RANK ORDER POLYNOMIAL DECOMPOSITION FOR IMAGE COMPRESSION

Olivier Egger[1], Reto Grüter, Jean-Marc Vesin and Murat Kunt

Signal Processing Laboratory, Swiss Federal Institute of Technology, Lausanne
[1] currently with the Audio Visual Communications Lab (LCAV)
1015 Lausanne, Switzerland

ABSTRACT

In this paper, a novel decomposition scheme for image compression is presented. It is capable to apply any nonlinear model to compress images in a lossless way. Here, a very efficient polynomial model that considers spatial information as well as order statistic information is introduced. This new rank order polynomial decomposition (ROPD) that allows also for a progressive bitstream is applied to various images of different nature and compared to the morphological subband decomposition (MSD) and to the best prediction mode for lossless compression of the international standard JPEG. For all compressed images, ROPD provides better compression results than MSD and clearly outperforms the lossless mode of JPEG.

1. INTRODUCTION

Various image compression applications require a lossless representation of the reconstructed image. A lossless compression is often required for medical applications. Usually, a diagnostic on a digitized image must be done on the original image. This type of compression does not permit a high compression ratio. However, the possibility of browsing is a very efficient way of saving time if the pictures are located on a distant server. Indeed, a lossless representation of the image is certainly not necessary for all the images which are displayed. Therefore, the possibility of progressively decoding a picture adds a useful functionality for these compression schemes.

In this paper, a novel nonlinear decomposition scheme is presented, allowing for progressive coding of the bitstream. After broad experiments in which various nonlinear models have been examined, a novel polynomial expansion model that considers spatial information as well as morphological information of an image has been defined. The so-called rank order polynomial decomposition (ROPD) scheme is applied to various images and is compared to the lossless mode of JPEG as well as to the morphological subband decomposition (MSD). Further improvement of the compression quality of ROPD is shown on an example of an ultrasound image where arbitrarily–shaped regions are processed.

2. THE POLYNOMIAL PREDICTION METHOD

In our decomposition framework presented in the next section any nonlinear prediction method can be applied. In this context, we have compared various nonlinear approximations such as multilayer perceptrons (MLP) [6], radial basis function (RBF) networks [4] and polynomial approximations [5]. Although the neural networks have provided good prediction results on the test set used, their model complexity turned out to be enormous. On the other hand, much simpler models with fewer parameters but similar approximation qualities have been found using polynomial approximation. Moreover, any function can be arbitrarily well approximated by polynomial models. In this context, the goal is to predict an image pixel value as a function of its neighboring pixel values. The residual error constitutes the high subband in the decomposition scheme. In order to reduce this approximation error a suitable polynomial model must be built. In the following, we propose an efficient method based on an orthogonal search procedure to search for the best polynomials.

In the context of prediction of an image pixel value we want to consider a simplified auto–regressive with exogenous inputs (NARX) model as introduced in [5] as follows:

$$x(n) = \sum_{m=0}^{M} a_m p_m(n) + e(n) \quad n = 1, \ldots, N \qquad (1)$$

where $x(n)$ is the value to predict, $\{p_m(n)\}$ are polynomials (terms) of $x(n-1)$, …, $x(n-D)$ that constitute the neighboring pixel values, $\{a_m\}$ are the unknown parameters, and $e(n)$ is the modeling error.

The image structure is a priori unknown. The number of possible models for polynomials of a higher order grows exponentially and it is therefore not appropriate to apply an exhaustive search method. We propose to search for the best fitting polynomials by starting with the simplest model and expanding it by adding further polynomials. First we test all possible models with one polynomial only. The term providing the lowest mean square error (MSE) is retained. In a next step, we expand our model by adding a second term to the polynomial retained previously. The second term results again from an exhaustive search over all possible polynomials up to a given order. The expansion continues until the optimum model complexity is attained according to the minimum description length (MDL) criterion [7]. Although this expansion search method may be suboptimal it is exhaustive per considered term. Extensive experiments with time series [3] have been performed to explore the good prediction performance. However, since the presented method is very suitable to describe spatial information in data, possible morphological patterns in the data cannot be captured. Therefore, we propose an extension of the existing method that is capable to describe spatial information as well as order statistic information.

