \boldsymbol{\omega})\mathbf{A} + \mathbf{N} \ , \end{aligned} \tag{4}$$

where

$$\begin{aligned} \boldsymbol{\tau} &= [\tau_1 \ \ \ldots \ \ \tau_d]^T \\ \boldsymbol{\omega} &= [\omega_{D_1} \ \ \ldots \ \ \omega_{D_d}]^T \\ \mathbf{A} &= [\mathbf{a}_1 \ \ \cdots \ \ \mathbf{a}_d]^T \\ \mathbf{S} &= \text{diag}(\mathbf{s}) \\ \mathbf{D} &= \text{diag}(\mathbf{d}) \\ \mathbf{V}(\boldsymbol{\tau}) &= [\mathbf{v}(\tau_1) \ \ \ldots \ \ \mathbf{v}(\tau_d)] \\ \mathbf{v}(\tau) &= [\exp(-j\omega_1\tau) \ \ \ldots \ \ \exp(-j\omega_N\tau)]^T \\ \mathbf{\Phi}(\boldsymbol{\omega}) &= \text{diag}(\boldsymbol{\omega}) \end{aligned}$$

and where, for example, $\text{diag}(\boldsymbol{\omega})$ is a diagonal matrix with the elements of the vector $\boldsymbol{\omega}$ along its diagonal. The columns of $\mathbf{Q}(\boldsymbol{\tau}, \boldsymbol{\omega})$ have the following form:

$$\mathbf{q}(\tau_k, \omega_{D_k}) = \mathbf{S}\mathbf{v}(\tau_k) - \omega_{D_k}\mathbf{D}\mathbf{v}(\tau_k) \ .$$

By interchanging the roles of the samples in time and space, the delay and doppler estimation problem can be cast into the more familiar framework of DOA estimation. To see this, compare (4) with the standard model used in DOA estimation:

$$\mathbf{X} = \mathbf{A}(\boldsymbol{\theta})\mathbf{S} + \mathbf{N}, \tag{5}$$

where $\boldsymbol{\theta}$ is a vector containing the DOAs of the signals. In (5), $\mathbf{A}$ is a known function of the $d$ parameters in $\boldsymbol{\theta}$, and $\mathbf{S}$ is usually treated as an unknown unstructured matrix. On the other hand, in (4) it is $\mathbf{Q}$ that is parameterized and $\mathbf{A}$ that is unstructured. In essence, the roles of time (frequency) and space have thus been reversed. Instead of the array manifold $\mathbf{a}(\theta)$ in $m$-space employed in the DOA model, the delay/doppler model uses a "signal" manifold $\mathbf{q}(\tau_k, \omega_{D_k})$ in $N$-space. A further parallel is drawn in [7] by comparing (4) with the generalized array manifold that is associated with polarized antenna arrays [4, 5] and signals with angular spread [6].

In practice, $\mathbf{X}$ is obtained by performing a DFT on the time domain data in $\mathbf{X}_t$. As such, the translation of time delays into a linearly increasing phase shift $e^{-j\omega\tau}$ does not hold exactly, except in certain special cases involving, for example, a periodic signal or a signal with finite time support. However, if $t_N - t_1 \gg \max_k \tau_k$ and the signal is sampled at least at the Nyquist rate, then the error induced by the finite length DFT will be small, and the frequency domain model will be a reasonable approximation (this is illustrated by the simulation results in Section 4).

## 3. SUBSPACE-BASED ESTIMATION METHODS

In this section we describe algorithms for time delay and frequency/doppler offset estimation based on Noise Subspace Fitting (NSF) [8, 9], Signal Subspace Fitting (SSF) [8, 10, 9], MUSIC [3] and ESPRIT [11]. It will be shown that due to the special structure of the signal manifold in the frequency domain, both NSF and SSF reduce to a $d$-dimensional search for the delay parameters. Of the two, SSF is expected to be more robust when the spatial signature matrix $\mathbf{A}$ is nearly rank-deficient, or when the time-delay differences are very small [9]. Both methods require initial estimates of the $\boldsymbol{\tau}$ parameters, and for this purpose the MUSIC estimator and an ESPRIT-based estimator are described. The MUSIC estimator requires $d$ one-dimensional searches, while the ESPRIT estimator, which ignores the doppler shifts, does not require any search.

