IMPROVING SIGNAL SUBSPACE ESTIMATION FOR BLIND SOURCE SEPARATION IN THE CONTEXT OF SPATIALLY CORRELATED NOISES.

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ABSTRACT

In this paper we address the issue of Orthogonal Techniques for Blind Source Separation of periodic signals when the mixtures are corrupted with spatially correlated noises. The noise covariance matrix is assumed to be unknown. This problem is of major interest with experimental signals. We first remind that the Principal Components Analysis (PCA), cannot provide a correct estimate of the signal subspace when the noises are spatially correlated or when their power spectral densities are different. We then introduce a new estimator of the unnoisy spectral matrix using delayed blocks. The only assumption is that the noise correlation and cross-correlation lengths must be shorter than the source correlation lengths. Simulation results show the efficiency of the new method.

1. INTRODUCTION

Blind Source Separation consists in recovering the signals emitted by p sources from n $(n \geq p)$ linear and stationary mixtures of these signals. The n sensors are receiving convolutive mixtures corrupted with additive noises. The observation vector $\underline{x}(t)$ is modelled as:

$$\underline{x}(t) = \sum_{i=1}^{p} \underline{h}_i(t) * s_i(t) + \underbrace{\sum_{j=1}^{n} \underline{g}_j(t) * n_j(t)}_{\underline{b}(t)} \quad (1)$$

• the $s_i(t)$ are periodic sources of different fundamental frequencies,
• the k-th component of $\underline{h}_i(t)$ is the impulse response characterizing the propagation from the i-th source to the k-th sensor,
• the elements of $\underline{b}(t)$ result from the filtering of n spectrally white gaussian noises $n_j(t)$.
• the sources are mutually independent and independent from the noises.

In the frequency domain, the convolutive mixture becomes an instantaneous mixture at each frequency bin:

$$\underline{X}(f) = \underbrace{\underline{\underline{H}}(f).\underline{S}(f)}_{\underline{Y}(f)} + \underline{B}(f) \quad (2)$$

$\underline{\underline{H}}(f)$ is the $n \times p$ matrix whose columns are the Fourier Transforms of vectors $\underline{h}_i(t)$. In order to simplify the notation, (f) will be omitted.
The orthogonal techniques use the PCA as a first step, to whiten the observations. This relies on projecting the observations on an orthonormal base of the signal subspace $\mathbb{E}_S$. Further separation is achieved with the use of 4-th order information [1] or joint diagonalisation [3] to find the exact base of the sources thanks to Givens' rotation matrices. Obviously, the efficiency of the whole process depends on the accuracy of the first step, since it provides an estimation of the number of sources (using MDL, AIC or other criteria) and a base of the signal subspace. In this paper we just address the signal subspace estimation issue. The whitening matrix is built with the eigenvalues and eigenvectors of $\underline{\underline{\gamma}}_Y = E\{\underline{Y}.\underline{Y}^+\}$ which is the spectral matrix of the unnoisy mixtures. Unfortunately $\underline{\underline{\gamma}}_Y$ is estimated from $\underline{\underline{\gamma}}_X$ ($\underline{\underline{\gamma}}_X = \underline{\underline{\gamma}}_Y + \underline{\underline{\gamma}}_B$) where $\underline{\underline{\gamma}}_B$ is unknown. When the noise spectral matrix is not proportional to the identity matrix (i.e. when the noises are not spatially white), $\underline{\underline{\gamma}}_Y$ is ill estimated. Consequently the source number and the signal subspace are not correctly estimated. The PCA looses efficiency. This problem is reminded in the second part of this paper. In the third part we introduce a new estimator of $\underline{\underline{\gamma}}_Y$, using delays to eliminate the noise influence. In the fourth part we choose a criterion of distance to the signal subspace and the performance of the proposed method is compared with the PCA.

