A HYBRID EQUALIZER MERGING THE ADVANTAGES OF BAUD SPACED AND FRACTIONALLY SPACED EQUALIZERS

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ABSTRACT

A transversal equalizer with half Baud spaced taps in the center and extended with Baud spaced taps on both sides is presented. This hybrid equalizer combines the benefits of Baud spaced equalizers - like superior equalization of notches in the middle of the transmission band - and fractionally spaced equalizers, which have a superior performance when equalizing asymmetric notches in the slope of the transmission band, when the same number of coefficients are used. The hybrid equalizer offers the reduced sensitivity to sampling time changes and the ability to model the matched filter in the receiver as the fractionally spaced equalizer. The problem of tap-wandering, which is present in fractionally spaced equalizers, is reduced due to the reduced degree of freedom in the coefficient adjustment.

1. INTRODUCTION

Adaptive equalizers are needed for example to mitigate intersymbol interference originating from multipath propagation in a mobile communication environment or to establish high data transmission rates over twisted pair wire channels.

Transversal equalizers are widely used in QAM demodulators for radio-link systems and voice line modems [1]. In mobile communication environments their adaptation speed may be increased to that of recursive least-squares based algorithms [3]. The input signal to these equalizers may be sampled with the symbol rate in so-called Baud spaced equalizers (BSE) or with a multiple of the symbol rate in fractionally spaced equalizers (FSE). In many applications half Baud spaced equalizers (HBSE) are used. They are FSEs for an input signal which is sampled with twice the symbol rate [4].

The BSE is usually adjusted according to the zero-forcing algorithm, which minimizes the peak distortion at the equalizer output. Half baud spaced equalizers are normally adjusted according to the mean-square-error (MSE) criterion, that minimizes the mean-square value of the error.

The properties of the BSE and the HBSE concerning their equalization capability, sampling time sensitivity and coefficient stability are complementary in some aspects. In a digital radio-link system, the BSE has a superior performance when equalizing notches in the center of the transmission band because of its doubled time-span compared to a HBSE with the same number of taps. As the input signal of the BSE is sampled with the symbol rate $1/T$ the Nyquist criterion does not hold. This results in aliasing. The BSE can only try to compensate for the frequency response characteristic of the aliased input spectrum and therefore suffers from performance degradation if there is an asymmetric notch of the channel in one slope of the transmission band. The BSE is optimal when it is preceeded by a filter, matched to the received pulse that has been distorted by the channel. When the channel characteristics are unknown the receiver filter is usually matched to the transmitted pulse, which results in a performance very sensitive to the choice of the sampling time [4, pp.617-621].

The performance of the HBSE with the same number of taps is not as high as that of the BSE, when the notch of the channel is in the middle of the transmission band because of the halved time-span of the HBSE. When the notch is in the slope of the transmission band the performance is better than that of the BSE, as there is no aliasing due to the doubled sample rate, which fulfills the Nyquist criterion. The HBSE is able to incorporate the matched receiver filter and its performance is almost insensitive to the sampling time. One drawback of the HBSE is its tendency to tap-wandering [2], which may lead to performance degradation due to coefficient saturation or overflow.

The new hybrid equalizer is presented in Sec. 2, and hereafter it is compared to the BSE and HBSE for a digital radio-link channel. Sec. 4 compares the HBSE and the hybrid equalizer concerning their tap-wandering tendencies. Finally, the results are summarized.

2. THE HYBRID EQUALIZER

The idea for the hybrid equalizer is based on the observation, that the distance between the optimum sampling time and the actual sampling time in the BSE is in the range of $[-T/2 \ldots +T/2]$, when the sampling time changes between $\pm T/2$. In the HBSE this maximum distance is halved due to the doubled sampling rate resulting in a significantly reduced sensitivity to changes of the sampling time. As sampling time changes are limited to the interval $[-T/2 \ldots +T/2]$ only the taps in the HBSE that are next to the center tap are close to the optimum sampling point. They are therefore most important for the reduced sampling time sensitivity of this equalizer. Hence, an equalizer with a few half Baud spaced taps in the center and Baud spaced taps next to them should have at least the reduced sampling time sensitivity of the HBSE.

