BAYESIAN ANALYSIS FOR THE FAULT DETECTION OF THREE-PHASE INDUCTION MACHINE

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ABSTRACT

One of the most widely used techniques for obtaining information on the health state of three-phase induction machines is based on the processing of stator current. In fact, in the case of steady state operations, anomalous current spectral components, that increase if a fault occurs, allow to diagnose the presence and, in some case, the type of fault. In this paper, a Bayesian approach is proposed using a simulation technique, the Markov chain Monte Carlo (MCMC), to estimate the amplitude of some spectral components modified by machine faults and the slip, a parameter related to the load conditions, with a view to automatically detecting faults. Results on real stator current waveform are given.

1. INTRODUCTION

The problem treated in this paper is the monitoring of a three-phase induction machine with a view to automatically detecting faults.

In the literature, several parameters within this motor have been monitored by researchers as a means of obtaining useful healthy information : axial vibrations, axially directed flux [2]. But, one of the most widely used techniques for obtaining information on the health state of three-phase induction machines is based on the processing of stator current. In fact, in the case of steady state operations, anomalous current spectral components, that increase if a fault occurs, allow to diagnose the presence and, in some case, the type of fault [3]. Stator current monitoring is usually based on the classical spectral analysis using FFT algorithm.

In this paper, a Bayesian approach using simulation method is proposed to take into account a priori knowledge about stator current and to eliminate all the unmodified parameters. Since all the frequencies present in the stator current depend linearly on the slip, a parameter related to the load, our objective is to estimate it and the amplitudes modified by machine fault.

The paper is organized as follows. In section 2, some results about stator current spectral analysis are introduced and a general spectral characterization is given allowing to propose a stator current model. In section 3, the Bayesian solution to estimate parameters under interest using the *one-variable-at-a-time random walk* Metropolis-Hastings (M-H) algorithm is described. Finally, the proposed method is performed on real stator current data.

2. STATOR CURRENT ANALYSIS

First, study conditions of the stator current for the supposed healthy three-phase induction machine are given. In order to facilitate successful detection of machine faults, the motor have been monitored whilst running under full load conditions during steady state operation. Therefore, the slip s is constant. From [1], [5], we can notice that all the principal frequencies present in the stator current are defined in function of some known physical machine parameters like number of pole pairs or number of rotor slots, f_0 the known supply frequency and the slip s assumed to be unknown. That allows to ensure the stationarity of the treated sequence of stator current samples.

The principal spectral components of the stator current are given for the normalized supply frequency $f_0 = 0.05$ and $s = 0.04$ in fig. (1) and (2) (see [7] for more details). It is made of the supply frequency harmonics and spectral components depending on s. Fig. (2) is a zoom around f_0 where $x = \frac{(1-s)f_0}{2}$ and $y = 2sf_0$.

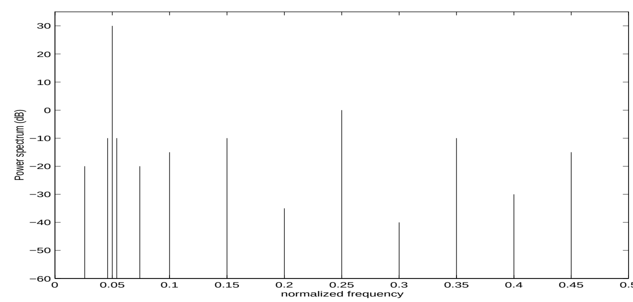

Figure 1: Principal spectral components of stator current

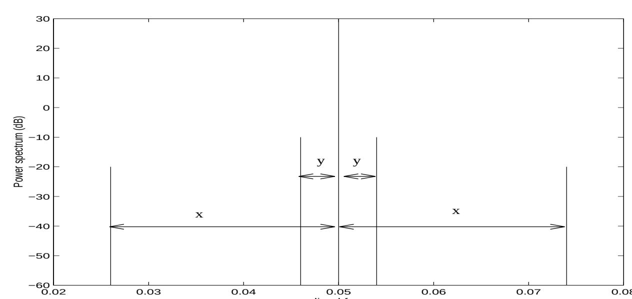

Figure 2: Zoom around the supply frequency, $f_0 = 0.05$

Motor fault modifies the spectrum of the healthy motor by changing amplitude of some spectral components and these characteristic frequencies are coupled to particular faults, they have been determined for air gap eccentricities, broken bars in the rotor cage, rolling-element bearing failures and electrical based faults like stator voltage unbalance [7].

