USING SIGNAL CANCELLATION FOR OPTIMUM BEAMFORMING IN A CELLULAR CDMA SYSTEM

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ABSTRACT

We propose a new algorithm for estimating the interference--plus-noise covariance matrix for beamforming in a cellular CDMA system in a fading channel. The method uses direct PN sequence signal cancellation. We show in theory that our method outperforms that of [1,2] for finite input data. The results, confirmed by simulation, show that we get improved DOA estimates and SINR with lower computational requirements.

1. INTRODUCTION

Using a base station antenna array in cellular code-division multiple-access (CDMA) communications systems can potentially increase system capacity by several fold [1]. Beamforming shows great potential for improving signal to interference and noise ratios (SINR) which in turn increases cell capacity. To perform optimum SINR beamforming, we need to estimate an array response vector and an interference-noise (IN) covar iance matrix [3]. Currently, estimation of the IN covariance matrix for optimum beamforming requires great computation [1,2]. As a result, sub-optimum beamforming (maximum SNR) is used which does not require the IN matrix . However, when the number of users is not very large and the distribution of users is not uniform, there is a large gap between maximum SINR and maximum SNR beamforming. We propose a direct method to estimate the interference-noise covariance matrix which inc reases SINR and decreases computation compared with [1,2]. Since DOA estimation of mobiles is also improved, the method can potentially be applied to transmit beamforming in the down-link. This paper is organized as follows: Section II describes our system model and in Section III our algorithm is compared with [1] through analysis and s imulation.

2. SYSTEM MODEL

Here we consider the reverse (mobile to base station) link with Rayleigh amplitude fading, path loss, shadowing, and perfect power control in a generic cellular CDMA system. First we consider single path case. Assuming a narrow band signal model, at time t, the baseband signal received at the M-element antenna array for the ith user is:

$$\mathbf{x}_i(t) = \sum_{j=1}^{N} c_j(t-\tau_{i,j}) b_j(t-\tau_{i,j}) \sqrt{P_j}(t) \mathbf{a}_j(t) + \mathbf{n}(t) \quad (1)$$

where N is the total number of in-band mobiles, $c_j(t)$ is the pseudo noise (PN) sequence for the j^{th} mobile defined as

$$c_j(t) = \sum_{l=-\infty}^{\infty} c_{j,l}\, p(t - lT_c) \quad (2)$$

T_c is the chip period, p(t) is the chip pulse assumed to be an arbitrary time -limited waveform. PN chips are modelled as i.i.d random variables taking values ± 1 with equal probability, $b_j(t)$ is the information bit sequence of the j^{th} mobi le, $\tau_{i,j}$ is the differential time delay of the j^{th} mobile relative to that of the i^{th} mobile , vector $\mathbf{n}(t) \sim \mathcal{N}(0, \sigma^2 I)$ represents i.i.d Gaussian thermal noise, P_j is the total power received at the base station of the j^{th} mobile, and $\mathbf{a}_j(t)$ is the array response vector of j^{th} mobile whose time-varying DOA is $\theta_j(t)$. Without loss of generality, for all i=1,2,...... N and j=1,2,...... N, we assume self-synchronization, i.e. $\tau_{i,i}$=0, and the signal $b_j(t-\tau_{i,j})$, chips $c_j(t)$ and noise $\mathbf{n}(t)$ are mutually uncorrelated. Chips $c_j(t-\tau_{i,j})$ and $c_k(t-\tau_{i,k})$ are assumed mu tually uncorrelated as well as bits $b_j(t-\tau_{i,j})$ and $b_k(t-\tau_{i,k})$ for all k=1,2,......N and $k \neq j$. The array response vector $\mathbf{a}_j(t)$ is assumed to be unchanged over one information bit period T_b. The spreading gain L is defined as T_b/T_c . From [2],

$$\mathbf{R}_{xx_i}(t) = P_i \mathbf{a}_i(t) * \mathbf{a}_i(t)^{\mathbf{H}} + \sum_{j=1, j\neq i}^{N} P_j \mathbf{a}_j(t) * \mathbf{a}_j(t)^{\mathbf{H}} + \sigma^2 \mathbf{I} \quad (3)$$

