MULTISTAGE CANCELLATION OF TERRAIN SCATTERED JAMMING AND CONVENTIONAL CLUTTER

Daniel J. Rabideau

Massachusetts Institute of Technology, Lincoln Laboratory
Lexington, Massachusetts 02173, USA
danr@LL.MIT.EDU

ABSTRACT

This paper addresses the problem of adaptively canceling both conventional clutter and terrain-scattered jamming (TSJ) in airborne radar systems. Existing algorithms for this type of interference adapt first in space/fast-time to cancel the TSJ, then in space/slow-time to cancel the conventional clutter. Unfortunately, the rapid weight updating required to cancel the nonstationary TSJ will modulate the clutter and targets, making the cancellation of conventional clutter extremely difficult and reducing the accuracy of the reported target locations. This paper proposes a multi-stage beamformer that prevents modulated clutter from degrading cancellation performance. The processor is formulated and its properties are described. The application of this beamformer to site-specific simulated data sets is used to illustrate its performance.

1. INTRODUCTION

The data received at the output of a sensor array during a radar system's coherent processing interval (CPI) can be represented as a cube of size $N \times M \times R$. Each element in this cube is a sample taken from a specific point in {element, pulse, range delay} space, where N is the number of sensors, M is the number of pulses transmitted within the CPI (slow-time), and R is the total number of range delays (fast-time). In the absence of interference (other than thermal noise), the optimal receiver for this data cube is a sequence of matched filters tuned to the target signal in angle (space), Doppler (slow-time) and range (fast-time) respectively.

When interference with unknown structure is present, it can be removed by substituting adaptive filters for some or all of these matched filters. In the simplest case where a passive radar is subjected to conventional (barrage) jamming, the jamming source is well-modeled as a far-field point source. Since the jammer is broadband (i.e. in all Dopplers) and is present at all range delays, spatial-only adaptive processing is used to form an N-dimensional adaptive weight vector $\mathbf{w}(\theta)$ to replace the spatial matched filter.

In the more typical case of active radar, a portion of the transmitted energy bounces off the earth and is scattered back into the radar's receiver. This interference, called clutter, contains a Doppler shift that depends on the angle of the scattering source and the radar's speed. Since this energy is distributed in space and Doppler, adaptive processing is used to form a $N \cdot M \times 1$ dimensional adaptive weight vector $\mathbf{w}(\theta, f)$, replacing the spatial and slow-time matched filters. Since the number of adaptive degrees of freedom (DOFs) can be large, dimension reducing transforms are often used [1].

In the most stressful case, the jammer is airborne and the radar is faced with a three tiered problem. Not only does it receive the "direct-path" jammer energy and conventional clutter, it also must contend with jammer energy that is scattered from the earth (a.k.a. hot clutter or Terrain Scattered Jamming -- TSJ). This multipath energy arrives from many angles and is partially coherent with the direct-path jamming signal. This suggests the TSJ cannot be canceled using only spatial DOFs since multipath components are present at the same angle as the target. Furthermore, the TSJ decorrelates across large time intervals (such as pulses) due to jammer motion. Thus processing across slow-time is usually ineffective. The TSJ is, however, correlated in fast-time due to the finite bandwidth of the jammer waveform. As a result, the multipath jamming component of the interference can be canceled by adapting in space/fast-time together. Due to the nonstationarity of the TSJ, these adaptive weights must be updated frequently within a CPI. Again, the number of adaptive DOFs can be large and dimension reducing transforms are often employed [2].

When the receiver is simultaneously subjected to all three types of interference (TSJ, direct-path jamming, and clutter), DOFs are needed in all three dimensions of the data cube [3]. However, simultaneous adaptation in three dimensions leads to processors of extremely high complexity and severe training requirements. Reduced DOF techniques are often required [4].