Therefore, with a view to detecting faults, a stator current model made of M superimposed sinusoids in additive Gaussian noise is proposed. The sinusoids frequencies are related linearly to the slip s and p of them correspond with p sinusoids amplitudes modified by machine faults.

3. BAYESIAN APPROACH

3.1. Data Model

Let $\mathbf{d} = \{d_0, \cdots, d_{N-1}\}$ be a sequence of N observed stator current samples which is assumed to be made of superimposed sinusoids in additive noise, that is :

$$d(n) = \sum_{k=1}^{p} a_{ck}\cos(\omega_k n) + a_{sk} sin(\omega_k n) + \sum_{k=p+1}^{M} a_{ck}\cos(\omega_k n) + a_{sk} sin(\omega_k n) + e(n), \quad (1)$$

where $e(n)$ is a sample of a zero-mean, i.i.d., white, Gaussian noise with variance σ^2 unknown. $a_{ck} = A_k\cos(\phi_k)$, $a_{sk} = -A_k\sin(\phi_k)$ where A_k and ϕ_k are, respectively, the unknowns amplitude and phase of the k^{th} sinusoid.
ω_k is the frequency of the k^{th} sinusoid defined by :

$$\omega_k = \alpha_k s + \beta_k$$

, where α_k and β_k are known, α_k being equal to zero for some frequencies and s is the parameter to estimate.

The amplitudes of the first p sinusoids are to estimate and the amplitudes of the $(M-p+1)$ remaining ones are to eliminate.

Model (1) can be written in the following matrix equation:

$$\mathbf{d} = \mathbf{Ga} + \mathbf{Db} + \mathbf{e}, \quad (2)$$

where $\mathbf{e}$ is an $N \times 1$ vector of Gaussian noise samples, $\mathbf{a}$ is an $2p \times 1$ vector of sinusoid amplitudes to estimate such as

$$\mathbf{a}^T = \begin{bmatrix} a_{c1} & a_{s1} & a_{c2} & a_{s2} & \cdots & a_{cp} & a_{sp} \end{bmatrix}.$$

$\mathbf{G}$ is an $N \times 2p$ matrix defined by :

$$\mathbf{G} = \begin{bmatrix} \mathbf{f}_{c1} & \mathbf{f}_{s1} & \mathbf{f}_{c2} & \mathbf{f}_{s2} & \cdots & \mathbf{f}_{sp} \end{bmatrix},$$

where

$$\mathbf{f}_{ck}^T = \begin{bmatrix} 1 & \cos(\omega_k) & \cdots & \cos(\omega_k(N-1)) \end{bmatrix},$$

$$\mathbf{f}_{sk}^T = \begin{bmatrix} 0 & \sin(\omega_k) & \cdots & \sin(\omega_k(N-1)) \end{bmatrix},$$

$\mathbf{b}$ is an $2(M-p+1)\times 1$ vector of sinusoid amplitudes to eliminate by integration such as :

$$\mathbf{b}^T = \begin{bmatrix} a_{c(p+1)} & a_{s(p+1)} & \cdots & a_{cM} & a_{sM} \end{bmatrix},$$

and $\mathbf{D}$ is an $N \times 2(M-p+1)$ matrix defined by:

$$\mathbf{D} = \begin{bmatrix} \mathbf{f}_{c(p+1)} & \mathbf{f}_{s(p+1)} & \cdots & \mathbf{f}_{cM} & \mathbf{f}_{sM} \end{bmatrix}.$$

3.2. Bayesian solution

The objective is to estimate the slip s and sinusoids amplitude vector $\mathbf{a}$. In the Bayesian framework, two principal steps are necessary:

- First, to determinate the analytic expression of the posterior density of s and $\mathbf{a}$ only given the data $\mathbf{d}$ and the prior information I, $p(s, \mathbf{a}|\mathbf{d}, I)$.
- Second, to evaluate the statistics of interest from the posterior density like posterior mean (MMSE estimator) or posterior maximum (MAP estimator).

