

A GENERALIZED SCHUR ALGORITHM IN THE KREIN SPACE AND ITS APPLICATION TO H^∞ -FILTERING

Kyungsup Kim and Joohwan Chun

Department of Electrical Engineering
Korea Advanced Institute of Science and Technology
373-1 Yusong-gu, Kusung-dong, Taejon, 303-701, Korea
kskim@sclab.kaist.ac.kr and chun@pisces.kaist.ac.kr

ABSTRACT

This paper introduces a generalized Schur algorithm in the Krein space with an indefinite inner product. Concepts such as Carathéodory classes and Schur classes used in the classical Schur algorithm cannot be applied in the Krein space since the positive-definiteness corresponds merely to the nonsingularity in the Krein space. We note also that these problems appear when fast algorithms for suboptimal H^∞ filtering are implemented. We shall derive the extended Chandrasekhar algorithm which is a fast implementation of H^∞ filtering, and explain the connection between the generalized Schur algorithm and the Chandrasekhar algorithm. Using this result it is possible to derive a fast algorithm for suboptimal H^∞ filtering.

1. INTRODUCTION

The Riccati equation approach for solving the H^∞ optimal control problem has recently emerged in [7, 13]. The same technique has been applied to solve the H^∞ filtering problem [6, 8, 13]. This equation is different from the classical Riccati equation derived from the Kalman filtering or H^2 problems [6, 7] since the equations have the indefinite structure. It is well-known that there are fast algorithms such as Chandrasekhar-type algorithms, which are closely related to the Schur algorithm [10, 11]. We note that when fast algorithms for H^∞ filtering are derived by using the similar concepts as classical Schur algorithms, they have different structures because of the lack of positive-definite properties. We extend the Schur algorithm in the Krein space which can be applied to the H^∞ filtering problems. We derive an extended Chandrasekhar algorithm and explain the connection between the extended Chandrasekhar algorithm and the generalized Schur algorithm.

Let us introduce some basic notations and concepts used in this paper. The regions \mathbb{D} , \mathbb{E} , and \mathbb{T} denote, respectively, the open unit disk set, the open set outside the unit circle, and the unit circle. For each rational matrix valued function $H(z)$, \tilde{H} denotes $\tilde{H}(z) = H^*(z^{-1})$ where $*$ is a complex matrix conjugate. An inner product space is a complex vector space \mathcal{V} with a complex-valued function $\langle \cdot, \cdot \rangle : \mathcal{V} \times \mathcal{V} \rightarrow \mathbb{C}$ such that $\langle af + bg, h \rangle = a\langle f, h \rangle + b\langle g, h \rangle$ (linearity), and $\langle f, g \rangle = \langle g, f \rangle^*$ (symmetry) for any $f, g \in \mathcal{V}$ and for any scalar a, b . The Krein space is an inner product space \mathcal{V} over complex field with the above mentioned condition. See [15] for details. In the Hilbert space, the positivity of $\langle f, f \rangle$ for all $f \in \mathcal{V}$ is added in the above condition. We now introduce some notations and terminologies for functional spaces

on the unit circle \mathbb{T} . For any function F defined on the unit circle \mathbb{T} , let $\|F\|_{L^\infty} = \sup\{\|F(z)\| : |z| = 1\}$. The space $L_{p \times q}^\infty$ is the set of all measurable $p \times q$ matrix valued functions with $\|F\|_{L^\infty} < \infty$. The subspace $H_{p \times q}^\infty$ is the set of all $F \in L_{p \times q}^\infty$ with bounded analytic continuation to the unit disk \mathbb{D} . The linkage between a rational transfer matrix and its state-space realization $[A, B, C, D]$ will be denoted by

$$[A, B, C, D](z) = C(z^{-1}I - A)^{-1}B + D, \quad (1)$$

where $A \in \mathbb{C}^{n \times n}$, $B \in \mathbb{C}^{n \times r}$, $C \in \mathbb{C}^{m \times n}$, and $D \in \mathbb{C}^{m \times r}$. In the complex analysis and approximation theory, the concepts of the Carathéodory class and the Schur class are introduced in the Hilbert space [3, 4, 5]. We shall extend these concepts in the Krein space which is an extended space of the Hilbert space. A $d \times d$ matrix function $F(z)$ is said to be in the Carathéodory class \mathcal{C} in the Hilbert space if $F(z)$ is analytic in the open unit disk and $(F + \tilde{F})/2$ is nonnegative definite in this domain.

