HARMONIC PHASE COUPLING FOR BATTLEFIELD ACOUSTIC TARGET IDENTIFICATION

Douglas E. Lake

Army Research Laboratory
AMSRL-SE-SA
2800 Powder Mill Road
Adelphi, MD 20783-1197
dlake@arl.mil

ABSTRACT

Target identification using battlefield acoustic sensor arrays is an important problem for the Army ([6]). Acoustic signatures of targets of interest, such as tanks and trucks, exhibit time-varying patterns of harmonic amplitudes that facilitate target ID. In this paper, harmonic phase coupling is shown to also be present in these acoustic signatures though issues remain to fully exploit these relationships for target ID. With all else being equal, frequencies with simple harmonic relationships, such as 2 to 1 or 3 to 2, are preferred to produce stable features. Also, naive use of FFT approaches to estimate phase leads to erratic estimates. Cramer-Rao bounds (CRB) are presented and provide insights into the limitations of the accuracy of phase coupling estimates and suggests the value of developing nonstationary methods.

1. COUPLED HARMONIC SIGNAL MODEL

The spectrogram from an acoustic sensor taken by the Acoustic Signal Processing branch of the Army Research Laboratory (ARL) during recent field tests at Aberdeen Proving Ground (APG) is shown in Figure (1). The harmonic structure and the time-varying nature of the signal is very apparent. A coupled harmonic signal model accurately models a large portion of a vehicle's acoustic signature, especially that coming from the engine, and is given by

$$s(t) = \sum_{k=1}^{m} A_k \cos(h_k \omega t + \phi_k) \qquad (1)$$

where ω is called the *fundamental frequency*, h_k is an integer indicating the k^{th} harmonic number (assumed known). In (1), $A_k > 0$ and ϕ_k are respectively the amplitude and phase of the k^{th} harmonic. It is sometimes preferable to write (1) as

$$s(t) = \sum_{k=1}^{m} u_k \cos(h_k \omega t) + v_k \sin(h_k \omega t) \qquad (2)$$

where $u_k = A_k \cos(\phi_k)$ and $v_k = -A_k \sin(\phi_)$. The parameterization of fundamental frequency $f = \omega/(2\pi)$ can also be used in (1) and (2).

The data received at the acoustic sensor is a sampled version of the continuous-time model in (1) with additive noise consisting of the cumulative effect of ambient noise such as wind, non-target sources. Also, the target's acoustic signature is not completely described by 1). For the data in Figure (1), the fundamental frequency can be assumed constant for short time windows (on the order of a second) and the stationary methods to be presented can be applied. The sampling rate is $T = 2000$ samples per second and FFT's were taken with $n = 2048$ samples. The strongest line at approximately 90 Hz is the 6^{th} harmonic so that the fundamental frequency is approximately 15 Hz. For the current ARL target ID system ([9]), the second through twelfth harmonics are utilized so that $\mathbf{h} = (2,3,4,5,6,7,8,9,10,11,12)^T$ and $m = 11$ in (1).

2. MAXIMUM LIKELIHOOD ESTIMATION

Let $\mathbf{y} = (y_1, y_2, \ldots, y_n)^T$ be a vector denoting a given set of n samples at times $t(1), t(2), \ldots, t(n)$. An example of data from a short time window (approximately a 1/4 a second) is shown in Figure (2) which is the first 512 samples from the data in Figure (1). The time indices are assumed to be $t(i) = i - (n+1)/2$ which corresponds to associating the phase with the middle of the time window and as discussed in [3] greatly simplifies the analysis.

Douglas Lake is an ASEE Postdoctoral Fellow with the Army Research Laboratory.