Alternatively, factored algorithms are attractive because breaking the problem down into several pieces reduces the number of DOFs, which lowers the computational cost and required training set size. These algorithms first adapt in space/fast-time to cancel the TSJ, then adapt in space/slow-time to cancel conventional clutter. Unfortunately, the rapid weight updating required to cancel the nonstationary TSJ will modulate the clutter and targets, making the cancellation of conventional clutter difficult and reducing the accuracy of the reported target locations [5]. This paper discusses how a special pre-filter can be used to reduce the clutter before it is modulated, so that after modulation it lies below the noise floor and consequently does not degrade system performance.

2. MATHEMATICAL FRAMEWORK

As we have seen, TSJ cancellers often use rapid weight updating

This work was sponsored by DARPA under Air Force Contract #F19628-95-C-0002. Opinions, interpretations, conclusions and recommendations are those of the author are not necessarily endorsed by the United States Air Force.

to track the nonstationary interference. This section examines how rapid weight updating interacts with coherent interference, giving rise to a set of adaptive weights that modulate signals received through their sidelobes. Understanding the mechanisms responsible for this modulation is an important step in formulating an algorithmic approach to deal with it [4, 5].

2.1 TRAINING-INDUCED MODULATIONS

There are two types of modulations that arise when adaptive weights are rapidly updated. The first results from the differences in training sets used in each successive adaptation, even if the interference is stationary. Suppose the underlying interference covariance matrix is $\mathbf{R}_{ideal}$ and we wish to cancel this interference using the optimal Wiener filter, $\mathbf{w} = \mathbf{R}_{ideal}^{-1}\mathbf{d}$ (where $\mathbf{d}$ is the normalized array response to a signal from θ_{look}). In practice, $\mathbf{R}_{ideal}$ is estimated from the data using the maximum likelihood estimator $\hat{\mathbf{R}} = T^{-1} \cdot \mathbf{X}_i \cdot \mathbf{X}_i^H$ (where the T columns of $\mathbf{X}_i$ consist of training snapshots). Suppose this estimate is calculated repeatedly using M independent sets of training snapshots $\mathbf{X}_i$, $i = 1,\ldots,M$, each with the same ideal covariance $\mathbf{R}_{ideal}$ (which might occur if one wished to update the adaptive weights on each of the M pulses within a CPI). Then the distribution of the i[th] element of the adaptive weight vector is:

$$w_i \overset{d}{=} w_{quiescent,i} + c_i t_i \tag{1}$$

where

$$\mathbf{w}_{quiescent} = \frac{\mathbf{R}_{ideal}^{-1}\mathbf{d}}{\mathbf{d}^H \mathbf{R}_{ideal}^{-1}\mathbf{d}}$$

and c is a deterministic quantity as derived in [6]. In [7], it is shown that the random components in (1) compose a linearly transformed, multivariate t-distributed random vector. As a result, we can state that the adaptive response in direction $\mathbf{v}_a$ is given by:

$$\boxed{\mathbf{w}^H \mathbf{v}_a \overset{d}{=} q + c \cdot t} \tag{2}$$

where q is the quiescent response, $q = \mathbf{w}_{quiescent}^H \mathbf{v}_a$.

The random component of (2) degrades the Doppler localization of incoming signals. To see this, let $\mathbf{w}_p$ represent the adaptive weights computed using $\mathbf{X}_p$. Let $\mathbf{W} = [\mathbf{w}_1 \;\; \ldots \;\; \mathbf{w}_M]$ be the full set of weight vectors computed on each of the M pulses within a CPI. Suppose these weights are applied to a target (or clutter discrete) from direction $\mathbf{v}_d \otimes \mathbf{v}_a$ ($\mathbf{v}_d$ is the Doppler component, $\mathbf{v}_a$ is the space/fast-time component). Then the output gain on this target across pulses forms the vector:

$$\mathbf{v}_d \circ \left(\mathbf{W}^H \mathbf{v}_a\right) \overset{d}{=} q\mathbf{v}_d + c \cdot \mathbf{v}_d \circ \begin{bmatrix} t_1 \\ \vdots \\ t_M \end{bmatrix}$$

and we clearly see that the target's output response (in slow-time) is no longer matched to its input Doppler response vector, $\mathbf{v}_d$. This variation in the adaptive sidelobe levels can be reduced by increasing the size of the training set or using a constrained filter design technique [8].