In Eq. 1 the polynomial $p_m(n)$ consists of a nonlinear combination of the pixel values $x(n-1), \ldots, x(n-D)$. In a more general sense, these values can be written as an input vector $\mathbf{x} = (x_1, x_2, \ldots, x_D)$. By ordering the observation values by rank we obtain $\mathbf{x}_{(r)} = (x_{(1)}, x_{(2)}, \ldots, x_{(D)})$; $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(D)}$ are the sample order statistics. $\mathbf{x}_{(r)}$ is called the rank order vector. By concatenating the two vectors we get the final input vector:
$\mathbf{x_{in}} = (x_1, \ldots, x_D, x_{(1)}, x_{(2)}, \ldots, x_{(D)})$. The search algorithm is now capable to find polynomials that are spatially ordered (built of values of the first half of $\mathbf{x_{in}}$)), statistically ordered (using the second half) or even a mixture of the two. Due to this extension, morphological operators such as for example median filters can be modeled perfectly.

3. THE SUBBAND DECOMPOSITION SCHEME

In [2] the morphological subband decomposition (MSD) has been introduced. It has been shown that this predictive subband decomposition enjoys some interesting properties. Two reasons have made the MSD a powerful tool for image coding. The first reason is the absence of any ringing effect in the reconstructed images due to the nonlinear characteristics of the filters. The second reason is its ability to define a lossless scheme as described in [1].

The idea behind using rank order polynomial approximation (ROP) instead of the morphological filters is that it might be possible to find more suitable parameters to represent all the images containing edges and textures. Indeed, the MSD is based on the model of contours and flat regions whereas a more general approximation should be able to represent a more general model of a natural image.

It is clear that for a given application one may need specific parameters for this task. For example, the statistics of X–ray images are completely different from the statistics of images of people or landscapes. Therefore, a morphological filter might perform sub–optimally in comparison with a specifically optimized ROP.

The proposed nonlinear lossless subband decomposition is shown in Fig. 1. The block $\mathcal{P}(\mathbf{x})$ denotes the nonlinear prediction scheme. The vector $\mathbf{x}$ is formed by the D neighboring pixels in the same way as for the MSD. The parameter D has been chosen to be 6 which leads to the same region of support as for the best morphological filter. Therefore, this scheme enjoys the same properties as the MSD. This decomposition has been adapted to define a lossless coding scheme in the same way as presented in [1]. This is done by means of the operator $\mathcal{Q}(\cdot)$ which denotes a quantization procedure with a quantization step of 1. In such a framework all kinds of nonlinear approximation methods can be applied. It is clear that a better prediction leads to a better decorrelation, and, finally, better compression performances can be obtained.

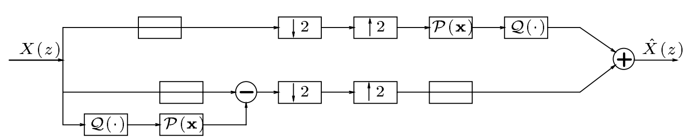

Figure 1: The nonlinear subband decomposition allowing for a lossless compression.

3.1. Compression Results

Test images of different types have been used for comparison purposes. Two of the medical images come from magnetic resonance imaging (MRI) and one is an X–ray. The three medical images are shown in Fig. 2. Three images of a general type are compared as well. They are shown in Fig. 3.

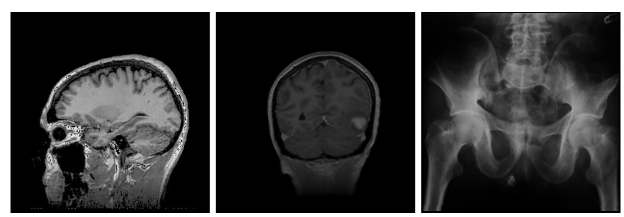

Figure 2: The three medical test images. (a) “Sagital”, of type MRI. (b) “Coronal”, of type MRI. (c) “Pelvis”, of type X–ray.