Let's consider the forms of rational matrix functions which appear in many applications such as Kalman filtering, control theory and H^∞ optimization problems. Define a matrix

$$\Sigma = \begin{bmatrix} Q & L \\ L^* & R \end{bmatrix}, \quad (2)$$

where Q is a $p \times p$ matrix, R is a $q \times q$ matrix, and L is a $p \times q$ matrix. Consider a rational matrix function of the form

$$\Pi_\Sigma = [C^*(zI - A^*)^{-1} \quad I] \Sigma \begin{bmatrix} (z^{-1}I - A)^{-1}C \\ I \end{bmatrix}. \quad (3)$$

The rational matrix valued function Π_Σ is called the Popov function [9] associated to Σ . In the Kalman filtering problem and H^2 problem, the form of Popov functions is a little different from those in H^∞ filtering. Specifically the matrix Σ is nonnegative definite in H^2 filter, but it is not in H^∞ filtering. In these problems, it is important to derive the J -spectral factorization of the Popov function Π_Σ . We shall derive a fast algorithm of the J -spectral factorization via the generalized Schur algorithm and explain the linkage between it and the indefinite Riccati equation or Chandrasekhar-type algorithms in H^∞ filtering.

2. ELEMENTARY FACTORS IN KREIN SPACES

We shall introduce a few basic properties of the elementary J -lossless factors and the elementary Blaschke matrix factor which

will be used in the sequel in the Krein space. In the Hilbert space these factors are well-known, but since some properties of the Hilbert space factors cannot be inherited to the Krein space the new factors should be expanded. To begin with, we shall generalize the concept of the J -unitariness in the Krein space.

Definition 2.1. For given diagonal matrices J_1 and J_2 with 1 or -1 on the diagonal, a matrix U is (J_1, J_2) -unitary matrix if $U^* J_1 U = J_2$. Moreover, if $J_1 = J_2$, then U is J_1 -unitary.

Let \mathcal{K} denote a Krein space such that $\langle \cdot, \cdot \rangle$ inner product of \mathcal{K} satisfies $\langle v_1, v_2 \rangle = (Jv_1, v_2)$ for any $v_1, v_2 \in \mathcal{K}$ where J is a signature matrix and (\cdot, \cdot) is an inner product of the Hilbert space $|\mathcal{K}|$. We define a matrix conjugate A^* of A in the Krein space such that $A^* = JA^*J$, where J is a signature matrix, since for any $v_1, v_2 \in \mathcal{K}$, A satisfies that $\langle Av_1, v_2 \rangle = \langle v_1, JA^*Jv_2 \rangle$ where $*$ is a matrix conjugate in Hilbert space. Define a signature matrix J_1 such that $J_1 = J \oplus -J$ where \oplus stands for the direct sum. For any given matrix w , we derive a J_1 -unitary matrix with respect to w as follows. Define $H(w)$ such that

$$H(w) = \begin{bmatrix} M_1^{-1} & O \\ O & M_2^{-1} \end{bmatrix} \cdot \begin{bmatrix} I & -w^* \\ -w & I \end{bmatrix}, \quad (4)$$

where M_i satisfy that $J - w^*Jw = M_1^*JM_1$ and $J - JwJw^*J = M_2^*JM_2$. We can check easily that $\begin{bmatrix} I & w^* \\ -w & I \end{bmatrix} \tilde{U} = \begin{bmatrix} X & O \end{bmatrix}$. Consider the block matrix L defined by

$$L = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (5)$$

where A, B, C and D are $n \times n$ matrices. We define a homographic transformation acting on $n \times n$ matrices X by

$$X \mapsto L(X) = (AX + B)(CX + D)^{-1}, \quad (6)$$

where $CX + D$ is nonsingular.