Fig. 1 shows the block diagram of a hybrid equalizer with three half Baud spaced center taps and two Baud spaced taps on the sides.

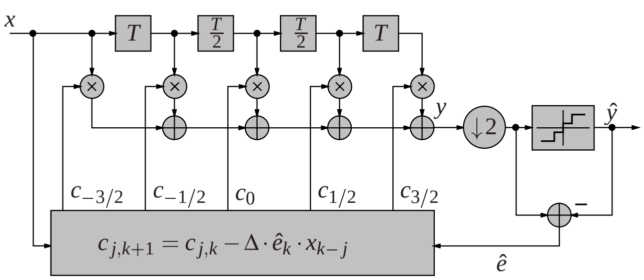

Figure 1: Hybrid 3/5 equalizer with three half Baud spaced taps and two Baud spaced outer taps

3. COMPARISON FOR A DIGITAL RADIO-LINK CHANNEL

We used a two path channel model with a delay of the second path of $\Delta\tau = 6.3ns$ [5]. For a symbol rate of 27.6 MBaud, typical for a radio link channel, $\Delta\tau = 0.174T$ holds. The channel has a transfer function of

$$S(\omega) \quad = \quad \alpha \left[1 - \beta \cdot e^{-j(\omega \cdot \Delta\tau - \Delta\phi)}\right] \quad , \tag{1}$$

where $\Delta\phi$ is the phase of the delayed path. This spectrum has notches, i.e. local minima, where $(\omega \cdot \Delta\tau - \Delta\phi) = (2n+1)\pi\,;\ n = 0, \pm 1, \pm 2, \ldots$. The position of the notch in the spectrum is determined by $\Delta\phi$. The relative notch depth A_{notch} resulting from Eqn. 1 is

$$A_{notch} \quad = \quad -20dB \cdot \log(1-\beta) \quad . \tag{2}$$

If the amplitude β of the echo approaches one, A_{notch} gets large and the spectrum is severely distorted.

Tab. 1 shows the features of the equalizers that were compared. The hybrid 5/11 equalizer can be interpreted as a Baud spaced equalizer with 9 taps, which is augmented by two half baud spaced taps $c_{\pm 1/2}$ next to the center tap.

The BSE was evaluated assuming an optimally matched filter before the input signal was sampled. Raised cosine pulses were used as its input signal. The equalizers with half Baud spaced sampling were analysed by using root raised cosine pulses, as their ability to incorporate the receiver pulse-forming should be reflected in the results.

Equalizer	# taps	# $\frac{T}{2}$ taps	outer taps
Baud spaced	11	-	$\pm 5T$
Half Baud spaced	11	11	$\pm 2.5T$
Hybrid 3/11	11	3	$\pm 4.5T$
Hybrid 5/11	11	5	$\pm 4T$

Table 1: Number of taps, half Baud spaced center taps and position of outer taps of the compared equalizers

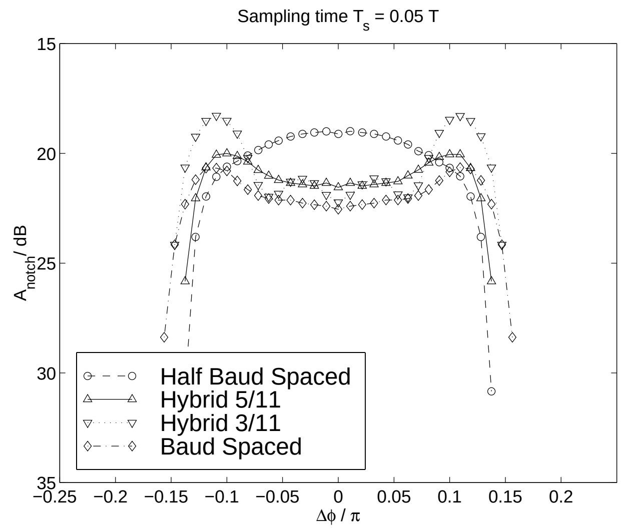

Figure 2: Signature patterns for optimum sampling time $T_{s,opt} = 0.05T$, $BER = 10^{-3}$