Figure 3: The three natural test images. (a) “Lena”. (b) “Pepper”. (c) “Weather”.

Image	Size	Type	JPEG	MSD	ROPD
		Med.:	b/p	b/p	b/p
“Sagital”	256×256	MRI	4.37	3.67	3.40
“Coronal”	256×256	MRI	2.13	1.45	1.39
“Pelvis”	448×448	X–ray	2.69	2.47	2.32
		Gen.:			
“Lenna”	512×512	Natural	4.70	4.43	4.33
“Pepper”	512×512	Natural	4.99	4.76	4.66
“Weather”	352×288	Natural	4.92	4.72	4.52

Table 1: Compression comparison of the proposed compression scheme with the international standard JPEG.

All test images have been compressed in a lossless way with the two proposed decompositions (MSD and ROPD) and with the standard JPEG. The results are summarized in Tab. 1. It is shown that for all the test images, the proposed algorithms perform better than the standard JPEG. An improvement of up to 53% is achieved for MRI type of images. From the table one can also clearly see the marked superiority of the ROPD in comparison with the MSD. For natural images an improvement of around 9% is achieved with the proposed ROPD in comparison with the standard JPEG.

3.2. Illustration of the Progressive Bitstream

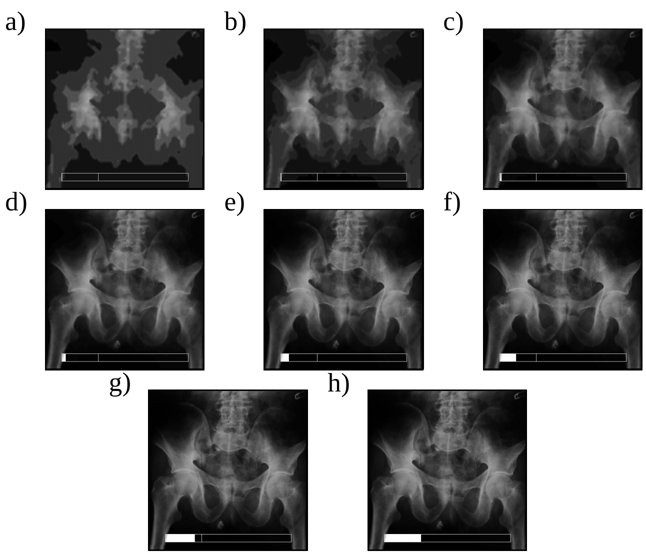

Figure 4: “Pelvis” 448×448 pixels, Lossless rate 2.32 b/p. Each picture has been decoded with only part of the bitstream. (a) With 1.3% of the bitstream, 24.29 dB. (b) 2.6%, 28.49 dB. (c) 5.6%, 33.61 dB. (d) 11.2%, 38.58 dB. (e) 22.4%, 43.21 dB. (f) 44.8%, 46.33 dB. (g) 81.0%, 55.97 dB. (h) Original picture: 100%, ∞ dB. The full bar represents the amount of information used to reproduce the image, the line represents the lossless rate and the empty bar represents the amount of information of the uncompressed original picture.

The proposed lossless compression scheme has the functionality of having a completely embedded bitstream. That means that a picture compressed in a lossless way can be decompressed at any bitrate. In order to recognize the picture, only a small part of the bitstream is necessary. This is illustrated in Fig. 4 for the X–ray image “Pelvis” using the ROPD. It is demonstrated that only a small part of the bitstream is necessary to get a good picture quality (about 5%). Fig. 5 shows the rate–distortion curve of the two proposed coding schemes. It can be observed on the figure that the quality is increasing with an increasing bitrate. Also, as expected, the PSNR tends to ∞ at the lossless bit rate. The lossless rates are represented in the graph by the two arrows. One can see that the ROPD is superior to the MSD at any bitrate. At low bitrates the improvement with respect to the ROPD is around 3 dB. This means that the ROPD has a better lossless rate than the MSD but has also a much better browsing quality if only a part of the bitstream is decoded.