### 3.1. Noise Subspace Fitting

The NSF loss function for the problem at hand may be written as [5, 8]

$$\mathbf{V}_{NSF}(\boldsymbol{\tau}, \boldsymbol{\omega}) = \text{tr}\left\{ \mathbf{Q}^*(\boldsymbol{\tau}, \boldsymbol{\omega})\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^*\mathbf{Q}(\boldsymbol{\tau}, \boldsymbol{\omega})\hat{\mathbf{U}} \right\}, \tag{6}$$

where $\text{tr}\{\cdot\}$ denotes the matrix trace, $(\cdot)^*$ the conjugate transpose, $(\cdot)^\dagger$ the pseudo inverse, and

$$\hat{\mathbf{U}} = \mathbf{Q}^\dagger(\hat{\boldsymbol{\tau}}, \hat{\boldsymbol{\omega}})\hat{\mathbf{E}}_s\mathbf{W}\hat{\mathbf{E}}_s^*\mathbf{Q}^{\dagger *}(\hat{\boldsymbol{\tau}}, \hat{\boldsymbol{\omega}}) \ , \tag{7}$$

$\hat{\boldsymbol{\tau}}$ and $\hat{\boldsymbol{\omega}}$ are consistent (initial) estimates of $\boldsymbol{\tau}$ and $\boldsymbol{\omega}$, $\mathbf{W}$ is a diagonal weighting matrix, $\hat{\mathbf{E}}_s$ is the matrix whose columns are the left singular vectors corresponding to the $d$ largest singular values of $\mathbf{X}$, and $\hat{\mathbf{E}}_n$ is a $N \times (N-d)$ matrix whose columns are orthogonal to those of $\hat{\mathbf{E}}_s$. The choice of the matrix $\mathbf{W}$ depends on whether it is desired to approximate the so-called deterministic or stochastic ML solution (see [8, 9] for details). In the simulations presented later, we use the stochastic ML weighting

$$\mathbf{W} = (\hat{\mathbf{\Lambda}}_s - \hat{\sigma}^2\mathbf{I})^2\hat{\mathbf{\Lambda}}_s^{-1} \ , \tag{8}$$

where $\hat{\mathbf{\Lambda}}_s$ is a diagonal matrix formed from the $d$ largest squared singular values of $\mathbf{X}$, and $\hat{\sigma}^2$ is a consistent estimate of the noise variance (obtained, for example, as the average of the $m-d$ smallest squared singular values of $\mathbf{X}$). Introduce

$$\mathbf{M}(\boldsymbol{\tau}) = \begin{bmatrix} \mathbf{V}^*\mathbf{D}^*\mathbf{P}\mathbf{D}\mathbf{V} & -\mathbf{V}^*\mathbf{D}^*\mathbf{P}\mathbf{S}\mathbf{V} \\ -\mathbf{V}^*\mathbf{S}^*\mathbf{P}\mathbf{D}\mathbf{V} & \mathbf{V}^*\mathbf{S}^*\mathbf{P}\mathbf{S}\mathbf{V} \end{bmatrix}$$

$$\odot \begin{bmatrix} \hat{\mathbf{U}}^T & \hat{\mathbf{U}}^T \\ \hat{\mathbf{U}}^T & \hat{\mathbf{U}}^T \end{bmatrix} \ , \tag{9}$$

where $\boldsymbol{\eta}(\boldsymbol{\omega}) = \begin{bmatrix} \boldsymbol{\omega}^T & \mathbf{e}^T \end{bmatrix}$, $\mathbf{e} = [1 \quad \cdots \quad 1]^T$ is $d \times 1$ and $\mathbf{P} = \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^*$. Setting $\partial \mathbf{V}_{NSF}/\partial \boldsymbol{\omega} = \mathbf{0}$ yields

$$\hat{\boldsymbol{\omega}} = \mathbf{M}_{11}^{-1} \mathbf{M}_{12} \mathbf{e} , \tag{10}$$

where the real part of the matrix $\mathbf{M}(\boldsymbol{\tau})$ has been partitioned into $d \times d$ blocks:

$$\mathrm{Re}\, \mathbf{M}(\boldsymbol{\tau}) = \begin{bmatrix} \mathbf{M}_{11} & -\mathbf{M}_{12} \\ -\mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} .$$