Using code-filtering [2], The antenna outputs are correlated with PN codes to yield one sample vector per information bit. At information bit n

$$\mathbf{z}_i(n) = \sqrt{T_b}\sqrt{P_i} b_i(n) \mathbf{a}_i(n) + \frac{1}{\sqrt{T_b}} \int_0^{T_b} \mathbf{n}(t) c_i(t) dt$$

$$+ \sum_{j=1, j\neq i}^{N} \frac{1}{\sqrt{T_b}} \int_0^{T_b} \sqrt{P_j} b_j(t-\tau_{i,j}) c_j(t-\tau_{i,j}) c_i(t) \mathbf{a}_j(n) dt \quad (4)$$

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The post-correlation autocorrelation matrix can be defined as

$$\mathbf{R}_{zz_i}(n) = \frac{1}{T_c}\mathbf{E}\{\mathbf{z}_i(n) * \mathbf{z}_i(n)^{\mathbf{H}} \quad (5)$$

Using the result in [2,4], we have

$$\mathbf{R}_{zz_i}(n) = LP_i\mathbf{a}_i(n)\mathbf{a}_i(n)^{\mathbf{H}} + \beta \sum_{j=1,j\neq i}^{N} P_j\mathbf{a}_j(n)\mathbf{a}_j(n)^{\mathbf{H}} + \frac{\sigma^2}{T_c}\mathbf{I} \quad (6)$$

where β is a constant. If the signal is rectangular, it will be $\frac{2}{3}$. In reality, the channel is bandlimited, therefor, the assumption of a square wave does not hold. If this bandlimited channel has ideal low pass filter characteristics, β will be 1. In our proposed algorithm, it will not affect the estimation of array response vector as we use two correlators and their outputs. Whatever β is, it is the same for the two correlators and **MAI** from the two correlator's outputs will cancel each other perfectly. We notice that in (6), $\mathbf{R}_{zz_i}(n)$ is independent of the PN codes of user i as long as PN codes and information bits are random sequences. In addition to forming $\mathbf{z}_i(n)$, we propose to also despread the array output with the PN code:

$$cc_j(t) = \sum_{l=-\infty}^{\infty} (-1)^l c_{j,l}p(t - lT_c) \quad (7)$$

It is straightforward to show that $cc_j(t)$ is a random binary sequence where

$$\mathbf{E}[(-1)^{l+m}c_{j,l}c_{j,m}] = 0, l \neq m \quad (8)$$

If we apply (7) to (4) as a matched filter, we notice that after integration over T_b, the signal term vanishes as long as L is even. In a real system, it is uncommon to select L as an odd number. We obtain as output

$$\mathbf{y}_i(n) = \frac{1}{\sqrt{T_b}}\int_0^{T_b} \mathbf{n}(t)cc_i(t)dt + \sum_{j=1,j\neq i}^{N} \frac{1}{\sqrt{T_b}}\int_0^{T_b} \sqrt{P_j}b_j(t-\tau_{i,j})c_j(t-\tau_{i,j})cc_i(t)\mathbf{a}_j(n)dt \quad (9)$$

Using an analogous definition to Eq. (5), we obtain

$$\mathbf{R}_{yy_i}(n) = \beta \sum_{j=1,j\neq i}^{N} P_j\mathbf{a}_j(n) * \mathbf{a}_j(n)^{\mathbf{H}} + \frac{\sigma^2}{T_c}\mathbf{I} \quad (10)$$

which is the interference and noise portion of Eq.(6). Alternatively, in [1,2], $\mathbf{R}_{in}$ is estimated as

$$\mathbf{R}_{na_i}(n) = \alpha(\mathbf{R}_{xx} - \frac{1}{L}\mathbf{R}_{zz_i}) \quad (11)$$

where α depends on whether the channel is bandlimited or not. However, this constant will not change the output **SINR**.