2.2 COHERENCE-INDUCED MODULATIONS

The second type of modulation that results from the rapid weight updating process used for TSJ cancellation is related to the coherence of the interference itself. As such, it depends strongly on the extent to which TSJ is spread (spatially) and represented in the correlation matrix (beamspace).

Assume the array covariance matrix is the sum of an interference term and a noise term:

$$\mathbf{R}(t)_{ideal} = \mathbf{V} \cdot \mathbf{A}(t) \cdot \mathbf{V}^H + \sigma^2 \mathbf{I} \tag{3}$$

where $\mathbf{V} = \left[\mathbf{v}_1 \;\; \cdots \;\; \mathbf{v}_K\right]$. Without loss of generality, the noise power, σ^2, is assumed to be unity.

If the K interference components are perfectly coherent (a simplifying assumption that is necessary to make the analysis tractable and justified via time-series modeling), the interference correlation matrix is of the form:

$$\mathbf{A}(t) = \mathbf{a}(t) \cdot \mathbf{a}(t)^H \quad \text{where} \quad \mathbf{a}(t) = \left[\alpha_1 e^{j\omega_1 t} \;\; \cdots \;\; \alpha_K e^{j\omega_K t}\right]^T \tag{4}$$

and ω_i is the Doppler shift of the i^{th} interference component. The α_i terms represent the correlated interference amplitudes for sources $i = 1,\ldots,K$.

Using this model, we can now derive an exact expression for the adaptive beamformer's sidelobe response in the presence of coherent interference. First, apply the matrix inversion lemma to (3). Then, compute the adaptive weights at time t:

$$\mathbf{w}(t) \equiv \mathbf{R}(t)_{ideal}^{-1}\mathbf{d} = \mathbf{d} - \frac{\mathbf{V}\mathbf{a}(t)\mathbf{a}(t)^H \mathbf{V}^H \mathbf{d}}{1 + \mathbf{a}(t)^H \mathbf{V}^H \mathbf{V}\mathbf{a}(t)}. \tag{5}$$

The gain of this adaptive beam in the sidelobe direction θ_{SL} is given by:

$$\mathbf{w}(t)^H \mathbf{v}_{SL} = g - \frac{\mathbf{d}^H \mathbf{V}\mathbf{a}(t)\mathbf{a}(t)^H \mathbf{V}^H \mathbf{v}_{SL}}{1 + \mathbf{a}(t)^H \mathbf{V}^H \mathbf{V}\mathbf{a}(t)} \tag{6}$$

where g is the nonadaptive sidelobe level, $g = \mathbf{d}^H \mathbf{v}_{SL}$. After some algebraic manipulation, one can show that (6) is equivalent to:

$$\boxed{\mathbf{w}(t)^H \mathbf{v}_{SL} = g - \frac{\sum_{l=1}^{K}\sum_{m=1}^{K} \alpha_l \alpha_m^* e^{j(\omega_l - \omega_m)t} \left(\mathbf{d}^H \mathbf{v}_l\right)\left(\mathbf{v}_m^H \mathbf{v}_{SL}\right)}{1 + \sum_{k=1}^{K}\sum_{i=1}^{K} \alpha_k^* \alpha_i e^{j(\omega_k - \omega_i)t} \mathbf{v}_k^H \mathbf{v}_i}}. \tag{7}$$

In (7), the second term contains the ratio of the sum of complex exponentials. As such, it is responsible for signal modulation. Several dominant modes are often present as shown in [5]. The key observation here is that by transforming the data into beamspace prior to adaptation, the spatial extent of the TSJ can be limited, which causes the corresponding terms in (7) to be small [8]. Thus, beamspace cancellers can limit the regions where modulation takes place.

In Figure 1a, we illustrate the radar clutter after TSJ mitigation has taken place. Here the TSJ adaptation is performed in element space and modulated clutter is clearly seen at all angles. Figure 1b shows the same clutter spectrum after TSJ mitigation is performed in beamspace. Here, a single auxiliary beam containing multiple fast-time taps is used [2]. Observe that the modulated clutter is contained within a small set of angles near the auxiliary beam. In both cases, this modulated clutter will degrade end-to-end system performance. A technique that avoids clutter modulation is sought.