Substituting (10) into the cost function leads to the following criteria for estimating $\boldsymbol{\tau}$:

$$\hat{\boldsymbol{\tau}} = \arg \min_{\boldsymbol{\tau}} \ \mathbf{e}^T \left( \mathbf{M}_{22} - \mathbf{M}_{12}^T \mathbf{M}_{11}^{-1} \mathbf{M}_{12} \right) \mathbf{e} , \tag{11}$$

which is the sum of the elements of the Schur complement of $\mathbf{M}_{11}$ in Re $\mathbf{M}(\boldsymbol{\tau})$. It is worth mentioning that, since typically $N \gg d$, it is advantageous to compute $\mathbf{P}$ as $\mathbf{P} = \mathbf{I} - \hat{\mathbf{E}}_s \hat{\mathbf{E}}_s^*$ rather than $\mathbf{P} = \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^*$.

The NSF algorithm is implemented by performing a $d$-dimensional search of the criterion in (11). As mentioned above, consistent initial estimates of $\boldsymbol{\tau}$ and $\boldsymbol{\omega}$ are required for computing the matrix $\hat{\mathbf{U}}$ used in the NSF criterion. One way of obtaining $\hat{\mathbf{U}}$ would be to first implement the NSF algorithm with $\hat{\mathbf{U}} = \mathbf{I}$, and use the resulting estimates to form the optimal $\hat{\mathbf{U}}$. Setting $\hat{\mathbf{U}} = \mathbf{I}$ is equivalent to using the MUSIC approach described later in this section. There are two drawbacks associated with the NSF algorithm: first, the algorithm is not always able to resolve closely spaced components in $\boldsymbol{\tau}$, and second, the algorithm's performance may deteriorate when the rows of $\mathbf{A}$ are linearly dependent, which can occur when either $d > m$, or two arrivals with different delays share the same spatial signature. The SSF algorithm presented in the next section overcomes these two drawbacks.

### 3.2. Signal Subspace Fitting

The SSF estimates of the delays and frequency/doppler offsets can be found by minimizing [8, 9, 10]

$$\mathbf{V}_{SSF}(\boldsymbol{\tau}, \boldsymbol{\omega}) = \mathrm{tr} \left\{ \boldsymbol{\Pi}_{\mathbf{Q}}^{\perp} \hat{\mathbf{E}}_s \mathbf{W} \hat{\mathbf{E}}_s^* \right\} , \tag{12}$$

where the diagonal weighting $\mathbf{W}$ is as defined in (8). As shown below, the doppler parameters can also be explicitly estimated using SSF, but only for the case where $d < N/2$, which is not a serious restriction in most cases. Define $\mathbf{C} = \begin{bmatrix} \mathbf{SV} & -\mathbf{DV} \end{bmatrix}$, and suppose that $d < N/2$ and $\mathbf{C}$ has full column rank. Introduce

$$\mathbf{T}^{-1} = \begin{bmatrix} \boldsymbol{\Phi} & -\mathbf{I} \end{bmatrix} (\mathbf{C}^* \mathbf{C})^{-1} \begin{bmatrix} \boldsymbol{\Phi}^* \\ -\mathbf{I} \end{bmatrix} , \tag{13}$$

and let $\boldsymbol{\Gamma}_{ij}$ be the $d \times d$ blocks of the matrix

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\Gamma}_{11} & \boldsymbol{\Gamma}_{12} \\ \boldsymbol{\Gamma}_{12}^* & \boldsymbol{\Gamma}_{22} \end{bmatrix} \triangleq \mathbf{C}^{\dagger} \hat{\mathbf{E}}_s \mathbf{W} \hat{\mathbf{E}}_s^* \mathbf{C}^{\dagger *} . \tag{14}$$

Also, let $\boldsymbol{\alpha}$ be the vector formed from the real part of the diagonal elements of $\boldsymbol{\Gamma}_{12} \hat{\mathbf{T}}$, and define

$$\boldsymbol{\Omega} = \mathrm{Re} \left( \hat{\mathbf{T}} \odot \boldsymbol{\Gamma}_{11}^T \right) \tag{15}$$

$$\rho = \mathrm{tr} \left\{ \boldsymbol{\Gamma}_{22} \hat{\mathbf{T}} \right\} . \tag{16}$$

Then, minimization with respect to $\boldsymbol{\omega}$ yields the estimate

$$\hat{\boldsymbol{\omega}} = \boldsymbol{\Omega}^{-1} \boldsymbol{\alpha} . \tag{17}$$