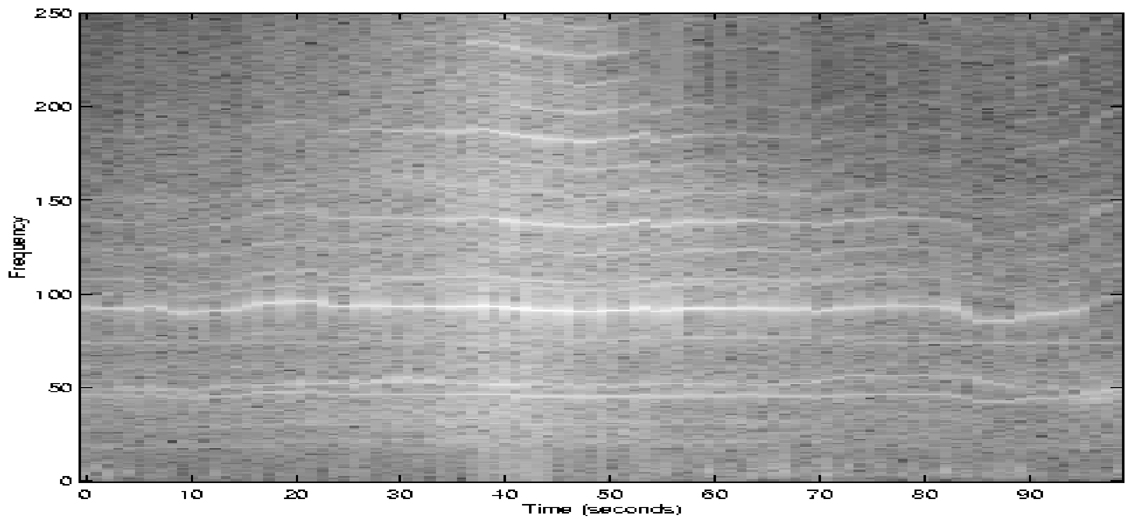

Figure 1: Battlefield Acoustic Data with Time-varying Harmonic Signal

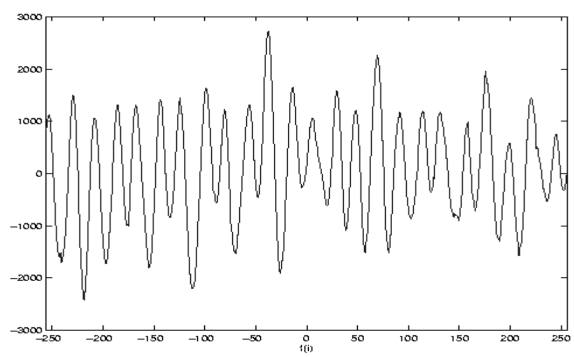

Figure 2: Acoustic Data with Coupled Harmonic Signal

Using (2), the coupled harmonic signal parameter estimation problem can be written as a linear model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{z} \tag{3}$$

where $\mathbf{X} = \mathbf{X}(\boldsymbol{\omega})$ is a $n \times 2m$ matrix and the parameter

$$\boldsymbol{\beta} = \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} \tag{4}$$

One can write $\mathbf{X} = [\mathbf{C}\ \mathbf{S}]$ where $\mathbf{C}$ and $\mathbf{S}$ are $n \times m$ matrices given by

$$\begin{aligned} \mathbf{C}_{ij} &= \cos(h_j \omega t(i)) \\ \mathbf{S}_{ij} &= \sin(h_j \omega t(i)) \end{aligned} \tag{5}$$

so that (3) becomes

$$\mathbf{y} = \mathbf{C}\mathbf{u} + \mathbf{S}\mathbf{v} + \mathbf{z} \tag{6}$$

Assuming the noise is white, the noise vector $\mathbf{z}$ is multivariate Gaussian, $N_n(\mathbf{0}, \sigma^2\mathbf{I})$ and the maximum likelihood estimate (MLE) of $\boldsymbol{\beta}$ is given by

$$\hat{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}}(\omega) = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y} \tag{7}$$

leading directly to the MLE's of $\mathbf{u}$ and $\mathbf{v}$ by (4). The MLE's of the amplitude and phase are obtained via

$$\begin{aligned} \hat{A}_k &= (\hat{u}_k^2 + \hat{v}_k^2)^{1/2} \\ \hat{\phi}_k &= \arctan(-\hat{v}_k, \hat{u}_k) \end{aligned} \tag{8}$$

where arctan is the four quadrant arc tangent function (MATLAB function ATAN2).

Accurate approximate MLE solutions for $\mathbf{u}$ and $\mathbf{v}$ can be found for large n and ω not too close to 0 and $h_k\omega$ not too close to π. It is well-known (for example [2]) that in this case

$$\hat{\boldsymbol{\beta}} \approx \frac{2}{n}\mathbf{X}^T\mathbf{y} \tag{9}$$

without matrix inversion. The MLE of $\mathbf{u}$ and $\mathbf{v}$ can also be efficiently and simply estimated using the FFT of $\mathbf{y}$. The continuous Fourier transform (CFT) of $\mathbf{y}$, denoted by $Y(f)$ is

$$Y(f) = \sum_{i=1}^{n} e^{-2\pi j f t(i)} y_i \tag{10}$$

for $0 \leq f \leq 1/2$. The FFT or more precisely the discrete Fourier transform (DFT), calculates (10) only at the center frequencies $F_k = k/n$ of the Fourier bins for $k = 0, 1, \ldots, (n-1)$. Approximate estimates using the FFT can be obtained by rounding the frequencies to the nearest frequency bins.