Theorem 2.1. Let L be a $2n \times 2n$ J_1 -unitary matrix and w is chosen such that $J - w^*Jw$ and $J - JwJw^*J$ are nonsingular. Assume that $L(w)$ is a homographic transformation acting on w . Then $L(w)^*L(w) - I$ and $L(w)L(w)^* - I$ are nonsingular.

Proof. Assume that $I - ww^*$ is nonsingular. Note that

$$\begin{aligned} I - L(w)^*L(w) &= J\{J - L(w)^*JL(w)\} \\ &= J(Aw + B)^{-1}(-w^*Jw + J)(Aw + B)^{-1}. \end{aligned} \quad (7)$$

Hence $L(w)^*L(w) - I$ is nonsingular. The proof of $L(w)L(w)^* - I$ is similar. \square

Let's consider a classical result of the Blaschke product and the matrix-valued Blaschke factor in the Krein spaces. It is well-known in complex analysis, circuit theory and interpolation theory that the convergence of the infinite Blaschke product is related to the convergence of the Nevanlinna-Pick interpolation [1, 2, 12]. Let's define a Blaschke factor of the form $\xi_s(z) = \frac{[\bar{z}_s]}{z_s} \frac{z_s - z}{1 - \bar{z}_s z}$. The matrix valued Blaschke factor is defined by $Y_s = J \oplus \xi_s^{-1}J$ where J is a signature diagonal matrix. Now we shall consider the J_1 -losslessness in the Krein space. A $2n \times 2n$ matrix θ is said to be J_1 -lossless in the Krein space if θ is analytic in the unit disk \mathbb{D} and J_1 -unitary on the unit circle \mathbb{T} .

3. THE J -SPECTRAL FACTORIZATION USING A GENERALIZED SCHUR ALGORITHM

We note that one of the properties of Caratheodory classes cannot be used in the Krein spaces because the positive-definite property doesn't apply. The positive-definiteness is relaxed to the singularity in the Krein spaces. This fact has been noted by many researchers who have studied the H^∞ optimization, the J -spectral factorization and the indefinite metric spaces [6, 8, 9, 14]. Let's consider $\Pi(z)$ defined by

$$\Pi(z) = F(z) + \tilde{F}(z), \quad (8)$$

where $F(z)$ is analytic in \mathbb{E} with $F(0) = J$, and J is the signature diagonal matrix. Without loss of generality, $F(0)$ can be normalized as $F(0) = J$ if $F(0)$ is nonsingular. We shall try to find the J -spectral factor of the form $\Pi(z) = \tilde{G}(z)JG(z)$ where $G^{-1}(z)$ is analytic in \mathbb{D} . A recursive J -spectral factorization algorithm can be derived similarly to the classical tangential Schur algorithm.

Algorithm 3.1. Assume that the initial graph function $g_1(z)$ is defined by

$$g_1(z) = \begin{bmatrix} J + \tilde{F}(z) \\ J - \tilde{F}(z) \end{bmatrix} = \begin{bmatrix} D_1(z) \\ N_1(z) \end{bmatrix}. \quad (9)$$

Then it is easy to check that g_1 satisfies $\Pi(z) = \tilde{g}_1(z)J_1g_1(z)$ and that $N_1(0) = O$. Assume that for each $s = 1, 2, \dots$, the graph functions g_s are defined in the form $g_s = \begin{bmatrix} D_s^* & N_s^* \end{bmatrix}^*$. We define c_s such that

$$c_s = \lim_{z \rightarrow 0} z^{-1}N_s(z)D_s^{-1}(z). \quad (10)$$

With $H(c_s)$ in (4), by the next step of the recursion can be computed by

$$g_{s+1} = H(c_s) \begin{bmatrix} J & O \\ O & z^{-1}J \end{bmatrix} g_s = \begin{bmatrix} D_{s+1}^* \\ N_{s+1}^* \end{bmatrix}, \quad (11)$$

where $N_{s+1}(0) = O$. By continuing the recursion, g_s converges to $\begin{bmatrix} G^* & O \end{bmatrix}^*$ as $s \rightarrow \infty$

