TRANSMISSION OF TWO USERS BY MEANS OF PERIODIC CLOCK CHANGES

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ABSTRACT

In modern telecommunications, it is often necessary to transmit several informations at the same time. It corresponds to multi-user transmission. In this paper, we present a new multi-user method by means of linear periodic time-varying filters. For two users, it is seen that the use of periodic clock changes simplifies the reconstruction. We apply this method to transmission of two stationary binary signals. Simulations show that perfect renconstruction is possible.

1. INTRODUCTION

These days, spread spectrum technics are often used for multi-user transmissions [1]. In [2] and [3], we showed that a linear periodic time-varying filter applied to a stationary signal introduces spread spectrum and permits multiple reconstructions of the initial signal. In this paper, the authors generalize these results and propose a new multi-user transmission method by means of linear periodic filters. When the initial signals are band-limited, we show that (in theory) perfect reconstructions can be obtained. It is realised submitting two stationary signals to particular linear periodic filters, the periodic clock changes [4]. An example is given for the transmission of binary signals.

2. MULTI-USER TRANSMISSION AND LINEAR PERIODIC TIME-VARYING FILTERS

2.1. Problem formulation

Let $Z_j = \{Z_j(t), \quad t \in \mathbf{R}\}$ with $j \in \{1..N\}$ be N random stationary processes of zero mean and mean square continuous. They admit respectively the Cramér-Loève spectral representations $\Theta_{Z_j}(\omega)$ [5], determined by:

$$Z_j(t) = \int_{-\infty}^{+\infty} e^{i\omega t} d\Theta_{Z_j}(\omega) \tag{1}$$

The principle of the new multi-user transmission proposed in this paper is firstly to subject each process Z_j to a linear periodic time-varying filter $\tilde{h}_j$ of period $T = 2\pi/\omega_0$ [6]. We note $H_{j\ t}(\omega)$ the frequency response of $\tilde{h}_j$. Secondly, the sum of the outputs of these filters is transmitted. We observe the zero mean process $X = \{X(t), \quad t \in \mathbf{R}\}$ of spectral representation $\Theta_X(\omega)$ given by:

$$X(t) = \sum_{j=1}^{N} \int_{-\infty}^{+\infty} e^{i\omega t} H_{j\ t}(\omega) d\Theta_{Z_j}(\omega) \tag{2}$$

Lastly, a linear reconstruction method of the Z_j is done when the parameters of the filters are known:

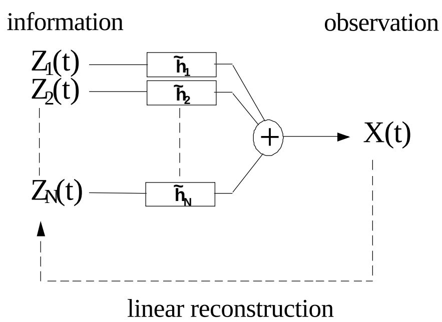

Fig.1, Multi-user system

2.2. Linear reconstruction

As $\tilde{h}_j$ is a linear periodic filter, its frequency response $H_{j\ t}(\omega)$ is periodic in t [6]. If we suppose that it admits a Fourier development, assumed to be sufficiently regular, we note:

$$H_{j\ t}(\omega) = \sum_{k=-\infty}^{+\infty} \psi_{j\ k}(\omega) e^{ik\omega_0 t} \tag{3}$$

where the $\{\psi_{j\ k}(\omega)\}_{k\in\mathbf{Z}}$ are the coefficients of the Fourier decomposition of $H_{j\ t}(\omega)$ given by:

$$\psi_{j\ k}(\omega) = \frac{1}{T}\int_{-T/2}^{T/2} H_{j\ t}(\omega)e^{-ik\omega_0 t}dt \qquad (4)$$

From (2) and (3), we deduce that X can be expressed as:

$$X(t) = \sum_{j=1}^{N}\sum_{k=-\infty}^{+\infty} e^{ik\omega_0 t}\int_{-\infty}^{+\infty} e^{i\omega t}\psi_{j\ k}(\omega)d\Theta_{Z_j}(\omega) \qquad (5)$$