3.2.1. Computation of the posterior density

The likelihood of data is given by the joint probability of the noise samples :

$$p(\mathbf{d}|\mathbf{a}, \mathbf{b}, \sigma, s, I) = (2\pi\sigma^2)^{-\frac{N}{2}} \times \exp\left[-\frac{1}{2\sigma^2}(\mathbf{d} - \mathbf{Ga} - \mathbf{Db})^T(\mathbf{d} - \mathbf{Ga} - \mathbf{Db})\right]. \quad (3)$$

From Bayes' theorem, the joint posterior probability density of all of the parameters, $p(\mathbf{a}, \mathbf{b}, \sigma, s|\mathbf{d}, I)$, is :

$$p(\mathbf{a}, \mathbf{b}, \sigma, s|\mathbf{d}, I) = \frac{p(\mathbf{d}|\mathbf{a}, \mathbf{b}, \sigma, s, I)p(\mathbf{a}, \mathbf{b}, \sigma, s|I)}{p(\mathbf{d}|I)}, \quad (4)$$

where $p(\mathbf{a}, \mathbf{b}, \sigma, s|I)$ corresponds with prior density of the parameters given prior information I and $p(\mathbf{d}|I)$ is the normalization constant. As $(\mathbf{a}, \mathbf{b}, s, \sigma)$ are i.i.d, we can write :

$$p(\mathbf{a}, \mathbf{b}, \sigma, s|I) \propto p(\mathbf{a}|I)p(\mathbf{b}|I)p(s|I)p(\sigma|I) \quad (5)$$

The priors we have chosen are as follows :

- Non informative uniform priors for each of the elements of $\mathbf{a}$ and $\mathbf{b}$,
  $p(a_i|I) = k_i \; i = 1, \cdots, p$ where k_i are constants.
  $p(b_i|I) = l_i \; i = 1, \cdots, M-p+1$ where l_i are constants.
- Jeffrey's prior for σ : $p(\sigma|I) \propto \frac{1}{\sigma}$
- Informative Gaussian prior for s:

  $$p(s|I) = \frac{1}{\sigma_s^2}\exp(-\frac{1}{2\sigma_s^2}(s-\mu_s)^2) \quad (6)$$

  where μ_s and σ_s^2 are given by the experiment.

The nuisance parameters $\mathbf{b}$ and σ are eliminated by integration:

$$p(s, \mathbf{a}|\mathbf{d}, I) = \int_0^{\infty}\int_{-\infty}^{\infty} p(\mathbf{a}, \mathbf{b}, \sigma, s|\mathbf{d}, I)d\sigma d\mathbf{b}. \quad (7)$$

After computation, the posterior density of s and $\mathbf{a}$ has the following expression:

$$p(s, \mathbf{a}|\mathbf{d}, I) \propto \frac{1}{\sigma_s^2}\exp(-\frac{1}{2\sigma_s^2}(s-\mu_s)^2) \times \frac{\left[(\mathbf{d} - \mathbf{Ga})^T\mathbf{Q}(\mathbf{d} - \mathbf{Ga})\right]^{-\frac{(N-2(M-p+1))}{2}}}{\sqrt{\det(\mathbf{D}^T\mathbf{D})}} \quad (8)$$

where $\mathbf{Q} = \mathbf{I}_N - \mathbf{D}(\mathbf{D}^T\mathbf{D})^{-1}\mathbf{D}^T$ with $\mathbf{I}_N$ is the $(N\times N)$ identity matrix.

From (8), the evaluation of the posterior mean or the posterior maximum are required but the high dimension and the complexity of the posterior density make its exploitation not possible by exact analytic approach and difficult by conventional numerical methods.

The proposed solution is to generate samples from this posterior density by using a MCMC algorithm, straightforward to implement and not requiring the knowledge of the normalization constant

3.2.2. *Simulation using Metropolis-Hastings algorithm*

A MCMC method is a simulation technique that generate a sample from a target distribution $\pi(.)$ by specifying the transition probability of a Markov process. The Markov chain is then iterated a large number of times in computer-generated Monte Carlo simulation, see [4].