$$\begin{aligned} \hat{u}_k &= (2/n)\mathrm{Re}(Y(\mathrm{round}(nh_k f)/n) \\ \hat{v}_k &= -(2/n)\mathrm{Im}(Y(\mathrm{round}(nh_k f)/n) \end{aligned} \qquad (11)$$

where round(x) is the nearest integer to x. The MLE of amplitude can be obtained by (8) but to account for using the center frequency of the Fourier bins rather than the exact frequencies, the estimate of phase is

$$\begin{aligned} \hat{\phi}_k &= \arctan(-\hat{v}_k, \hat{u}_k) \\ &- (nh_k f - \mathrm{round}(nh_k f))\pi(n-1)/n \end{aligned} \qquad (12)$$

which can be a correction of almost $\pi/2$ for harmonic frequencies that are half a bin away from the center. As discussed later, the correction in (12) is vital for accurate estimation of harmonic phase coupling. It also should be noted that the approximate estimate (9) is equivalent to using the CFT in (10) rather than the DFT which leads to

$$\begin{aligned} \hat{u}_k &= (2/n)\mathrm{Re}(Y(h_k f)) \\ \hat{v}_k &= -(2/n)\mathrm{Im}(Y(h_k f)) \end{aligned} \qquad (13)$$

which is just (11) without rounding. An important point about these approximate MLE's is they remain valid (perhaps requiring a larger n) in the colored noise case.

3. HARMONIC PHASE COUPLING

Using methods to estimate fundamental frequency described in [3], the least squares estimates of amplitude and phase for the data in Figure (1) were obtained. The estimated amplitudes of the third and sixth harmonic are shown in Figure (3) which, despite the variation in power as a function of range due to acoustic propagation effects, have a fairly constant difference. This suggests that the ratio of the third and sixth harmonic (the strongest line) is a more robust feature for target ID. For this reason, normalizing the amplitudes by the strongest line is part of the current target ID system

There also exists a phase coupling between the third and sixth harmonic as shown in Figure (4) which is a plot of the difference

$$d_{36} = \mathrm{mod}(2\phi_3 - \phi_6, 2\pi) \qquad (14)$$

versus time. The value of d_{36} is relatively constant over short time windows. However, the difference is not zero as predicted in some models, e.g. [8], and does appear to vary slightly over the entire time period. This apparent anomaly is perhaps due to propagation effects

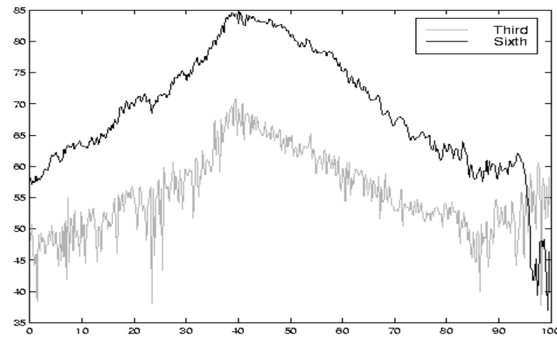

Figure 3: Harmonic Amplitude Estimates

and/or variations in the target's aspect or operation characteristics . To what extent differences such as in (14) are useful in target identification remains an open issue. In addition, there is also an issue of how accurate these differences can be estimated. One reason, the harmonic numbers 3 and 6 were chosen to show coupling is the simple 2 to 1 ratio between them that minimizes the variance of the corresponding difference. With harmonics 5 and 7, for example, the corresponding difference is $d_{57} = \mathrm{mod}(7\phi_5 - 5\phi_7, 2\pi)$ which, with all else being equal, would have a variance 18.5 that of (14). So for targets where 5 and 7 are dominant harmonic lines, the harmonic coupling will be more difficult to detect and track and may not be stable features.

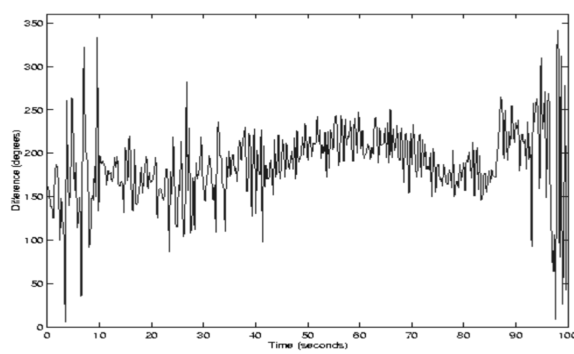

Figure 4: Phase Coupling between Third and Sixth Harmonic

The importance of using the corrected phase in (12) can be demonstrated in the calculation of (14). Without the correction, empirical results showed that the calculated differences were about 180 degrees off (i.e. the worst possible error) half of the time and nearly correct the other half. The reason for this is, assuming uniform distribution within the Fourier frequency bins, that half the time the error in the phase estimates are in opposite directions and add together destructively in (14) and half the time they cancel each other out.