In order to continue the recursion each graph function g_s should have the proper property. In classical Nevanlinna algorithms, it is called to be admissible if g_s are analytic matrices in \mathbb{D} , D_s is nonsingular in \mathbb{D} , and $N_sD_s^{-1}$ is a contractive matrix in the \mathbb{D} . In Krein spaces the third property is relaxed to the nonsingularity. It is well-known that g_s converges to $\begin{bmatrix} G^* & O \end{bmatrix}^*$ in Hilbert spaces. We are also able to prove similar convergence theorems [16] in the Krein space.

In Kalman filter, H^∞ -filter and control theory, a rational matrix valued function $\Pi_\Sigma(z)$ called the Popov function associated to Σ is defined as in (3) after setting $W(z) = C^*(zI - A)^{-1}$. In these problems, it is important to obtain the spectral factorization of the Popov function Π_Σ . We note that the Popov function Π_Σ can be decomposed into the form of Caratheodory classes. Here Q and R in Σ are positive-definite in the classical theory, but in the Krein space it isn't necessary.

Lemma 3.1. Define Popov function Π_Σ as in (3) and assume that there is a solution P of the Lyapunov equation of the form

$$P = APA^* + Q \quad (12)$$

where A is asymptotical stable, i.e., all of the eigenvalues of A are in the unit disk, and Q is a symmetry matrix. Then Π_Σ can be decomposed as in (8). Moreover, if we assume that $F(z) = [\hat{A}, \hat{B}, \hat{C}, \hat{D}](z)$ is defined as the equation (1), then $\hat{A}, \hat{B}, \hat{C}$, and \hat{D} has the following forms:

$$\hat{A} = A, \hat{B} = C^*PA + L, \hat{C} = C, \hat{D} = \frac{R + C^*PC}{2}. \quad (13)$$

Proof. First, change the term $P - APA^*$ of (12) by adding the terms $zI - A, z^{-1}I - A^*$ such that

$$P - APA^* = (zI - A)P(z^{-1}I - A^*) + AP(z^{-1}I - A^*) + (zI - A)PA^*. \quad (14)$$

Replace Q of the right side of the equation (12) by the equation (14) and reformulate Π_Σ such that

$$\begin{aligned} \Pi_\Sigma(z) = & R + C(zI - A)^{-1}L + L^*(z^{-1}I - A^*)^{-1}C^* + \\ & CPC^* + C(zI - A)^{-1}APC^* + CPA^*(z^{-1}I - A^*)^{-1}C. \end{aligned} \quad (15)$$

Hence $F(z)$ satisfies the equation (13). \square

In result, we can transform a Popov function Π_Σ to a Caratheodory form. Now we shall derive a state-space interpretation of the Schur recursion. Let $\hat{D} = MJM^*$. By multiplying M^{-1}, M^{*-1} on both side of $F(z)$, we can normalize $F(z)$. Hence without loss of generality, we can set $F(z) = [A, B, C, J](z)$, $D_1(z) = [A, B, C, 2J](z)$ and $N_1(z) = [A, B, C, O](z)$ in (9). Let's review the basic operations of the state-space realization which are needed in the recursive procedure. Assume that $[A, B, C, D](z)$ is a state-space realization representation. Let's define $F_i(z) = [A_i, B_i, C_i, D_i](z)$ for each $i = 1, 2$. Then the following properties are satisfied:

1. $F_1^{-1}(z) = [A_1 - B_1D_1^{-1}C_1, B_1D_1^{-1}, D_1^{-1}C_1, D_1^{-1}](z)$,
2. if $F(z) = [A, B, C, O](z)$,
then $z^{-1}F(z) = [A, BA, C, CB](z)$.