2. MISMATCHING OF THE USUAL PCA IN SEVERE NOISE CONDITIONS

The vector of unnoisy mixtures can be written as:

$$\underline{Y} = \underline{\underline{H}}'.\underline{S}' \quad \text{with} \quad E\{\underline{S}'.\underline{S}'^{+}\} = \underline{\underline{I}}_p \tag{3}$$

$^{+}$ stands for the transconjugate and $\underline{\underline{I}}_p$ for the identity matrix of rank p. The Singular Value Decomposition of $\underline{\underline{H}}'$ is:

$$\underline{\underline{H}}' = \underline{\underline{V}}.\underline{\underline{D}}^{1/2}.\underline{\underline{\Pi}} \tag{4}$$

- $\underline{\underline{V}}$ and $\underline{\underline{\Pi}}$ are two unitarian matrices respectively $n \times n$ and $p \times p$,
- $\underline{\underline{D}}^{1/2}$ is a $n \times p$ diagonal matrix with elements $\sqrt{\lambda_i}, \quad i = 1 \dots p$. When the first mixing matrix $\underline{\underline{H}}$ is unitarian, the λ_i are exactly the Power Spectral Densities (PSD) of the sources at the frequency f.

The Eigenvalue Decomposition of $\underline{\underline{\gamma}}_Y$ can be written, using the singular elements of $\underline{\underline{H}}'$:

$$\underline{\underline{\gamma}}_Y = \underline{\underline{V}}.\underline{\underline{D}}^{1/2}.\underline{\underline{\Pi}}.\underline{\underline{\Pi}}^{+}.\underline{\underline{D}}^{1/2^{+}}.\underline{\underline{V}}^{+} = \underline{\underline{V}}.\underline{\underline{D}}.\underline{\underline{V}}^{+} \tag{5}$$

$$\underline{\underline{D}} = \text{diag}(\lambda_1, \dots, \lambda_p, 0, \dots, 0) \tag{6}$$

The eigenvalues λ_i are assumed to be ranged in decreasing order. The first p eigenvalues and their corresponding eigenvectors are representative of $\mathbb{E}_{\mathbf{S}}$.

The PCA consists in projecting $\underline{X}$ on an orthonormal base of the signal subspace with a matrix $\underline{\underline{W}}$ verifying

$$\underline{\underline{W}}.\underline{\underline{\gamma}}_Y.\underline{\underline{W}}^{+} = \underline{\underline{I}}_p \tag{7}$$

Let us denote $\underline{\underline{D_s}}$ the square sub-matrix containing the first p diagonal elements of $\underline{\underline{D}}$ and $\underline{\underline{V_s}}$ the rectangular submatrix containing the first p columns of $\underline{\underline{V}}$. The whitening matrix is equal to

$$\underline{\underline{W}} = \underline{\underline{D_s}}^{-1/2}.\underline{\underline{V_s}}^{+} \tag{8}$$

Unfortunately, in a noisy context, one can only access to $\underline{\underline{\gamma}}_X$. When the noises are spatially white, the noise spectral matrix is on the form (9) and the Eigenvalue Decomposition of $\underline{\underline{\gamma}}_X$ provides the same eigenvectors as for $\underline{\underline{\gamma}}_Y$ (10).

$$\underline{\underline{\gamma}}_B = \sigma_b^2.\underline{\underline{I}}_n \tag{9}$$

$$\underline{\underline{\gamma}}_X = \underline{\underline{V}}.\underbrace{(\underline{\underline{D}} + \sigma_b^2.\underline{\underline{I}}_n)}_{\underline{\underline{\Omega}}}.\underline{\underline{V}}^{+} \tag{10}$$

In these conditions, σ_b^2 can be estimated from the $n-p$ last eigenvalues of $\underline{\underline{\Omega}}$ and substracted to the p first eigenvalues to get an estimate of $\underline{\underline{D_s}}$.