In [2], the array response vector is estimated as the generalized eigenvector corresponding to the largest eigenvalue of the Hermitian-definite matrix pencil $\mathbf{R}_{zz_i} - \eta\mathbf{R}_{xx_i}$. In [3], it is shown that the beamformer that will maximize the SINR has the form $\mathbf{w}_i = \gamma\mathbf{R}_{in}^{-1}\mathbf{a}_i$, where we have dropped the time dependence to simplify notation. γ is a constant which will not affect SINR and can be omitted. Using our method, we calculate the optimum weights as

$$\hat{\mathbf{w}}_i = \mathbf{R}_{yy_i}^{-1}\mathbf{a}_i \quad (12)$$

while using the method in [1,2], the weights are given by

$$\tilde{\mathbf{w}}_i = \mathbf{R}_{na_i}^{-1}\mathbf{a}_i \quad (13)$$

We should point out that only the phase of $\hat{\mathbf{w}}_i$ and $\tilde{\mathbf{w}}_i$ affect the final **SINR**, their magnitude have no effect on output **SINR** [3]. For the case of multipath delay spread, generalization of the above is straightforward. Since (12) only requires updating one matrix, it can be shown that (12) requires only about 70% of the computation as compared with square root updating of (13) [1,2].

3. FINITE-SAMPLE PERFORMANCE

First, we show that our new method converges to the optimum solution. Define $(SINR_i)_{max}$ to be the SINR for the true array response vector $\mathbf{a_i}$ and true interference-noise covariance matrix $\mathbf{R}_{IN}$.It can be shown that [3]

$$(SINR_i)_{max} = LP_i\mathbf{a_i}^{\mathbf{H}}\mathbf{R_{IN}}^{-1}\mathbf{a_i} \quad (14)$$

To simplify the problem, we assume we have perfect array vector estimates using both methods. Let $(SINR)_1$ denote the SINR of the proposed method and $(SINR)_2$ denote the SINR of the proposed method in [1,2]. Normalizing $(SINR)_{max}$ [3,5], let

$$\hat{\eta}_i \equiv \frac{SINR_1}{SINR_{max}} = \frac{|\hat{\mathbf{w}}_\mathbf{i}^{\mathbf{H}}\mathbf{a_i}|^2}{\hat{\mathbf{w}}_\mathbf{i}^{\mathbf{H}}\mathbf{R}_{IN}\hat{\mathbf{w}}_\mathbf{i}}\frac{1}{\mathbf{a_i}^{\mathbf{H}}\mathbf{R}_{IN}^{-1}\mathbf{a_i}} \quad (15)$$

and

$$\tilde{\eta}_i \equiv \frac{SINR_2}{SINR_{max}} = \frac{|\tilde{\mathbf{w}}_\mathbf{i}^{\mathbf{H}}\mathbf{a_i}|^2}{\tilde{\mathbf{w}}_\mathbf{i}^{\mathbf{H}}\mathbf{R}_{IN}\tilde{\mathbf{w}}_\mathbf{i}}\frac{1}{\mathbf{a_i}^{\mathbf{H}}\mathbf{R}_{IN}^{-1}\mathbf{a_i}} \quad (16)$$

In [5], it is shown that $\hat{\eta}_i$ is Beta-distributed, i.e.,

$$\hat{\eta}_i \sim \beta(N - M + 2, M - 1) \quad (17)$$

Where N is the number of samples used to estimate covariance matrix and M is the number of antennas. According to the Beta distribution,

$$\mathbf{E}[\hat{\eta}_i] = \frac{N - M + 2}{N + 1} \quad (18)$$

$$\mathbf{Var}(\hat{\eta}_i) = \frac{(N - M + 2)(M - 1)}{(N + 1)^2(N + 2)} \rightarrow \frac{1}{N^2}, \text{as } N \rightarrow \infty \quad (19)$$

For optimality, $\hat{\eta}_i = 1$ and as $N \rightarrow \infty$,

$$\mathbf{E}\{|\hat{\eta}_i - 1|\} = 1 - \mathbf{E}\{\hat{\eta}_i\} \rightarrow 0 \quad (20)$$

implying that $\hat{\eta}_i$ converges in the mean and in the probability to optimum SINR, which means that the proposed method's estimate of SINR is consistent. Also

$$\mathbf{E}\{|\hat{\eta}_i - 1|^2\} \rightarrow \mathbf{E}\{|\hat{\eta}_i - \mathbf{E}\{\ hat\eta_i\}|^2\} = \mathbf{Var}(\hat{\eta}_i) \rightarrow 0 \quad (21)$$

implying that $\hat{\eta}_i$ converges in the mean-square sense to the optimum S INR.