Inserting (17) into the cost function leads to the following criteria for estimating $\boldsymbol{\tau}$:

$$\hat{\boldsymbol{\tau}} = \arg \min_{\boldsymbol{\tau}} \ \rho - \boldsymbol{\alpha}^T \boldsymbol{\Omega}^{-1} \boldsymbol{\alpha} - \mathrm{tr} \left\{ \mathbf{C}\mathbf{C}^{\dagger} \hat{\mathbf{E}}_s \mathbf{W} \hat{\mathbf{E}}_s^* \right\} . \tag{18}$$

Note that the computation required to evaluate the SSF criterion can be significantly simplified by performing the trace calculation in (18) as

$$\mathrm{tr} \left\{ \mathbf{C}\mathbf{C}^{\dagger} \hat{\mathbf{E}}_s \mathbf{W} \hat{\mathbf{E}}_s^* \right\} = \mathrm{tr} \left\{ \left( \mathbf{C}^{\dagger} \hat{\mathbf{E}}_s \right) \mathbf{W} \left( \hat{\mathbf{E}}_s^* \mathbf{C} \right) \right\} . \tag{19}$$

The SSF algorithm is implemented by performing the $d$-dimensional search in (18). As with NSF, the SSF method requires consistent initial estimates of both $\boldsymbol{\tau}$ and $\boldsymbol{\omega}$ to form $\hat{\mathbf{T}}$, which is then used in calculating $\boldsymbol{\Omega}$ and $\boldsymbol{\alpha}$. Such estimates can be obtained using either the MUSIC or ESPRIT approaches presented in the following subsections, or by an initial application of SSF with $\hat{\mathbf{T}} = \mathbf{I}$.

### 3.3. MUSIC

In the standard MUSIC algorithm [3] for DOA estimation, the DOAs are determined to be the $d$ values of $\theta$ that make $\mathbf{a}(\theta)$ nearly orthogonal to $\hat{\mathbf{E}}_n$, according to the following measure:

$$\mathbf{V}_M(\theta) = \frac{\mathbf{a}^*(\theta) \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \mathbf{a}(\theta)}{\mathbf{a}^*(\theta) \mathbf{a}(\theta)} . \tag{20}$$

In the delay and doppler estimation problem, assuming that rank$(\mathbf{A}) = d$, we replace $\mathbf{a}(\theta)$ with the signal's frequency signature

$$\begin{aligned} \mathbf{q}(\tau, \omega_D) &= \mathbf{S}\mathbf{v}(\tau) - \omega_D \mathbf{D}\mathbf{v}(\tau) \\ &\triangleq \mathbf{G}(\tau) \mathbf{g}(\omega_D) , \end{aligned} \tag{21}$$

where $\mathbf{g}(\omega) = \begin{bmatrix} 1 & \omega_D \end{bmatrix}^T$, and

$$\mathbf{G}(\tau) = \begin{bmatrix} \mathbf{S}\mathbf{v}(\tau) & -\mathbf{D}\mathbf{v}(\tau) \end{bmatrix} .$$

For this case, the MUSIC loss function becomes

$$\mathbf{V}_M(\tau, \omega) = \frac{\mathbf{g}^*(\omega) \left[ \mathrm{Re} \left( \mathbf{G}^*(\tau) \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \mathbf{G}(\tau) \right) \right] \mathbf{g}(\omega)}{\mathbf{g}^*(\omega) \left[ \mathrm{Re} \left( \mathbf{G}^*(\tau) \mathbf{G}(\tau) \right) \right] \mathbf{g}(\omega)} , \tag{22}$$

since $\mathbf{g}(\omega)$ is real-valued. The MUSIC criterion in (22) is seen to be a ratio of quadratic forms in $\mathbf{g}(\omega)$, and thus minimizing $\mathbf{V}_M(\tau, \omega)$ with respect to $\mathbf{g}(\omega)$ is equivalent to finding, as a function of $\tau$, the minimum generalized eigenvalue and associated eigenvector of the following $2 \times 2$ matrices:

$$\mathrm{Re} \left( \mathbf{G}^*(\tau) \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \mathbf{G}(\tau) \right) \boldsymbol{\gamma}_{\min} = \lambda_{\min} \mathrm{Re} \left( \mathbf{G}^*(\tau) \mathbf{G}(\tau) \right) \boldsymbol{\gamma}_{\min} .$$