Fig. 2 shows the signature patterns for a bit error rate $BER = 10^{-3}$ of the four equalizers for a signal to noise ratio of $36dB$, 64 QAM transmission and an optimized sampling time of $T_s = 0.05T$. The roll-off factor of the raised cosine pulses was 0.33. The ideal signatures should be narrow, and the tolerated notch depth A_{notch} should be high for low system failure probabilities.

The signature of the HBSE is typical for this equalizer architecture. When the notch is in the middle of the transmission band ($\Delta\phi = 0$) it tolerates a notch depth of only $19.0dB$, because of its small time span of $5T$ (see Tab. 1). When the notch is moved to the slope of the transmission spectrum the HBSE performs better than the BSE.

The BSE has a superior performance when the notch is in the middle of the transmission band ($A_{notch} = 22.5dB$), but due to the aliasing of the overlapping spectra the tolerated notch depth decreases until $A_{notch} = 20.7dB$. The signature of the BSE is wider than the one of the HBSE.

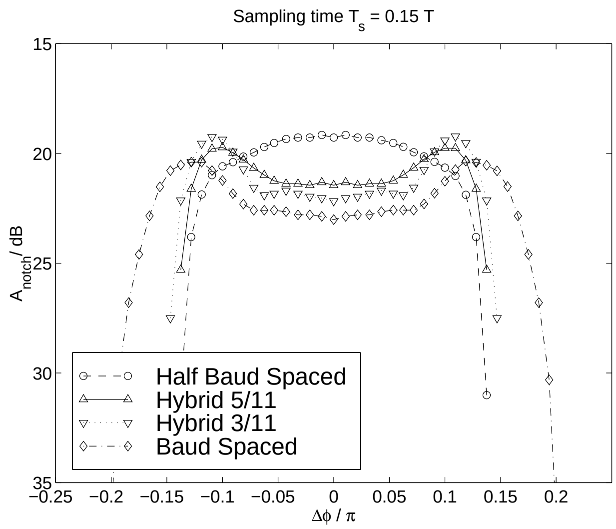

Figure 3: Signature patterns for sampling time $T_s = T_{s,opt} + 0.1T$, $BER = 10^{-3}$

The signature of the hybrid 5/11 equalizer is a composition of the behaviour of the BSE for in-band notches, where it tolerates a notch depth $A_{notch} = 21.5dB$, $1dB$ less than the BSE but $2.5dB$ better than the HBSE. When the notch moves into the slope of the spectrum the tolerated notch depth degrades to $A_{notch} = 20.0dB$. The performance in this area is only a little less than that of the HBSE.

Fig. 3 shows the signature patterns when the sampling time is increased by $0.1T$ from its optimum, and Fig. 4 when it is decresed by $0.1T$, respectively.

The patterns show that the performance of the BSE is

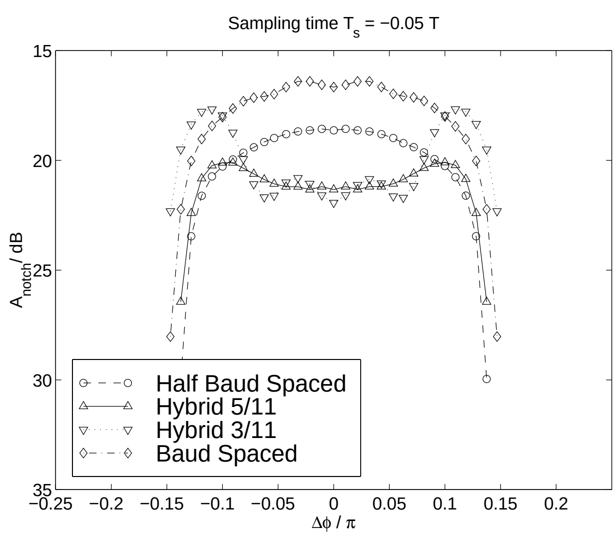

Figure 4: Signature patterns for sampling time $T_s = T_{s,opt} - 0.1T$, $BER = 10^{-3}$