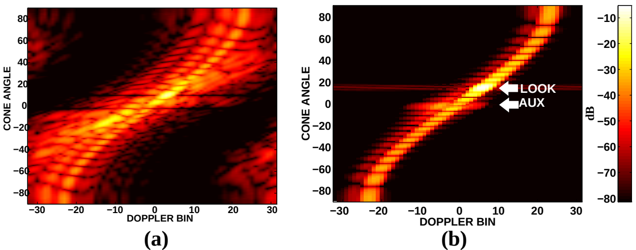

Figure 1 Modulated Clutter at the output of TSJ mitigation (beam steered to 10°). *(a)* Element Space *(b)* Beamspace (selected auxiliary).

3. MULTISTAGE APPROACH

Next, we propose a new multistage technique for canceling TSJ and clutter. Our approach involves exploiting the structure of the weight modulation (described above) to build special pre-filters that largely eliminate modulated clutter. The method is both simple and computationally efficient. Furthermore, it is shown to offer substantially better performance compared with conventional factored algorithms for canceling TSJ and clutter, as measured by the achievable Signal to Interference plus Noise Ratio (SINR).

3.1 Anti-Modulation Pre-filters

Our basic approach to preventing clutter modulation is to cancel "clutter of interest" prior to TSJ mitigation, as illustrated in Figure 2. The key phrase here is "clutter of interest"; i.e., the objective of this pre-filter is not to remove all of the clutter. Doing this is a difficult task requiring many adaptive DOFs [1] and is complicated by the presence of TSJ. Instead, the pre-filter only needs to remove enough clutter so that when the remainder is modulated, it lies below the noise floor at its destination frequency. Moreover, we do not require the pre-filter to remove *any* clutter from directions absent of modulation. This clutter will be canceled later (in stage 3) in the usual fashion using Space-Time Adaptive Processing (STAP). As a practical matter, we seek pre-filters that have only a small number of slow-time DOFs because combining too many pulses decorrelates the TSJ, causing large losses in SINR. Finally, the filter itself may be adaptive; however, adequate nonadaptive filters can often be designed by using measurements of the radar's velocity and crab to predict the clutter ridge's location.

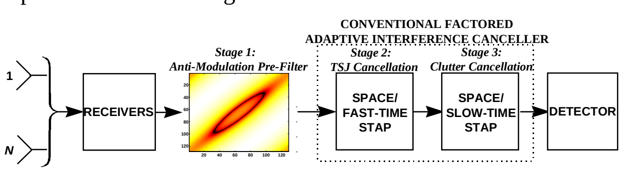

Figure 2. Multistage beamformer for canceling TSJ and clutter.

3.2 Representative Pre-Filter Implementations

The goal of the initial stage of processing is to cancel clutter that will be modulated by the subsequent TSJ cancellation stage. Since, every TSJ cancellation algorithm modulates clutter in a different fashion, the pre-filter's specification depends on the architecture chosen for stage 2. In this section, we consider two representative TSJ cancellation architectures and some pre-filter specifications that are appropriate for them.

Pre-filter's for the Selected Auxiliary TSJ Canceller:

First, consider the selected-auxiliary TSJ sidelobe canceller of [2]. As stated in section 2, this algorithm uses TSJ from a "reference" direction to cancel TSJ in the main beam. As such, we expect clutter received from this reference direction to modulate in Doppler (see Figure 1b). To prevent this modulation from degrading SINR, we can use a 2-D FIR notch filter (applied to spatial/slow-time dimensions of the data cube) to reduce the clutter received from this direction prior to TSJ cancellation. Figure 3a illustrates the modulated clutter at the output of a selected auxiliary TSJ canceller when one such pre-filter is used (compare to Figure 1b).