As in the algorithms of [3, 4], the time delays can be found by viewing $\lambda_{\min}$ as a function of $\tau$, and searching for the $d$ deepest minima of $\lambda_{\min}(\tau)$. The corresponding frequency offsets are then calculated using the generalized eigenvector associated with $\lambda_{\min}(\hat{\tau})$:

$$\hat{\omega}_{D_k} = \frac{\boldsymbol{\gamma}_{\min,2}(\hat{\tau}_k)}{\boldsymbol{\gamma}_{\min,1}(\hat{\tau}_k)} , \tag{23}$$

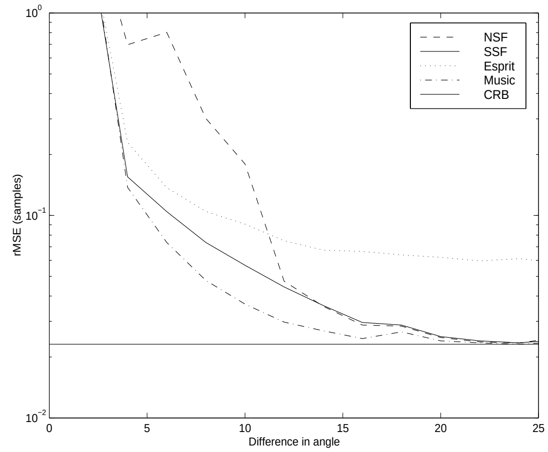
where $\boldsymbol{\gamma}_{\min,i}$ is element $i$ of $\boldsymbol{\gamma}_{\min}$.

### 3.4. ESPRIT

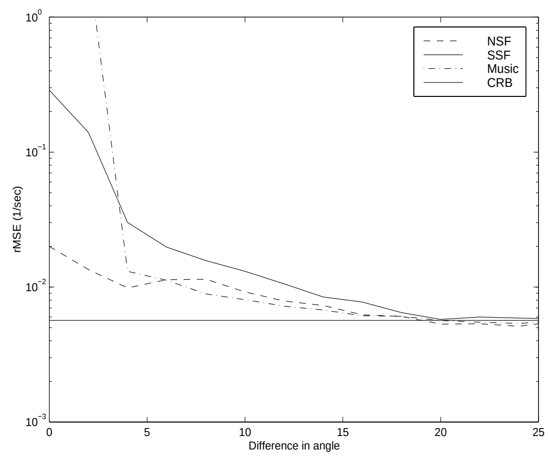
A fast algorithm based on ESPRIT was presented in [1, 2] for estimating time delays in cases where the frequency/doppler offset is zero. Our empirical results indicate that this approach still gives reasonable time delay estimates even when the frequency offset is non-zero but small. The fact that the algorithm yields the desired estimates in closed form (i.e., without search) makes it an attractive alternative for initializing the SSF and NSF searches.

## 4. NUMERICAL EXAMPLES

In this section we study how the performance of the estimators depends on the assumption of a full rank spatial signature matrix. Simulation data was generated using (1) for two multipath signals ($d = 2$) with time-delays $\boldsymbol{\tau} = [\ 0.5 \quad 3\ ]^T$, and DOAs $[\ 0^\circ \quad \theta\ ]$, where the DOA of the second arrival, $\theta$, is varied from $0^\circ$ to $25^\circ$. The data was corrupted by spatially and temporally white circular Gaussian noise with zero mean and standard deviation $\sigma$. The two columns of the signature matrix, $\mathbf{A}$, were given by the array response of a 5-element, half-wavelength spaced ULA. The signal sequence was chosen to be a unit power raised cosine function. Here, $N = 101$ samples are assumed to be taken from the array. Figure 1(a) compares the estimation errors, calculated from 500 Monte Carlo simulations, for the first time-delay estimates, and as can be seen from the figure, the NSF, the SSF and the MUSIC estimates achieve the CRB when the angular difference is above $15^\circ$. Furthermore, it is seen that all of the algorithms degrade significantly for angular differences lower than $5^\circ$. Figure 1(b) compares the rMSE for the first doppler shift estimates with the corresponding CRB. Here, the NSF and the MUSIC estimates are found to have a somewhat lower rMSE than the SSF estimates.




(a)




(b)

Figure 1: The rMSE of the proposed estimators as a function of the DOA difference compared with the corresponding CRB. (a) Time-delay, $\boldsymbol{\tau}_1$. (b) Doppler shift, $\boldsymbol{\omega}_1$.

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