With experimental signals $\underline{\underline{\gamma}}_B$ is hardly ever on the form of (9) and the factorization (10) is not possible anymore. Consequently, in the Eigenvalue Decomposition of $\underline{\underline{\gamma}}_X$ (11), the first p column vectors of $\underline{\underline{U}}$ don't span only the signal subspace but a p dimensional subspace in the $n+p$ dimensional space spanned by the signal and the noise. Moreover the eigenvalues are ill estimated too and algorithms (such as AIC, MDL,...) for estimating the source number fail. Consequently the whitening matrix $\underline{\underline{W}}$ is ill estimated.

$$\underline{\underline{\gamma}}_X = \underline{\underline{U}}.\underline{\underline{\Delta}}.\underline{\underline{U}}^{+} \quad \text{with} \quad \begin{cases} \underline{\underline{U}} & \neq & \underline{\underline{V}} \\ \underline{\underline{\Delta}} & \neq & \underline{\underline{\Omega}} \end{cases} \tag{11}$$

The conclusion is that the method used for estimating $\underline{\underline{\gamma}}_Y$ is not appropriate in severe noise conditions. In the case of periodic sources, we propose in the next section, a new estimator of $\underline{\underline{\gamma}}_Y$, more robust to spatially correlated noise.

3. A DIRECT ESTIMATOR OF THE UNNOISY SPECTRAL MATRIX

The spectral matrix $\underline{\underline{\gamma}}_X$ is estimated from the N-point Discrete Fourier Transform of $\underline{x}$ on M sliding blocks. In the case of periodic signals it seems interesting to exploit the fact that the autocorrelation lengths of the sources are larger than the correlation lengths of all the noises. Let τ_b be the greater correlation or cross-correlation length of the n noises. Let $\underline{X}$ be the DFT of $\underline{x}$ on a temporal block and $\underline{X}^{\tau}$ the DFT on a block delayed of τ samples. If $\tau \geq \tau_b$, at each frequency bin, the covariance matrix (12) contains only information about the sources.

$$\underline{\underline{\gamma}}_X^{\tau} = E\{\underline{X}.\underline{X}^{\tau+}\} \tag{12}$$

Suppose that the i-th source has an harmonic frequency f_i close to the analysis frequency. After the source normalization introduced in the previous section $\underline{\underline{\gamma}}_X^{\tau}$ can be written as:

$$\underline{\underline{\gamma}}_X^{\tau} = \underline{\underline{H}}'.\underbrace{E\{\underline{S}'.\underline{S}'^{\tau+}\}}_{\underline{\underline{\theta}}}.\underline{\underline{H}}'^{+} \tag{13}$$

$$\underline{\underline{\theta}} = \begin{pmatrix} e^{-j2\pi f_1 \tau} & & 0 \\ & \ddots & \\ 0 & & e^{-j2\pi f_p \tau} \end{pmatrix} \tag{14}$$

From (4) and (13) we get:

$$\underline{\underline{\gamma}}_X^{\tau} = \underline{\underline{V}}.\underline{\underline{D}}^{1/2}.\underline{\underline{\Pi}}.\underline{\underline{\theta}}.\underline{\underline{\Pi}}^{+}.\underline{\underline{D}}^{1/2^{+}}.\underline{\underline{V}}^{+} \tag{15}$$

It is then theoretically possible to find back the Eigenvalue Decomposition of $\underline{\underline{\gamma}}_Y$ (5) with the use of a second spectral matrix obtained with delay -2τ. The final relation is:

$$\underline{\underline{\gamma}}_Y \quad = \quad \underline{\underline{\gamma}}_X^{\tau}.(\underline{\underline{\gamma}}_X^{\tau\,+})^{-1}.\underline{\underline{\gamma}}_X^{-2\tau} \tag{16}$$

We must pay attention to the fact that this expression involves the inverse of a matrix of size n but rank p. In a practical consideration, stability is improved using the pseudo-inverse algorithm, so that only the non trivial eigenvalues of $\underline{\underline{\gamma}}_X^{\tau\,+}$ are inversed.

