JOINT MULTIPATH-DOPPLER DIVERSITY IN FAST FADING CHANNELS

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ABSTRACT

We present a new framework for code-division multiple access (CDMA) communication over fast fading mobile wireless channels. The performance of the RAKE receiver, which is at the heart of existing CDMA systems, degrades substantially under fast fading encountered in many mobile scenarios. Due to the time-varying nature of the fast fading channel, we employ joint time-frequency processing, which is a powerful approach to time-varying signal processing. Whereas the RAKE receiver exploits multipath diversity to combat fading, our framework is based on joint multipath-Doppler diversity facilitated by a fundamental time-frequency decomposition of the channel into independent flat fading channels. Diversity processing is achieved by a time-frequency generalization of the RAKE receiver which can be leveraged into several important aspects of system design. Performance analysis shows that CDMA systems based on the time-frequency RAKE receiver, due to their inherently higher level of diversity, can potentially deliver significant performance gains over existing systems.

1. INTRODUCTION

Signal fading caused by the dynamics of the wireless channel is one of the most significant factors limiting the performance of mobile communication systems. *Diversity techniques* are used in practice to combat fading, which essentially amount to transmitting the signal over multiple independent fading channels. Systems based on CDMA, one of the leading technologies for multiuser communication, are well-known for their ability to combat fading. The RAKE receiver, which is at the heart of existing CDMA systems, is designed for exploiting multipath diversity in slow fading scenarios. However, fast fading is encountered in many mobile communication scenarios due to the motion of the users and results in rapid channel variations over time. The performance of the RAKE receiver degrades significantly under fast fading due to the Doppler shifts induced by the rapid channel variations.

In this paper, we present a new CDMA framework for wireless communication over fast fading channels. Due to the time-varying nature of the fast fading channel, we employ joint time-frequency processing, which is a powerful approach to time-varying signal processing. Our framework

is anchored on a canonical time-frequency channel decomposition into independent flat fading channels, which facilitates diversity signaling via joint time-frequency processing.

Unlike existing systems, our approach exploits the source of degradation in fast fading scenarios — Doppler shifts — to provide additional diversity, in conjunction with multipath diversity. The receiver structure that achieves joint multipath-Doppler diversity is a time-frequency generalization of the RAKE receiver which can be leveraged into several aspects of system design, including novel signaling and modulation techniques, multiuser interference suppression and timing acquisition, and information theoretic issues related to fast fading channels. Performance analysis shows that the inherently higher level of diversity afforded by the time-frequency RAKE receiver can potentially deliver substantial performance gains over existing systems, which are manifested in virtually all aspects of system performance.

The next section provides a brief relevant description of the channel. Section 3 develops the concept of joint multipath-Doppler diversity based on our canonical time-frequency channel decomposition. Section 4 briefly discusses some system design issues, followed by concluding remarks in Section 5.

2. CHANNEL DESCRIPTION

The complex baseband signal $r(t)$ at a mobile wireless receiver is given by

$$r(t) = s(t) + w(t) \tag{1}$$

where $w(t)$ is complex additive white Gaussian noise with power spectral density $\mathcal{N}_0$, and $s(t)$ is the channel output to the transmitted complex baseband signal $x(t)$

$$s(t) = \int h(t, \tau) x(t - \tau) d\tau . \tag{2}$$

The channel is represented by the time-variant impulse response $h(t, \tau)$, and admits the following equivalent time-frequency representation [1]

$$s(t) = \iint H(\theta, \tau) x(t - \tau) e^{j2\pi\theta t} d\theta d\tau , \text{ where} \tag{3}$$

$$H(\theta, \tau) \stackrel{def}{=} \int h(t, \tau) e^{-j2\pi\theta t} dt \tag{4}$$

is the time-frequency *spreading function* of the channel. The frequency variable θ corresponds to the Doppler shifts introduced by the channel, and the time variable τ corresponds to the multipath delays.