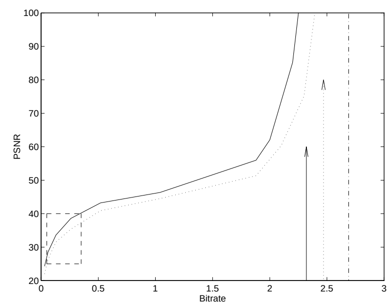

Figure 5: Rate–distortion curve of the proposed progressive lossless schemes for the picture “Pelvis”. (Full lines) ROPD, the arrow represents the lossless rate. (Dotted lines) MSD, the arrow represents the lossless rate. (Dash-dotted line) JPEG lossless rate.

3.3. Hybrid Lossy/Lossless Coding

Another type of application of the proposed schemes is oriented towards coding different objects of a picture in a different way. As an example, let us examine the nature of an ultrasound image. A typical image of this class is shown in Fig. 6. This image is composed of different components. The medical information coming from the ultrasound is the central triangularly shaped part. Let us call it the region of interest (ROI). Clearly, this part should be compressed in a lossless way. The electrocardiogram (ECG) below the ultrasound information contains also medical information which should not be distorted. The rest of the picture however, contains no medical information. Some regions contain text and other regions are part of the black background. The text regions do not need to be compressed graphically and the background contains no information in itself.

In this context, it is proposed to apply the proposed lossless coding scheme only to the region of interest while the

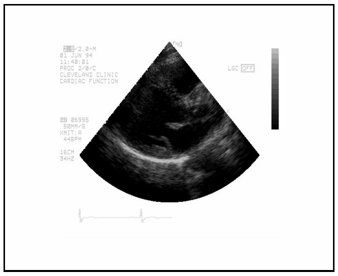

Figure 6: A typical ultrasound image, 720×576 pixels.

Method	Region	Lossless rate
Global	Whole image	0.95
Region–based	ROI	0.73
	ECG	0.02
	Text	0.09
	Background	0.05
	Total	0.89
JPEG	Whole image	1.69

Table 2: Comparison of the compression performances of the proposed scheme with JPEG on the "Ultrasound" picture.

rest of the image is processed in a different channel. The lossless scheme of JPEG is not able to process arbitrarily–shaped regions. Therefore, as a first comparison the ultrasound picture is compressed in a lossless way for two different cases. The first approach is to compress the total picture in one step. The second approach is to apply the ROPD to the different regions separately, compress them and add the four bitstreams together. The results are shown in Tab. 2. One can see that the best mode of JPEG is significantly inferior to the proposed methods. Indeed, an improvement of 78% is achieved by compressing losslessly the global image and an improvement of 90% is obtained by compressing the different objects separately. Coding the regions of interest and the background separately leads to a better coding performance than coding the whole picture in one bitstream. Notice that no shape information is taken into account assuming that such a segmentation would be fixed for this application. Indeed, the region of interest is at exactly the same location for all the ultrasound images coming from the same machine and thus, has not to be sent with the coded picture.

In most of the cases, ultrasound images are part of a complete sequence. The only part which is variable for each picture in the sequence is the medical part. The background information and the text is not changing from one image to another. Therefore, the background information can be sent only once, while the medical part has to be compressed separately for each picture. The lossless bit rate of the ROI and the ECG is 0.75 b/p. This is to be compared with the 0.89 b/p needed to transmit to complete picture losslessly. Clearly, there is a gain in a region–based coding of such an image.

4. CONCLUSION

A novel lossless compression scheme has been proposed based on a nonlinear decomposition allowing for the coding of a progressive bitstream. The ROPD, the implemented decomposition scheme, is based on a novel polynomial expansion model that considers both, spatial and morphological information of an image. It has been shown that the proposed scheme has a better performance than the MSD and that it outperforms the best prediction mode of the standard JPEG for all test images. An improvement of up to 53% is obtained on medical images, an improvement of around 9% can be achieved on natural images while an improvement of 90% can be achieved on ultrasound images. Additionally, the functionality providing good browsing quality has been demonstrated.

5. REFERENCES

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