Using results in [3,5], we now show that $\mathbf{E}[\hat{\eta}_i] > \mathbf{E}[\tilde{\eta}_i]$. Let $\hat{\mathbf{R}}_{zz_i}, \hat{\mathbf{R}}_{yy_i}$ denote the maximum likelihood estimates of $\mathbf{R}_{zz_i}$ and $\mathbf{R}_{yy_i}$ respectively, where $\hat{\mathbf{R}}_{yy_i}$ is an estimate of the IN matrix. Using the well-known Matrix Inversion Lemma,

$$\begin{aligned} \tilde{\mathbf{w}}_i &= \mathbf{R}_{na}^{-1}\mathbf{a_i} = [\hat{\mathbf{R}}_{zz_i} - LP_i\mathbf{a_i}\mathbf{a_i}^{\mathbf{H}}]^{-1}\mathbf{a_i} \\ &= \frac{1}{1 - LP_i\mathbf{a_i}^{\mathbf{H}}\hat{\mathbf{R}}_{zz_i}^{-1}\mathbf{a_i}}\hat{\mathbf{R}}_{zz_i}^{-1}\mathbf{a_i} = \beta\hat{\mathbf{R}}_{zz_i}^{-1}\mathbf{a_i} \end{aligned} \quad (22)$$

Where β is a scalar which will not affect the SINR. We therefore define

$$\dot{\mathbf{w}}_i = \hat{\mathbf{R}}_{zz_i}^{-1}\mathbf{a_i} \quad (23)$$

and so

$$\tilde{\eta}_i = \frac{|\dot{\mathbf{w}}_{\mathbf{i}}^{\mathbf{H}}\mathbf{a_i}|^2}{\dot{\mathbf{w}}_{\mathbf{i}}^{\mathbf{H}}\mathbf{R}_{IN}\dot{\mathbf{w}}_{\mathbf{i}}}\frac{1}{\mathbf{a_i}^{\mathbf{H}}\mathbf{R}_{IN}^{-1}\mathbf{a_i}} \quad (24)$$

Letting $\mathbf{R}_{zz_i}$ denote the true value of $\hat{\mathbf{R}}_{zz_i}$, We form another random variable $\eta_i^{'}$

$$\eta_i^{'} = \frac{|\dot{\mathbf{w}}_{\mathbf{i}}^{\mathbf{H}}\mathbf{a_i}|^2}{\dot{\mathbf{w}}_{\mathbf{i}}^{\mathbf{H}}\mathbf{R}_{zz_i}\dot{\mathbf{w}}_{\mathbf{i}}}\frac{1}{\mathbf{a_i}^{\mathbf{H}}\mathbf{R}_{zz_i}^{-1}\mathbf{a_i}} \quad (25)$$

and relationship between $\tilde{\eta}_i$ and $\eta_i^{'}$ is

$$\tilde{\eta}_i = \frac{\eta_i^{'}}{1 + (1 - \eta_i^{'})SINR_{max}} \quad (26)$$

Since algebraically the random variable $\eta_i^{'}$ is identical to $\hat{\eta}_i$[3,5], they have the same probability density function. Taking expectations,

$$\mathbf{E}[\tilde{\eta}_i] = \mathbf{E}[\frac{\eta_i^{'}}{1 + (1 - \eta_i^{'})SINR_{max}}] < \mathbf{E}[\eta_i^{'}] = \mathbf{E}[\hat{\eta}_i] \quad (27)$$

The above inequality means that on the average, the output SINR achieved by (12) is greater than the output SINR achieved by (13), particularly if $SINR_{max}$ is greater than 1. However, if $SINR_{max} < 1$, the difference between the two me thods becomes negligible.