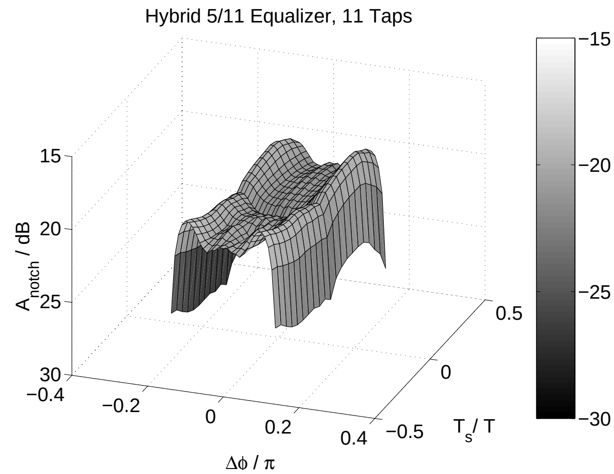

Figure 5: Signature patterns of hybrid 5/11 equalizer for sampling time $-T/2 \leq T_s \leq T/2$

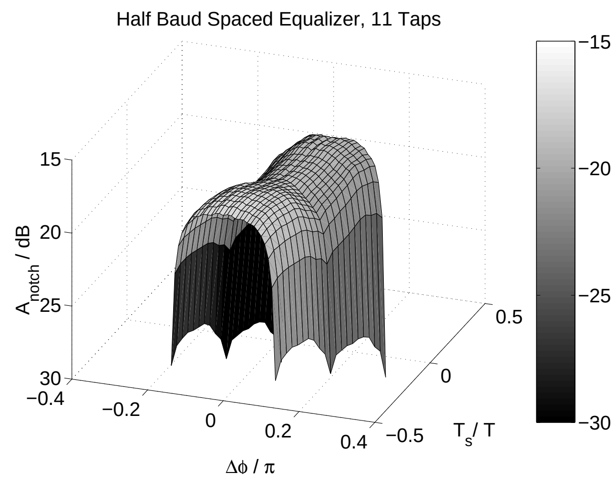

Figure 6: Signature patterns of HBSE for sampling time $-T/2 \leq T_s \leq T/2$

very sensitive to the sampling time, as it is expected. Its signature is getting much wider (Fig. 3) or the tolerated notch depth is significantly reduced (Fig. 4), when the sampling time is changed from its optimum value. The hybrid equalizers do not show these degradations. Fig. 5 shows the signature patterns of the hybrid 5/11 equalizer for sampling times $-T/2 \leq T_s \leq T/2$, which proof that its performance is nearly independent from the sampling time as that of the HBSE. The signature patterns of the HBSE are shown in Fig. 6.

4. TAP-WANDERING SENSITIVITY OF THE HYBRID EQUALIZER

Gitlin et. al. [2] have already shown that fractionally spaced equalizers may exhibit long-term instability when conven-

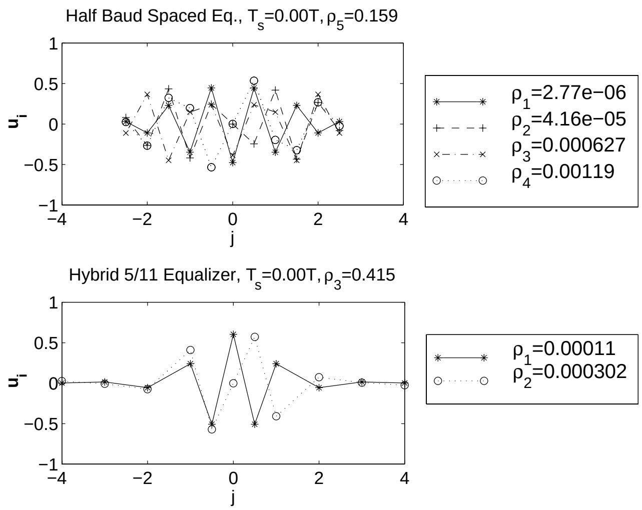

Figure 7: Smallest eigenvalues ρ_i and components of the corresponding eigenvectors u_i of a HBSE with 11 taps and a hybrid 5/11 equalizer