See [13] for details. Using these operations it is possible to obtain the following theorem.

Theorem 3.2. Assume that the initial graph function g_1 is defined by $g_1 = [(J + \tilde{F})^* \quad (-J + \tilde{F})^*]^*$. Assume that $g_k = [D_k^* \quad N_k^*]^*$ derived by the Schur recursions can be expressed in the form of state-space representation as $D_k(z) = [A, \Delta_k, C_k, \Omega_k](z)$ and $N_k(z) = [A, \Gamma_k, C_k, O](z)$. Let $H(c_k)$ be defined as in (4). Then Δ_k, Ω_k and Γ_k satisfy :

$$g_{k+1} = H(c_k) \begin{bmatrix} J & O \\ O & z^{-1}J \end{bmatrix} g_k, \quad (16)$$

$$\begin{bmatrix} \Omega_{k+1} & \Delta_{k+1} \\ O & \Gamma_{k+1} \end{bmatrix} = H(c_k) \begin{bmatrix} \Omega_k & \Delta_k \\ \Gamma_k & \Gamma_k A \end{bmatrix}. \quad (17)$$

4. AN APPLICATION OF THE SCHUR ALGORITHM TO THE H^∞ -FILTERING PROBLEM

In previous sections, the generalized Schur recursion has been proposed in indefinite metric spaces and we have shown that the Schur algorithm is related to the J -spectral factorization problem. It is

well-known that J -spectral factorization problems appear in the H^∞ control theory, the game theory and H^∞ suboptimal filtering problem [7, 8, 13]. We shall apply the results of the previous sections to the H^∞ suboptimal filtering problem.

Let's review the standard system model for the optimal filtering problem.

1. **Signal generation:** consider a time invariant state space model of the form

$$x(i+1) = Ax(i) + Bw(i) \quad (18)$$

$$y(i) = Cx(i) + Dw(i) \quad (19)$$

$$z(i) = Lx(i) \quad (20)$$

for $i = 1, 2, \dots$ where $x(t) \in R^n$ is the state, $w(i) \in R^q$ is the disturbance of noise, $y(i) \in R^p$ is the observation signal and $z(i) \in R^m$ is the output of interest.

2. **Filtering:** The results of the optimal filtering is described by:

$$\hat{x}(i+1) = A\hat{x}(i) + K_{f_2}(y(i) - C\hat{x}(i)) \quad (21)$$

$$\hat{z}(i) = L\hat{x}(i) \quad (22)$$

where $\hat{x}(i) \in R^n$ is the estimated state and $\hat{z}(i) \in R^m$ is the estimated output after filtering.

In the H^∞ suboptimal problem, our aim is to find an estimate of z in the form $\hat{z} = Fy$ such that $\|R\|_\infty < \gamma$ for some given γ where R is a function of the form : $R : w \mapsto z - \hat{z}$. The state-space based Riccati difference equation for H^∞ filtering problem similar to that which occurs in Kalman filtering, is introduced in [7, 8, 13]. The Riccati difference equation with stabilizing solution P_k is defined as

$$P_{k+1} = \hat{B}J\hat{B}^* + AP_kA^* - M_k\Sigma_k^{-1}M_k \quad (23)$$

for $k = 1, 2, \dots$, where $\Sigma(k)$ and $M(k)$ are defined as

$$\Sigma_k = \hat{D}J\hat{D}^* + \hat{C}P_k\hat{C}^* \quad (24)$$

$$M_k = \hat{B}J\hat{D}^* + AP_k\hat{C}^* \quad (25)$$

with

$$\begin{bmatrix} A & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} = \begin{bmatrix} A & B \\ L & C \end{bmatrix} \begin{bmatrix} B & O \\ O & \gamma I \\ D & O \end{bmatrix} \quad (26)$$

and $J = I \oplus -I$. Now we shall define a Popov function related to H^∞ filtering problems. As in the Kalman filtering problem where a Popov function is given in the form of the power spectrum of the stochastic process y_k and is derived from the Riccati equation, in H^∞ filtering problem a Popov function is given in the form of the quasi power spectrum of $[y(k)^* \quad z(k)^*]^*$. We will derive the Popov function Π_Σ in the H^∞ filtering problem by using the asymptotic solution of the Riccati difference equation.