The spreading function $H(\theta, \tau)$ is best modeled as a stochastic process to capture the statistical variability of the channel. The wide-sense stationary uncorrelated scatterer model is widely used in practice in which $H(\theta, \tau)$ is a Gaussian process characterized by its second-order statistics[1] [1]

$$\mathrm{E}\{H(\theta_1, \tau_1)H^*(\theta_2, \tau_2)\} = \Psi(\theta_1, \tau_1)\delta(\theta_1 - \theta_2)\delta(\tau_1 - \tau_2) \ , \quad (5)$$

where

$$\Psi(\theta, \tau) \stackrel{def}{=} \mathrm{E}\left\{|H(\theta, \tau)|^2\right\} \ , \quad (6)$$

and $\delta(t)$ denotes the Dirac delta function. The function $\Psi(\theta, \tau) \geq 0$ is called the *scattering function* and denotes the power in the different (independent) multipath-Doppler signal components. The maximum support of $\Psi(\theta, \tau)$ in the τ variable is termed the *multipath spread* and denoted by T_m. Similarly, the *Doppler spread* B_d is defined as the maximum support of $\Psi(\theta, \tau)$ in the θ variable.

3. JOINT MULTIPATH-DOPPLER DIVERSITY

In this section, we develop the concept of joint multipath-Doppler diversity that lies at the heart of our CDMA framework for fast fading channels. Diversity signaling is facilitated by a fundamental time-frequency decomposition of the channel into independent flat fading channels. For clarity of exposition, we focus our discussion on the single-user case, and binary signaling. The framework can be extended to the multiuser case as briefly discussed in Section 4.

3.1. Time-Frequency Channel Decomposition

A spread-spectrum signaling waveform $q(t)$ used in CDMA systems has the form

$$q(t) = \sum_{n=0}^{N-1} a[n]v(t - nT_c) \ , \quad 0 \leq t < T, \quad (7)$$

where T is the signaling duration, $v(t) = I_{[0,T_c)}(t)$ is the rectangular chip waveform of duration T_c, and $a[n]$ is the pseudorandom spreading code corresponding to $q(t)$ [1]. The signal bandwidth $B \approx 1/T_c$, and the parameter $N = T/T_c \approx TB \gg 1$ is called the spreading gain of the CDMA system.

The following time-frequency channel decomposition plays a fundamental role in our development [2].

Proposition. For a spread-spectrum input $x(t) = q(t)$, the output signal $s(t)$ in (3) admits the representation

$$s(t) \approx \frac{T_c}{T} \sum_{l=0}^{L-1} \sum_{p=0}^{P-1} \widehat{H}\left(\frac{p}{T}, lT_c\right) u_{p,l}(t) \quad (8)$$

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[1]We assume a zero-mean (Rayleigh fading) channel. Extension to non-zero mean situations (Rician fading) is straightforward.

where $L = \lceil T_m/T_c \rceil \approx \lceil T_m B \rceil$, $P = \lceil TB_d \rceil$, and $\widehat{H}(\theta, \tau)$ is a time-limited and (essentially) band-limited approximation of $H(\theta, \tau)$. The waveforms $u_{p,l}(t)$'s are time-frequency shifted copies of $q(t)$ defined as

$$u_{p,l}(t) \stackrel{def}{=} q(t - lT_c)e^{j\frac{2\pi pt}{T}}, \quad (9)$$

and are approximately orthogonal

$$\langle u_{p,l}, u_{p',l'} \rangle \approx \|q\|^2 \delta_{p-p'}\delta_{l-l'} \ , \quad (10)$$

where δ_m denotes the Kronecker delta function. □

The (θ, τ)-sampling in (8) is depicted in Figure 1. It is
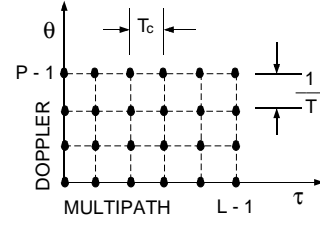

Figure 1: Sampling of the time-frequency plane inherent in the canonical channel representation.

worth noting that there is virtually no loss of information due to the (θ, τ)-sampling: due to the time- and (essentially) band-limited nature of $q(t)$, the receiver only "sees" a corresponding time- and band-limited version $\widehat{H}(\theta, \tau)$ of $H(\theta, \tau)$, which justifies the sampling in (8) [2]. In fact, (8) is a Karhunen-Loève-like expansion of the received signal $s(t)$: the $\widehat{H}(p/T, lT_c)$'s are uncorrelated (independent) random variables and the waveforms $u_{p,l}(t)$'s are (roughly) orthogonal. The orthogonality of $\{u_{p,l}(t)\}$ follows from the correlation properties of pseudorandom sequences [2, 1].