One of the usefulness MCMC method is the Metropolis-Hastings (M-H) algorithm. The transition kernel for the M-H chain is defined by:

$$P_{MH}(x,dy) = q(x,y)\alpha(x,y)dy + \left[1 - \int_{\mathcal{R}} q(x,y)\alpha(x,y)dy\right]\delta_x(dy), \quad (9)$$

where $\alpha(x,y)$ is the probability of move from x to y and is given by:

$$\alpha(x,y) = \begin{cases} \min[\frac{\pi(y)\,q(y,x)}{\pi(x)\,q(x,y)}, 1] & \text{if } \pi(x)q(x,y) > 0 \\ 1 & \text{otherwise} \end{cases} . \quad (10)$$

the density $q(x,y)$ is the candidate generating density straightforward to simulate. In our case, $q(x,y) = q_1(x-y)$ where $q_1(.)$ is a multivariate density such as the candidate y is drawn according to the process $y = x + z$, where z is called the increment random variable and follows q_1.

This *random walk M-H* algorithm is relevant in our problem since it does not require the precise location of the target density (eq. (8)).

Moreover, the *one-variable-at-a-time* version of this algorithm combining $(1+2p)$ updates at each iteration is proposed. In fact, it is easier to find several conditional kernels that converge to their respective conditional densities than to find one kernel that converges to the joint one. Convergence of this version is faster than one of the classical version of the M-H algorithm (see [6] for more details).

The *one-variable-at-a-time random walk* M-H method has been implemented following the algorithm:

find the initial values : $\theta^{(0)} = [s^{(0)}, \mathbf{a}^{(0)}]$
for $k = 1...T$ do :
  for $i = 1,..,2p+1$
    generate y_k from $q_i(y_k - \theta_i^{(k)})$
    generate u from $\mathcal{U}$(1,0)
    $\alpha_i(\theta_i^{(k)}, y_k) = \min\left\{1, \frac{p(y_k|\theta^{(k)}-\theta_i^{(k)},\mathbf{d},I)}{p(\theta_i^{(k)}|\theta^{(k)}-\theta_i^{(k)},\mathbf{d},I)}\right\}$
    if $u \leq \alpha_i(\theta_i^{(k)}, y_k)$ then
      $\theta_i^{(k+1)} = y_k$
    else
      $\theta_i^{(k+1)} = \theta_i^{(k)}$
    endif
  end
end

where $q_i(y_k - \theta_i^{(k)})$ is the candidate-generating density of the i^{th} parameter of θ and has been chosen as an independent normal univariate distribution of known variance δ_i^2 and zero mean.

After a sufficient number of iterations T, the generated samples θ_i are distributed following $p(\theta_i^{(k)}|\mathbf{d}, I)$ and posterior means are computed from them by:

$$\hat{\theta}_i = \frac{1}{T-T_0}\sum_{k=T_0}^{T}\theta_i^{(k)}. \quad (11)$$

4. RESULTS

In order to test the proposed method, laboratory experiments were performed with a 4 kW induction motor (a) for the supposed healthy machine and (b) for the machine where stator voltages are unbalanced by adding a 0.2 p.u. resistance to one phase, stator current data are plotted in fig. (3).

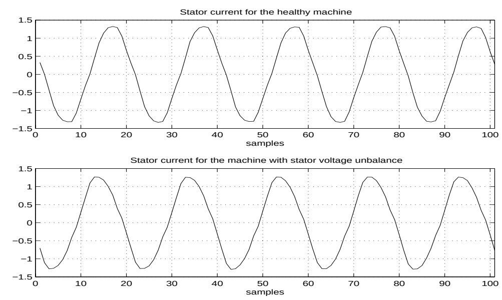

Figure 3: stator current data for the healthy and the damaged machine

The normalized supply frequency is $f_0 = 0.05$ and the slip provided by the experimenter is $s = 0.037$. The principal spectral component modified by this fault is the third harmonic of the supply frequency whose amplitude increases in significant way (see [7] for more details).

Sequences of N=100 stator current samples are assumed to be compounded of $M = 13$ principal spectral components. Amplitude of the third harmonic of f_0 has been monitored ($p = 1$). The prior parameters of (6) have been fixed at $\mu_s = 0.037$, $\sigma_s = 0.004$ and the standard deviation of $q_i(y_k - \theta_i^{(k)})$ $i = 1,2,3$ are $\delta_1 = 0.007$, $\delta_2 = 0.002$, $\delta_3 = 0.002$.