4. DISTANCE TO THE SIGNAL SUBSPACE - SIMULATIONS RESULTS

As we said in section 2, the efficiency of PCA relies on the estimation of the source number and the estimation of an orthonormal base of $\mathbb{E}_S$. We now need a criterion to measure jointly the accuracy of the estimated eigenvalues and the closeness to the signal subspace. Denote $||.||$ the matrix 2-norm, and $\widehat{\underline{\underline{\gamma}}_Y}$ the estimate of $\underline{\underline{\gamma}}_Y$. The distance $||\underline{\underline{\gamma}}_Y - \widehat{\underline{\underline{\gamma}}_Y}||$ is not appropriate since $\widehat{\underline{\underline{\gamma}}_Y}$ can be close to $\underline{\underline{\gamma}}_Y$ without the good eigenvalues and eigenvectors. Consequently the criterion must rely on the whitening matrix $\underline{\underline{W}}$. The usual rejection rates referred in [2] cannot be used here since the rotation matrix $\underline{\underline{\Pi}}$ is undetermined after the PCA. We must pay attention to the fact that the estimated whitening matrix $\widehat{\underline{\underline{W}}}$ is not uniquely determined: it can be left multiplied by a unitarian matrix [4]. Denote $\widehat{\underline{\underline{W}}} = \underline{\underline{\tilde{D_s}}}^{-1/2}.\underline{\underline{\tilde{V_s}}}^{+}$. If the column vectors in $\underline{\underline{\tilde{V_s}}}$ form an orthonormal base of $\mathbb{E}_S$ then $\underline{\underline{\tilde{V_s}}}^{+}.\underline{\underline{V_s}}$ is close to a unitarian matrix $\underline{\underline{P}}$. When the mixing matrix $\underline{\underline{H}}$ is unitarian, $\underline{\underline{P}}$ is diagonal. In this situation, when the estimated eigenvalues are close to the real ones, the norm of (17) is close to 1 (norm of $\underline{\underline{P}}$).

$$\widehat{\underline{\underline{W}}}.\underline{\underline{W}}^{\#} = \underline{\underline{\tilde{D_s}}}^{-1/2}.\underline{\underline{\tilde{V_s}}}^{+}.\underline{\underline{V_s}}.\underline{\underline{D_s}}^{1/2} \tag{17}$$

$\#$ denotes the pseudo-inverse. This considerations leads to the following distance criterion:

$$d(\widehat{\underline{\underline{W}}}, \underline{\underline{W}}) = | \quad ||\widehat{\underline{\underline{W}}}.\underline{\underline{W}}^{\#}|| - 1 \quad | \tag{18}$$

We show simulation results on figure 1, 2 and 3. Two sources are mixed and observed on 6 sensors. Each source is composed of 2 pure frequencies (0.14,0.36 and 0.15,0.37). The mixture is obtained from AR1 filters with coefficients in the range $[0, 1]$ so that some filters are low-pass and others are high-pass. The spatially correlated and spectrally colorated noises result from the filtering of white noises with AR1 filters too. As shown on the table below the noise power spectral densities on the frequency bin 0.15 are different on every sensor and the corresponding Signal to Noise Ratios are about -5 dB.

sensors	Noise PSD	SNR in dB
1	0.17	-5.01
2	0.32	-5.03
3	0.14	-5.01
4	0.21	-5.02
5	0.43	-5.00
6	0.09	-5.00

Computations are processed on 612 sliding blocks of 64 samples with $\tau = 70$ samples. The sliding step is 16 samples. Denote $\underline{\underline{\gamma_1}}$ and $\underline{\underline{\gamma_2}}$, the estimation of $\underline{\underline{\gamma}}_Y$ respectively with the usual PCA method, and the new method involving the delay τ. $d1$ and $d2$ are the corresponding distances to the signal subspace. In this simulation we assume that the number of sources is known, because we just study the signal subspace estimation. In figure 1 one can see that the eigenvalues of $\underline{\underline{\gamma_1}}$ in dotted line are very far from the eigenvalues of $\underline{\underline{\gamma}}_Y$ in solid line whereas in figure 2, the eigenvalues of $\underline{\underline{\gamma_2}}$ are very close to the eigenvalues of $\underline{\underline{\gamma}}_Y$. The distance $d2$ to the signal subspace is much lower than the distance $d1$ as shown in figure 3.