3.2. Time-Frequency Receiver Structure

For negligible intersymbol interference ($T_m \ll T$ and $B_d \ll B$), which is typically the case, the "one-shot detector" suffices in which each symbol can be decoded separately. Thus, without loss of generality, we assume that $r(t)$ in (1) is the received waveform for any one particular symbol.

The power of the representation (8) comes from the fact that it facilitates joint multipath-Doppler diversity to combat fading: the channel is decomposed into a finite number of flat fading channels that can be processed independently due to the orthogonality of the $u_{p,l}(t)$'s. The representation (8) also identifies the time-frequency receiver structure for diversity processing. Each diversity channel is defined by a particular $u_{p,l}(t)$, and the corresponding correlator outputs provide the sufficient statistics:

$$z_{p,l} = \langle r, u_{p,l} \rangle = \int r(t)q^*(t - lT_c)e^{-j2\pi\frac{p}{T}t}dt \ , \quad (11)$$

$0 \leq l \leq L - 1$, $0 \leq p \leq P - 1$. Note that (11) is a time-frequency correlation function[2] between $r(t)$ and $q(t)$, sampled according to the grid in Figure 1.

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[2]Identical to the cross-ambiguity function, or the short-time Fourier transform [3].

Diversity processing is achieved by combining the time-frequency correlator outputs $z_{p,l}$. For coherent signaling, the maximal ratio combiner is optimal which for binary signaling is given by [2, 4]

$$\widehat{b}_C = \text{sgn}\left(\text{real}\left\{\sum_{l=0}^{L-1}\sum_{p=0}^{P-1}\widehat{H}_{p,l}^*\left[\langle r, u_{p,l}^+\rangle - \langle r, u_{p,l}^-\rangle\right]\right\}\right)$$
(12)

where the $u_{p,l}^+(t)$'s are defined via (9) for the $b = 1$ symbol waveform $q^+(t)$, and $u_{p,l}^-(t)$'s correspond to the $b = -1$ symbol waveform $q^-(t)$. Note that the coherent detector (12) requires knowledge of the channel coefficients $\widehat{H}_{p,l}^* \stackrel{def}{=} \widehat{H}^*\left(\frac{p}{T}, lT_c\right)$, which may be estimated via a pilot transmission or training symbols.

If the channel coefficients are not available, but the channel *statistics* are known, the optimal noncoherent detector is given by [2, 4]

$$\widehat{b}_{NC} = \text{sgn}\left(\sum_{l=0}^{L-1}\sum_{p=0}^{P-1}\widehat{\Psi}_{p,l}\left[\left|\langle r, u_{p,l}^{+1}\rangle\right|^2 - \left|\langle r, u_{p,l}^{-1}\rangle\right|^2\right]\right)$$
(13)

where

$$\widehat{\Psi}_{p,l} \stackrel{def}{=} \frac{\|q\|^2\,\text{E}\left\{|\widehat{H}(\theta,\tau)|^2\right\}}{\|q\|^2\,\text{E}\left\{|\widehat{H}(\theta,\tau)|^2\right\} + \mathcal{N}_0}\,.$$
(14)

A special case of the noncoherent detector is the equal-gain square-law combiner [1] which assumes uniform power in all multipath-Doppler components; that is, $\widehat{\Psi}(\theta,\tau) = 1$.

Note that the receivers (12) and (13) are time-frequency generalizations of the RAKE receiver; restricting (12) and (13) to $P = 1$ yields the conventional RAKE receiver. Moreover, the time-frequency RAKE receiver can be efficiently realized via a bank of conventional RAKE receivers modulated to different Doppler frequencies [2, 4].

3.3. Potential Performance Gains

As evident from (8), the level of diversity is determined by the product $LP = L \times P$. The higher the level of diversity, the better the system performance against fading [1]. The product LP is proportional to the product of the signaling bandwidth $B \approx 1/T_c$ and the duration T: the larger the values of B and T, the better the performance against fading.