4. NUMERICAL AND SIMULATION RESULT

We do the chip level simulation [7], i.e, all the correlation matrices are derived from ON chip-level simulations. We use Eq(1) to calculate $\mathbf{x}_i(t)$, use Eq(4) to obtain $\mathbf{z}_i(n)$, use Eq(9) to acquire $\mathbf{y}_i(n)$. With these data vectors, we can obtain their autocorrelation matrix by using maximum likelihood estimation. In our simulation, we assume a 3-sector base station with a 5-element uniform linear array with half wavelength spacing in each sector. The cell radius is 500m, $1/T_b$=9600 bps, BPSK modulation is used, and the spreading gain L=128. A rectangular pulse shape is assumed. There are 25 mobiles randomly distributed in azimuth around the base station with uniform distribution in $[0°, 120°]$, and each mobile has three multipaths. The first path has SNR 7 dB, the second and third paths are 9.5 dB and 12 dB, respectively, less than the first path. The delay spread is assumed to be 7 chips over the three paths. We assume the fading channel is Rayleigh with a path loss exponent of four, perfect power control, random mobile speeds of less than 60km/hour and weight vector updates occur every T_b sec.

As shown in Figure 1, we observe DOA tracking of the first path (SNR 7dB) over 50 information bits, which is a clear improvement over the method in [1,2]. We employ a 2D-RAKE receiver as in [1,2], but with maximum-ratio combining to obtain a diversity gain [6]. In Figure 2, we see the SINR gain of our method as compared to [1,2], which shows that we would obtain increased cellular system capacity. We notice that for the first 10 bits, the performance gap is not clear. This is as expected because we need at least 2M (in our case, M=5) samples to get a good estimate of covariance matrix [5].

5. CONCLUSIONS

In this paper, we propose a new algorithm to directly estimate the interference-noise covariance matrix using PN signal cancellation. We obtain improved DOA estimation and an average increase of 2.5dB in output SINR compared w ith [1,2]. In addition, the computational complexity is less than that of [1,2].

6. REFERENCES

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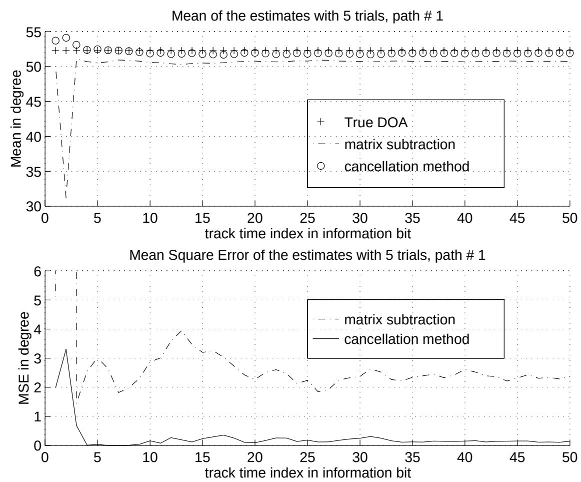

Figure 1: DOA of path 1 for 5 antenna elements and 25 mobiles each with 3 multipaths

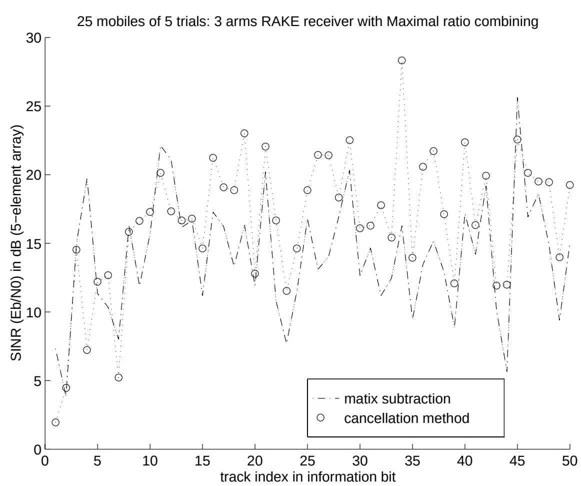

Figure 2: Output SINR for 5 antenna elements and 25 mobiles each with 3 multipaths