Although many 2-D filtering techniques could be applied to the design of this pre-filter, ones that result in low filter order are preferred (because they maintain aperture and avoid decorrelating the TSJ). For example, one such method takes a Taylor series expansion of the filter's frequency response and sets it equal to zero at points near the reference beam's mainlobe clutter. The filter coefficients are then found via least squares solution. The idea is analogous to [9] except we allow the filter response function to be non-elliptic and non-symmetric as needed.

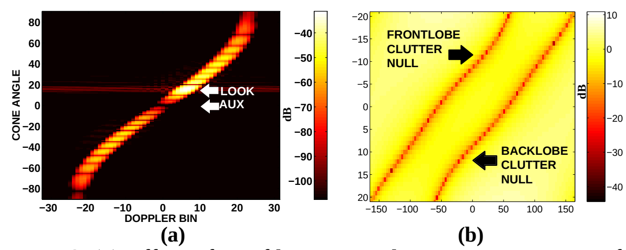

Figure 3 *(a)* Effect of pre-filtering on clutter seen at output of Sel. Aux. TSJ canceller. (Compare with Figure 1b) *(b)* Frequency response of an elliptic pre-filter for use with the 2D STAP TSJ canceller. Separate front and back lobe nulls are used to account for crab in the simulated data of Section 4.

Pre-filter's for the 2D STAP TSJ Canceller:

Second, let us consider a TSJ architecture that adapts on a set of adjacent input beams (with fast-time taps) spanning a sector of interest (e.g., where targets are being sought) [10]. The analysis of section 2 predicts that an entire region of the clutter ridge will undergo modulation during the TSJ mitigation (Figure 1a). To prevent this modulation from degrading the output SINR in these Doppler bins, we must create a filter that has its nulls aligned with the clutter ridge. Again, a good choice for this filter is to design a small elliptic pre-filter using the method of [9], with "tie down" points within the sector of interest. An example of a resulting filter response is illustrated in Figure 3b.

4. NUMERICAL EXAMPLES

In order to illustrate the effect pre-filtering has on end-to-end system performance, a time-series hot clutter simulation was employed. A uniform linear array operating at UHF and consisting of 28 elements, and 32 pulses was simulated. The radar bandwidth was 200kHz and the pulse repetition frequency was 333Hz. The simulation software has the ability to construct clutter and TSJ returns from any specific area of the world by using electronic mapping data provided by USGS. The data used below was derived from a simulation of the White Sands Missile Range facility in New Mexico and consisted of a mixed (mountain/prairie) terrain.

This data was first transformed into a beamspace consisting of 16 beams centered on broadside and spaced 3° apart. The data was then processed adaptively (using all 16 beams * 2 fast-time taps/beam = 32 DOFs) to cancel TSJ. Over-the-horizon data was used to obtain clutter-free snapshots for training the TSJ canceller. Next, the output data was processed using the adjacent-bin post-Doppler STAP algorithm to remove clutter [11]. Finally, the output SINR was computed and normalized by the optimal SNR achievable in the noise-only case (using matched filtering). The resulting metric is usually called "SINR Loss" in the literature. An SINR Loss of 0 dB indicates perfect interference rejection.

Figure 4 shows the SINR Loss at a range of 100 km when pre-filtering is not used. Observe that losses in excess of 25 dB are present over the majority of space. By comparison, Figure 5 shows the SINR Loss at the same range when the pre-filter described in Section 4 is used. Note that target visibility is greatly improved in regions away from the clutter ridge and jammer. This is quantified in Figure 6, which shows the percentage of this angle-Doppler space containing SINR losses of any given level. Observe that performance is poor for both algorithms at the extreme right of this plot as a result of the adaptive "notches" placed around clutter and the jammer's direct path. These will always be present. Moving left from this point, however, we observe increasingly better performance from the pre-filtered approach since these are the areas (away from clutter and jammer direct path) where modulated clutter was limiting visibility.