The worst-case situation occurs in a *time-nonselective* and *frequency-nonselective* channel in which no diversity is possible since B and T are small enough to yield $L = \lceil T_m B \rceil = 1$ and $P = \lceil TB_d \rceil = 1$. The result is the simple Rayleigh fading channel

$$r(t) = \frac{T_c}{T}\widehat{H}(0,0)q(t) + w(t) = \alpha q(t) + w(t).$$
(15)

In this case, the fluctuation in the signal-to-noise ratio (SNR) of the received signal, due to the statistical variation in α, has a tremendous effect on performance. Figure 2 compares the performance of coherent antipodal signaling over a nonfading (deterministic α) and a fading (random α; $M = 1$) channel [1]. The drastic performance degradation
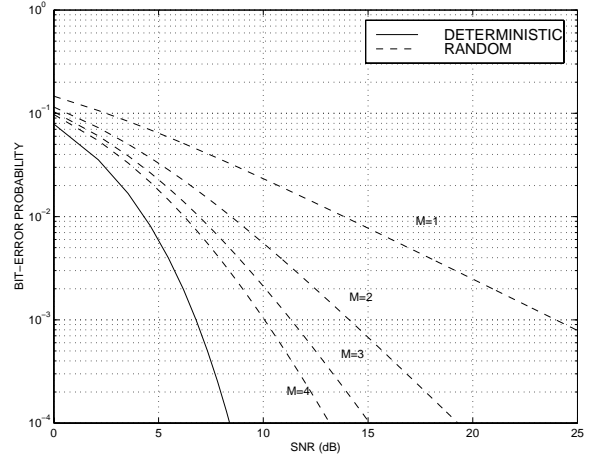

Figure 2: Bit-error probability as a function of SNR for antipodal signaling over a nonfading channel, and a Rayleigh fading channel with different levels (M) of diversity.

due to fading is evident — for instance, there is a loss of 18dB in SNR at a bit-error probability (BEP) of 10^{-3}.

The well-known ability of CDMA systems to combat fading comes from the large signaling bandwidth B which makes the channel frequency selective ($T_m B > 1$), thereby providing multipath diversity ($L \geq 2$). The RAKE receiver ((12) and (13) with $P = 1$) is used in practice to exploit multipath diversity, and corresponds to the grid samples on the multipath axis ($\theta = 0$) in Figure 1. Figure 2 shows the improvement in performance as a function of the number (M) of diversity components [1].[3] For example, going from to $M = L = 1$ (no diversity) to $M = L = 2$ (2-level diversity) improves the SNR by about 8dB at a BEP of $\approx 10^{-3}$. In general, the larger the number of diversity components, the closer the performance to the ideal unfaded channel.

However, the ability of existing CDMA systems to combat fading degrades significantly under fast fading which is encountered in many mobile communication scenarios. The RAKE receiver has been developed for slow fading scenarios ($TB_d < 1; P = 1$) in which the channel characteristics vary slowly over time. In fact, practical receiver structures are designed to take advantage of slow fading by averaging over several symbols to obtain better estimates of $\frac{T_c}{T}\widehat{H}^*\left(0, \frac{l}{B}\right)$. Doppler shifts encountered in fast fading result in degraded channel estimates which translate into significant losses in system performance — a noncoherent receiver is often used in practice for robustness to Doppler shifts, which typically incurs a direct loss of $3 - 6$dB in SNR.

Our approach to communication over fast fading channels exploits the cause of the problem — Doppler shifts — to provide additional diversity. Whereas the RAKE receiver relies only on multipath diversity, the time-frequency RAKE receiver exploits *joint multipath-Doppler diversity* by making the channel *time-selective* ($TB_d > 1; P \geq 2$) as well. The potential performance gains due to joint multipath-Doppler diversity are significant: by resolving one Doppler

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[3]The plots are based on an explicit expression for the BEP as a function of the number of diversity components [1].

component ($P = 2$), we double the level of diversity from $LP = L$ to $LP = 2L$. For example, for $L = 2$ resolvable multipath components, we can achieve 4-level diversity via the time-frequency RAKE receiver by resolving a Doppler component, as compared to the 2-level diversity achieved by the conventional RAKE receiver. As evident from Figure 2, going from $M = 2$ to $M = 4$ diversity components represents a gain of $4 - 6$dB in SNR at BEP of $10^{-3} - 10^{-4}$, which is large enough a gain to not only restore the performance of the conventional RAKE receiver, but potentially provide additional improvement as well.