Theorem 4.1. Assume that M_k, Σ_k and P_k converge to M, Σ and P when $n \rightarrow \infty$. Let's define $Q = BB^*, R = BD^*$ and $R = DD^*$ and define S_f and R_f such that

$$S_f = \begin{bmatrix} O & S \end{bmatrix} \quad (27)$$

$$R_f = \begin{bmatrix} -\gamma^2 I & O \\ O & R \end{bmatrix}. \quad (28)$$

Define the transfer function $W_c(z)$ and the gain K_f such that $W_c(z) = \hat{C}(z^{-1}I - A)^{-1}$ and $K_f = M\Sigma^{-1}$. Then the Popov function satisfies that

$$\begin{aligned}\Pi &= W_c Q W_c^* + S_f W_c^* + W_c S_f + R_f \\ &= (I + W_c K_f) \Sigma (I + K_f^* W_c^*)\end{aligned}\quad (29)$$

Moreover, if we set $Y(z) = I + W_c(z)K_f$, then Y^{-1} is analytic in \mathbb{E} .

Proof. As $n \rightarrow \infty$, the solution of the Riccati difference equation satisfies:

$$P - APA^* = Q - M\Sigma^{-1}M^*. \quad (30)$$

The left side of (30) can be replaced by (14), and then be pre- and post- multiplied by $W_c(z)$ and $\tilde{W}_c(z)$, respectively. From $\hat{C}P\hat{C}^* = \Sigma - R_f$ and $AP\hat{C}^* = M - S_f$, and hence substituting, we obtain (29). And it is easy to check that the right side of (29) is analytic. \square

Finally, a Popov function for H^∞ suboptimal filtering is derived of the form

$$\Pi(z) = \begin{bmatrix} W_c(z) & I \end{bmatrix} \begin{bmatrix} Q & S_f \\ S_f^* & R_f \end{bmatrix} \begin{bmatrix} \tilde{W}_c(z) \\ I \end{bmatrix}. \quad (31)$$

The middle term of the left side of (31) agrees with Σ of a Popov function of (2). Note that R_f is an indefinite matrix. Hence by using the generalized Schur algorithm for J spectral factorization, Y and K_f can be derived. As Kalman filtering, we can formulate a Chandrasekhar or a square-root for H^∞ filtering. Let P_k be the solution of the difference Riccati equation (23) with $P_1 = P$. Let $W_k J W_k^* = \{\hat{C}P_k\hat{C}^* + R_f\}$ and $L_k = \{AP_k\hat{C}^* + S\}W_k^{-1}$. Then for $k = 1, 2, \dots$, W_k , L_k and P_k satisfy the properties :

$$W_{k+1} J W_{k+1}^* = W_k J W_k^* + \hat{C}(P_{k+1} - P_k)\hat{C}^* \quad (32)$$

$$L_{k+1} J W_{k+1}^* = L_k J W_k^* + A(P_{k+1} - P_k)\hat{C}^* \quad (33)$$

$$\begin{aligned}(P_{k+2} - P_{k+1}) + L_{k+1} J L_{k+1}^* \\ = A(P_{k+1} - P_k)A^* + L_k J L_k^*\end{aligned}\quad (34)$$

From (32,33,34), following theorem can be proved immediately.