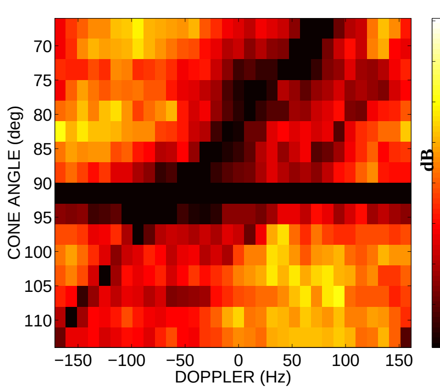

Figure 4 SINR Loss without pre-filter

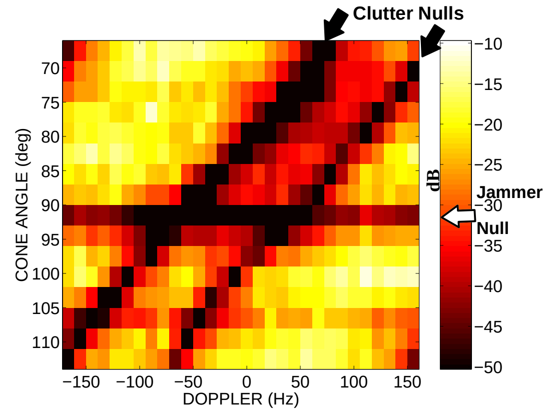

Figure 5 SINR Loss with pre-filtering.

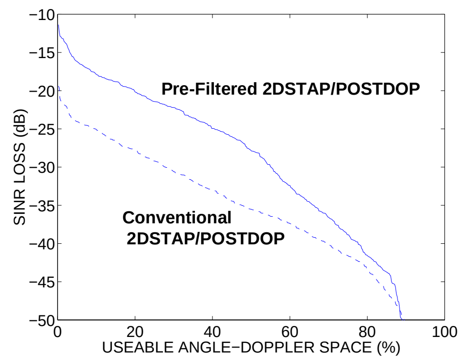

Figure 6 Aggregate SINR Loss Comparison.

5. REFERENCES

[1] J. Ward, "Space-Time Adaptive Processing for Airborne Radar," *IEEE Proc. ICASSP*, (Detroit, MI), pp. 2809-2812, 1995.

[2] D. Marshall, R. Gabel, "Simultaneous Mitigation of Multipath Jamming and Ground Clutter," Proc. 4th (DARPA) Adaptive Sensor Array Processing Workshop, MIT Lincoln Lab, pp. 285-315, 1996.

[3] R. Fante, J. Torres, "Cancellation of Diffuse Jammer Multipath by an Airborne Adaptive Radar," IEEE Trans. AES, pp. 805-820, 1995.

[4] D. Rabideau, "Revised STAP Strategies for Modulated Clutter," Proc. 5th DARPA Hot Clutter TIM, Rome Lab, Sept 1997.

[5] D. Rabideau, "Signal Modulation in Pulse-By-Pulse Adaptive Nulling Systems," Proc. 6th Adaptive Sensor Array Proc Workshop, MIT Lincoln Lab, pp. 387-415, 1997.

[6] S. Haykin, A. Steinhardt, "Adaptive Radar Detection and Estimation," John Wiley & Sons, New York, pp. 136-137.

[7] C. Richmond, "PDF's, Confidence Regions, and Relevant Statistics for a Class of Sample Covariance-Based Array Processors," IEEE Trans. SP, pp. 1779-1792, July 1996.

[8] D. Rabideau, "Modulation of Signals in Rapidly-Updated Adaptive Filters: Theory, Mitigation, and Applications," Proc. 30th Asilomar Conf. on Signal, Systems, and Computers, 1997.

[9] A. Saidi, J. McClellan, "Root Contours of Two Dimensional Prediction Polynomials," submitted to IEEE Trans. SP., 1997.

[10] S. Kogon, D. Williams, J. McClellan, "Factored Mitigation of Terrain Scattered Interference and Monostatic Clutter," Proc. 30th Asilomar Conf. on Signals, Sys., and Computers, pp. 526-530, 1996.

[11] J. Ward, *Space-Time Adaptive Processing for Airborne Radar*, Lincoln Laboratory Tech. Rpt. 1015, Dec. 1994.