In the sampling process, the first 1000 draws have been ignored and we collect the next 4000 (T=5000) ones to evaluate the posterior mean. The posteriors mean and variance of the generated samples for A_1 and s in each case (a) and (b) are given in the following array:

	(a)		(b)	
	mean	variance	mean	variance
A_1	1.16e-2	9.86e-7	6.84e-2	1.17e-6
s	0.0329	1.58e-5	0.0335	1.60e-5

Figures (4) and (5) show respectively the estimation of $p(A_1|s, \mathbf{d}, I)$ and $p(s|a_{c1}, a_{s1}, \mathbf{d}, I)$ in the healthy case whereas figures (6) and (7) show the estimation of $p(A_1|s, \mathbf{d}, I)$ and $p(s|a_{c1}, a_{s1}, \mathbf{d}, I)$ in the damaged case.

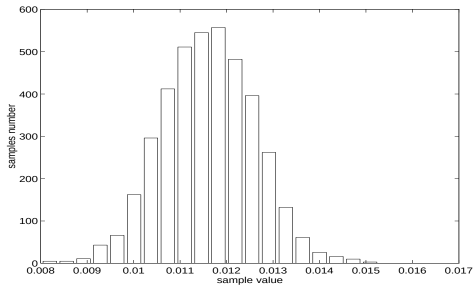

Figure 4: Estimation of $p(A_1|s, \mathbf{d}, I)$ for the healthy machine

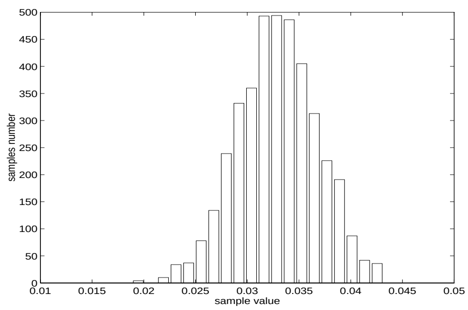

Figure 5: Estimation of $p(s|a_{c1}, a_{s1}, \mathbf{d}, I)$ for the healthy machine

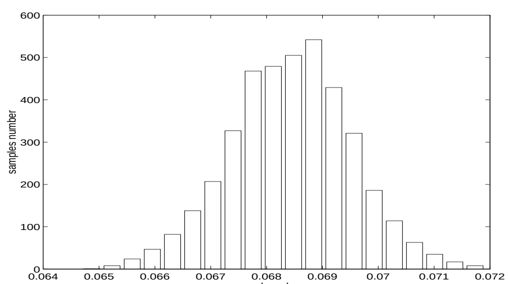

Figure 6: Estimation of $p(A_1|s, \mathbf{d}, I)$ for the damaged machine

The results obtained in each case for estimating the stator current parameters under study have been satisfactory. In fact, the stator current model choice is justified by two remarks : First, the slip estimation gives similar results in the healthy and damaged cases and, second, the estimated third harmonic amplitude A_1 has been increased in significant way in presence of the stator voltage unbalance.

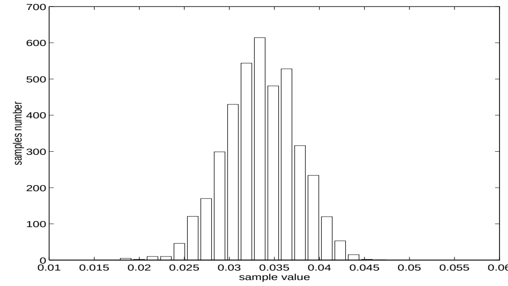

Figure 7: Estimation of $p(s|a_{c1}, a_{s1}, \mathbf{d}, I)$ for the damaged machine

5. CONCLUSION

The problem we have addressed is the fault detection of three-phase induction machine. A Bayesian approach associated with a MCMC algorithm has been proposed to analyse the stator current represented by noisy superimposed sinusoids. This algorithm allows to take into account a priori information on the data, given by the experiment, and to analyze only the components modified by the fault. It has been successfully applied on real stator current data, giving a first step to fault diagnosis.

6. REFERENCES

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