Theorem 4.2. Let W_k and L_k be as given in (32,33,34). Set $P_{k+1} - P_k = -\Gamma_k J \Gamma_k^*$ where P_k is the solution of the Riccati equation. Then W_k , L_k and Γ_k for $k = 1, 2, \dots$ can be computed recursively using the array

$$\begin{bmatrix} W_{k+1} & O \\ L_{k+1} & \Gamma_{k+1} \end{bmatrix} = \begin{bmatrix} W_k & \Gamma_k \\ L_k & A\Gamma_k \end{bmatrix} U \quad (35)$$

where U is J_1 -unitary.

5. CONCLUSION AND DISCUSSION

We propose a generalized Schur algorithm in the Krein space which enables us to obtain fast algorithms to J - spectral factorization problems or suboptimal H^∞ - filtering. Since the generalized Schur algorithm in the Krein space has different structures from the classical Schur algorithm, we also explore some problems in the underlying structure. Our new algorithm can be applied to the H^∞ filtering problems and J -spectral factorization problems.

6. REFERENCES

- [1] P. DEWILDE AND H. DYM, "Lossless chain scattering matrices and optimum linear prediction : The vector case", Circuit Theory Appl., vol. 9, pp. 137–175, 1981.
- [2] P. DEWILDE AND H. DYM, "Schur recursions, error formulas, and convergence of rational estimators for stationary stochastic sequences", IEEE Trans. Inf. Theory, vol. IT-27(4), pp. 446–461, 1981.
- [3] PH. DELSARTE, Y. GENIN, AND Y. KAMP, "The Nevanlinna-Pick problem for matrix-valued functions", SIAM J. Appl. Math, 36, pp. 47–61, 1979.
- [4] T. T. GEORGIOU AND P. P. KHARGONEKAR, "Spectral factorization of matrix-valued functions using interpolation", IEEE Trans. Circuits and Systems, vol. CAS-36(4), pp. 568–574, 1989.
- [5] C. CHANG AND T. T. GEORGIOU, "On a Schur-algorithm based approach to spectral factorization: Connection with the Riccati equation", Linear Algebra Appl., vol. 171, pp. 233–247, 1992.
- [6] M. J. GRIMBLE, "Polynomial matrix solution of the H_∞ filtering problem and the relationship to Riccati equation state-space results", IEEE Trans. Signal Processing, vol. 41(1), pp. 67–81, 1993.
- [7] J. C. DOYLE, K. GLOVER, P. P. KHARGONEKAR, AND B. A. FRANCIS, "State-space solution to standard H_2 and H_∞ control problems", IEEE Trans. Automat. Contr., vol. 34, pp. 831–847, 1989.
- [8] K. M. NAGPAL AND P. P. KHARGONEKAR, "Filtering and smoothing in an H^∞ setting", IEEE Trans. Automat. Contr., vol. 36, pp. 152–166, (1991).
- [9] V. IONESCU, M. WEISS, "Continuous and discrete-time Riccati theory : A Popov-function approach", Linear Algebra Appl., vol. 193, pp. 173–209, 1993.
- [10] A. H. SAYED, T. KAILATH, AND H. LEV-ARI, "Generalized Chandrasekhar recursions from the generalized Schur algorithm", IEEE Trans. Automat. Contr., vol. 39(11), pp. 2265–2269, 1994.
- [11] B. HASSIBI, "Indefinite metric spaces in estimation, control and adaptive filtering", PhD. Dissert. Dep. Elect. Eng., Stanford Univ., 1996.
- [12] N. YOUNG, "An introduction to Hilbert space", Cambridge, New York 1988.
- [13] M. GREEN AND D. J. N. LIMBEER, "Linear robust control", Prentice Hall, New Jersey, 1995.
- [14] I. GOHBERG, "Orthogonal matrix-valued polynomials and applications", Operator Theory, advance and application, vol. 34 1988.
- [15] I. GOHBERG, P. LANCASTER, AND L. RODMAN, "Matrices and indefinite scalar product", Operator Theory, advance and application, vol. 8, 1983.
- [16] K. KIM AND J. CHUN, "A Convergence proof of the tangential Schur algorithm in Krein space", to be submitted, 1997.