Figure 3 shows the close agreement between the theoretical and simulated performance of time-frequency RAKE receivers for both coherent and noncoherent signaling. The simulations are based on length-31 M-sequences [2].
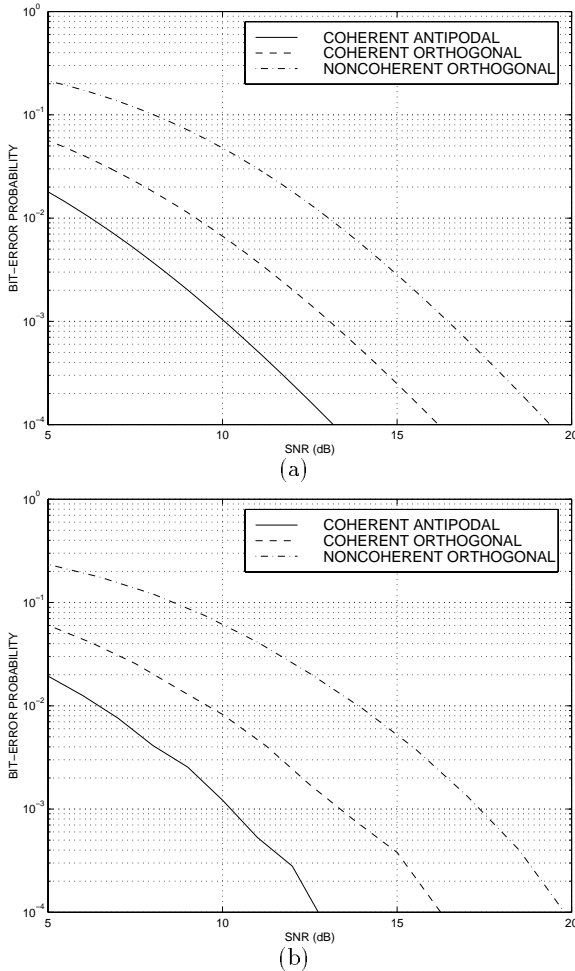

(a)

(b)

Figure 3: Performance comparison of time-frequency RAKE for $L = P = 2$: (a) theoretical, (b) simulated.

4. SYSTEM DESIGN ISSUES

Achieving the lucrative gains due to joint multipath-Doppler diversity in practice requires more work. As evident from (8), the channel has to be made time-selective ($TB_d > 1$) in order to achieve Doppler diversity ($P \geq 2$). Even though Doppler shifts typically encountered in mobile systems are large enough to degrade the performance of conventional receivers, they are not significant enough to directly yield $TB_d > 1$. One promising strategy for making the channel time-selective is to use signaling waveforms of sufficiently long duration T so that $TB_d > 1$ (or at least $TB_d \approx 1$; $P = 2$) [4]. Of course, in order to keep a fixed data rate, successive symbols might have to overlap in time, resulting in intersymbol interference (ISI). However, initial studies indicate that the excellent correlation properties of pseudorandom sequences make ISI virtually negligible, making time-selective signaling practically viable.

Our framework for exploiting joint multipath-Doppler diversity can be leveraged into several important aspects of CDMA system design. In particular, the interference due to multiple users is another significant performance-limiting factor. Our approach can be exploited to develop time-frequency generalizations of a variety of existing multiuser detectors, including the maximum likelihood, decorrelator, and minimum-mean-squared-error detectors, for operation under fast fading [5].

Another related important design issue is that of multiuser timing acquisition, which is particularly challenging under multipath fading. Existing acquisition techniques have been developed for slow fading and are either too complex, or do not satisfactorily account for the channel [4]. Moreover, none have shown promising results in practice. Our fundamental channel model suggests robust timing acquisition techniques that require only the second-order channel statistics as opposed to the *channel state* estimates that plaque many existing algorithms [4].

Finally, the time-frequency channel decomposition has interesting information-theoretic implications for fast fading channels that are also being explored currently [4].

5. CONCLUSION

We have presented the key ideas behind a new CDMA framework for fast fading channels that is based on the notion of joint multipath-Doppler diversity. The performance of existing systems based on the RAKE receiver degrades substantially under fast fading due to errors in channel estimation. The time-frequency RAKE receivers at the heart of our framework promise to deliver substantially improved performance due to their inherently higher level of diversity.

6. REFERENCES

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