4. CRAMER-RAO BOUNDS

The Cramer-Rao Bounds for the coupled harmonic parameters give insight into the practical matter of the accuracy of the features used for identification. The single sinusoid case, $m = 1$ and $h_1 = 1$, was first presented in [4] and is also discussed in detail in [2]. For large n and frequencies sufficiently separated, all the cross-terms in the information matrix for estimating arbitrary multiple frequencies involving different sinusoids are negligible ([7],[5]). Similarly, for the harmonically coupled case, the information matrix is approximately diagonal

$$I(\boldsymbol{\theta}) \approx \frac{n}{2\sigma^2} \begin{bmatrix} \mathbf{I} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{D} & 0 \\ 0 & 0 & \frac{n^2-1}{12} \sum_{k=1}^{m} h_k^2 A_k^2 \end{bmatrix} \tag{15}$$

where

$$\mathbf{D} = \mathrm{diag}(A_1^2, \ldots, A_m^2) \tag{16}$$

is a diagonal matrix Inverting (15) leads to the approximate CRB's for the amplitudes

$$\mathrm{Var}[\hat{A}_k] \geq \frac{2\sigma^2}{n} \tag{17}$$

the phases

$$\mathrm{Var}[\hat{\phi}] \geq \frac{1}{n\rho_k} \tag{18}$$

and fundamental frequency

$$\mathrm{Var}[\hat{\omega}] \geq \frac{12}{n(n^2-1)} \left(\sum_{k=1}^{m} h_k^2 \rho_k \right)^{-1} \tag{19}$$

where

$$\rho_k = A_k^2/(2\sigma^2) \tag{20}$$

is the SNR of the k^{th} sinusoid. As shown in ([1]) for example, an interesting result is that the CRB's for the colored Gaussian noise case are approximately equal to those above with $\sigma^2 = \sigma^2(h_k\omega)$ the local noise variance and $\rho_k = A_k^2/(2\sigma^2(h_k\omega)$ the local SNR.

For large n, the variance of the MLE's of amplitude, phase, and fundamental frequency are approximately equal to the CRB. Looking at (19), the variance of fundamental frequency goes down with the presence of high SNR tones with large harmonic numbers. For battlefield acoustics, this is not always the case because of increased propagation loss at higher frequencies. Also, the fundamental frequency can be estimated fairly accurately with small data sizes, but (18) shows that exploiting the harmonic phase coupling for target ID may require more time samples. However, because of the nonstationarity of the fundamental frequency it is not always possible to increase n arbitrarily and the accuracy of phase estimates is limited with stationary approaches. So, obtaining more accurate phase estimates with time-varying fundamental frequency is a subject for further research and presents an interesting problem in nonstationary signal processing.

5. REFERENCES

[1] M. Ghogho and A. Swami, "Lower bounds on the estimation of harmonics in colored noise," *Proc. Asilomar Conf on Sig Sys Comp, 1997.*

[2] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory* PTR Prentice Hall, Englewood Cliffs, New Jersey 07632, 1993.

[3] D. E. Lake "Efficient maximum likelihood estimation for multiple and coupled harmonics," submitted to *IEEE Trans. Signal Processing.*

[4] D.C. Rife and R.R. Boorstyn, "Single-tone parameter estimation from discrete-time observations," *IEEE Trans. Inform. Theory*, vol. IT-20,pp. 591-598, Sept. 1974.

[5] D.C. Rife and R.R. Boorstyn, "Multiple tone parameter estimation from discrete-time observations," *Bell Syst. Tech J.*, pp. 1389-1410, Nov. 1976.

[6] Srour, N. and Robertson, J., "Remote Netted Acoustic Detection System", Army Research Laboratory, ARL-TR-706, May 1995.

[7] P. Stoica, R. L. Moses, B. Friedlander, and T. Soderstrom, "Maximum likelihood estimation of the parameters of multiple sinusoids from noisy measurements," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 378-392, March 1989.

[8] A. Swami and M. Ghogho "Cramer-Rao bounds for coupled harmonics in noise," *Proc. Asilomar Conf on Sig Sys Comp, 1997.*

[9] Wellman M, Srour, N. and Hills D., "Acoustic Feature Extraction for a Neural Network Classifier", Army Research Laboratory, ARL-TR-1166, January 1997.