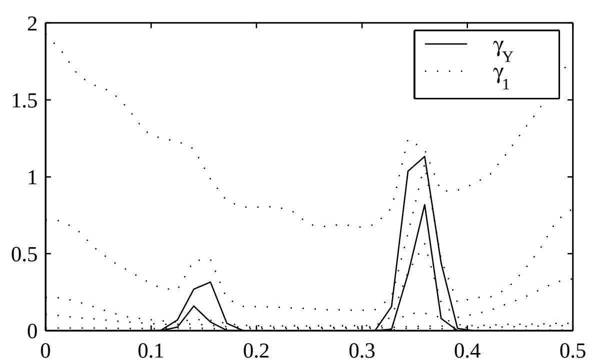

Figure 1: eigen values of $\underline{\underline{\gamma}}_Y$ and $\underline{\underline{\gamma_1}}$

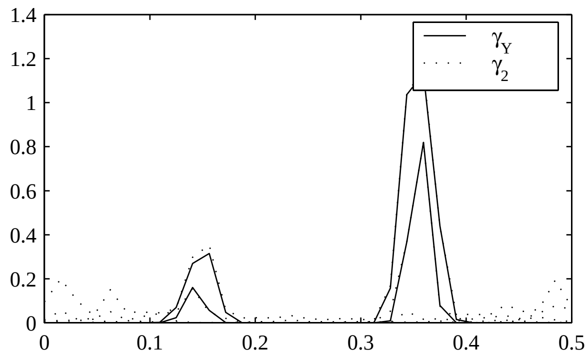

Figure 2: eigen values of $\underline{\underline{\gamma}}_Y$ and $\underline{\underline{\gamma_2}}$

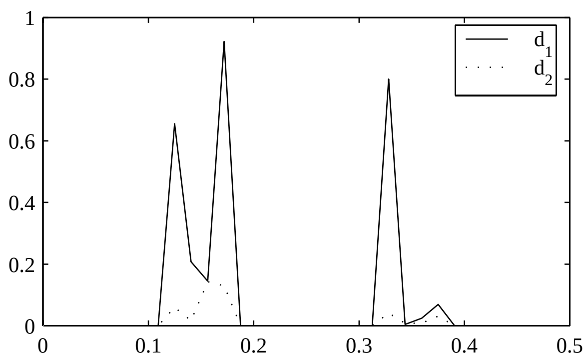

Figure 3: distance to $\mathbb{E}_S$ estimated from $\underline{\underline{\gamma_1}}$ and $\underline{\underline{\gamma_2}}$

5. CONCLUSIONS

In this paper we study the Principal Component Analysis issue in the context of spatially correlated noises when the covariance matrix is unknown. This approach is of major interest in Blind Source Separation of experimental signals. In this context the PCA fails because the spectral matrix of unnoisy mixtures is ill-estimated. We propose, in the case of periodic sources, a new estimator of this matrix computed from two interspectral matrices using two different delays. We choose a distance criteria to the signal subspace and show the efficiency of the method with a simulation in severe conditions (Signal to Noise Ratio around -5dB, spatially correlated and spectrally colorated noises). The results are encouraging but some further work has to be done to avoid the choice of a threshold (for the pseudo-inverse) and to apply the method to experimental signals.

6. REFERENCES

[1] M. Gaeta, J.L. Lacoume, 'Source separation without a priori knowledge : the maximum likelihood solution', in *Signal Processing*, pp 621-624, 1990.

[2] J.F. Cardoso, 'On the performance of orthogonal source separation algorithms', in *Proc. EUSIPCO*, pp 776-779, 1994.

[3] A. Belouchrani, K. Abed-Meraim, J.F. Cardoso and E. Moulines, 'A blind source separation technique using second order statistics', in *IEEE Trans. signal processing*, Vol. 45, pp 434-444, February 1997.

[4] J.F. Cardoso, A. Souloumiac, 'An efficient technique for the blind separation of complex sources', in *Proc. IEEE SP Workshop on Higher Order Stat.*, pp 275-279, 1993.