tional adaptation algorithms are used. The tendency to instability can be seen in almost vanishing eigenvalues of the channel-correlation matrix. In a finite-length HBSE, almost half of the eigenvalues are very small. When the coefficient vector is changed from its optimum in the direction of an eigenvector $C = C_{opt} + v \cdot u_i$, the MSE at the output increases to $MSE = MSE_{opt} + v^2 \cdot \rho_i$. Small eigenvalues therefore allow a significant deviation of the coefficient vector from its optimum without a substantial increase in the MSE. This results in large coefficient amplitudes which may finally lead to saturation or overflow.

Fig. 7 shows the smallest eigenvalues ρ_i and the components of the corresponding eigenvectors u_i of a HBSE with 11 taps in case of vanishing input noise, an undistorted channel and an optimum sampling time $T_s = 0T$. They are compared to the eigenvalues and corresponding eigenvectors of the hybrid 5/11 equalizer.

The HBSE has four small eigenvalues in the order of 10^{-3} down to 10^{-6}. The fifth eigenvalue $\rho_5 = 0.159$ is not critical. The eigenvectors $u_1, \ldots, u_4$ associated with $\rho_1, \ldots, \rho_4$ have significant components for all but the outermost coefficients $c_{\pm 5/2}$.

The hybrid 5/11 equalizer has only two small eigenvalues ρ_1 and ρ_2 in the order of 10^{-4}. The hybrid equalizer can be interpreted as a BSE, that has no critical eigenvalues, with two additional taps $c_{\pm 1/2}$ next to the center tap. The two taps add two degrees of freedom to the coefficient vector **C**, and as the samples at the taps $c_{-1/2}$, c_0 and $c_{1/2}$ are correlated, the eigenvalues for these additional eigenvectors are small. The eigenvectors u_1 and u_2 have significant components only within the five half Baud spaced center taps $c_{-1}, c_{-1/2}, \ldots, c_1$.

This property has been observed for sampling times $-T/2 \leq T_s \leq T/2$ as analyzed in Sec. 3 on the signature with BER=10^{-3}, and it can be understood by the fact, that the small eigenvalues belong to eigenvectors, that have a highpass transfer function in the band where the spectrum of the input signal vanishes. Only the $T/2$-spaced taps in the center of the hybrid equalizer are able to form such highpass functions.

The tendency for tap-wandering of the taps in the hybrid equalizer has been found to be reduced, as the smallest eigenvalue is increased by more than an order of magnitude compared to the HBSE. As the smallest eigenvalue is in the order of 10^{-4} the hybrid equalizer still requires the control of tap-wandering by e.g. tap-leakage. The property, that the components of the critical eigenvectors are concentrated to the center taps, may be exploited to concentrate the tap-leakage, that is usually applied uniformly to all taps in a HBSE, to the center taps in the hybrid equalizer.

Simulations using the L1-tap-leakage algorithm have shown, that tap-leakage concentrated on the five center taps can achieve the same minimum eigenvalues as conventional tap-leakage, at a significantly reduced output mean square error.

5. CONCLUSION

A hybrid equalizer using half Baud spaced center taps and Baud spaced border taps has been presented. The hybrid equalizer is a good compromise between a Baud spaced (BSE) and half Baud spaced equalizer (HBSE) with the same number of taps. It almost reaches the superior equalization capability of inband notches of the BSE, but without its severe sampling time sensitivity.

Notches in the slope of the transmission band are equalized nearly as good as with a HBSE, but the tendency to tap-wandering is reduced. The hybrid equalizer can incorporate the function of the receiver pulsforming as the HBSE.

6. REFERENCES

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