Its spectral representation becomes:

$$d\Theta_X(\omega) = \sum_{k=-\infty}^{+\infty}\sum_{j=1}^{N}\psi_{j\ k}(\omega-k\omega_0)d\Theta_{Z_j}(\omega-k\omega_0) \qquad (6)$$

For this multi-user transmission, the spectral representation of the received signal X is an infinite sum of weighted shifted versions around ω_0 multiples of each $d\Theta_{Z_j}(\omega)$, where j describe all the users. For example, if the supports of the $d\Theta_{Z_j}(\omega)$ are included in $[-5\omega_0/2, 5\omega_0/2[$, the modulus of (6) can be represented by:

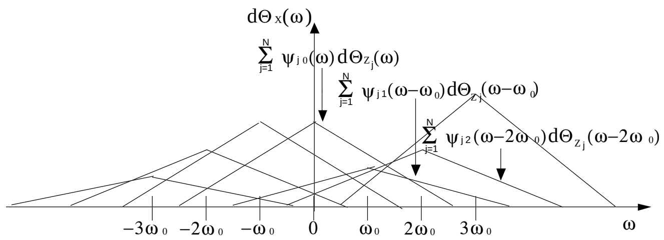

Fig.2, Observed spectral representation

We have now to reconstruct the Z_j only knowing the observed signal X and the linear periodic filters introduced $\tilde{h}_j$. In practice, we assume that the spectral support of the Z_j is included in $\Delta = [-\omega_0/2, \omega_0/2[$:

$$\forall\omega \notin [-\omega_0/2, \omega_0/2[, \quad d\Theta_{Z_j}(\omega) = 0 \qquad (7)$$

The equation (6) gives then:

$$\forall l \in \mathbf{Z}, \quad \forall\omega \in [-\omega_0/2, \omega_0/2[,$$
$$d\Theta_X(\omega + l\omega_0) = \sum_{j=1}^{N}\psi_{j\ l}(\omega)d\Theta_{Z_j}(\omega) \qquad (8)$$

We search the N spectral representations of the Z_j on Δ. We have N unknown parameters. Thus, N equations of (8), under some inversibility conditions, permit linear reconstruction of the Z_j.

3. APPLICATION FOR THE TRANSMISSION OF TWO USERS

3.1. General case

Let Z_1 and Z_2 be two stationary processes of spectral representation included in $\Delta = [-\omega_0/2, \omega_0/2[$. We transmit X, sum of Z_1 filtered by $\tilde{h}_1$ and of Z_2 filtered by $\tilde{h}_2$, where $\tilde{h}_1$ and $\tilde{h}_2$ are linear periodic time-varying filters of period $T = 2\pi/\omega_0$. Equations (8) give:

$$\forall l\in\mathbf{Z}, \quad \forall\omega\in[-\omega_0/2,\omega_0/2[,$$
$$d\Theta_X(\omega+l\omega_0)=\psi_{1\ l}(\omega)d\Theta_{Z_1}(\omega)+\psi_{2\ l}(\omega)d\Theta_{Z_2}(\omega) \qquad (9)$$

where the $\{\psi_{1\ k}(\omega)\}_{k\in\mathbf{Z}}$ and the $\{\psi_{2\ k}(\omega)\}_{k\in\mathbf{Z}}$ are the Fourier series development coefficients of $H_{1\ t}(\omega)$ and $H_{2\ t}(\omega)$, frequency responses of $\tilde{h}_1$ and $\tilde{h}_2$. Equations (9) permit to obtain $d\Theta_{Z_1}(\omega)$ and $d\Theta_{Z_2}(\omega)$ on Δ if and only if, for each ω of Δ, it exists two values of l for wich we can obtain two equations linearly independent. In practice, we attempt to verify that it exists two integers l_1 and l_2 such that, for each ω of Δ, $(\psi_{1\ l_1}(\omega), \psi_{2\ l_1}(\omega))$ and $(\psi_{1\ l_2}(\omega), \psi_{2\ l_2}(\omega))$ are not related:

$$\forall\omega\in[-\omega_0/2,\omega_0/2[, \quad \psi_{1\ l_1}(\omega)\psi_{2\ l_2}(\omega)-\psi_{1\ l_2}(\omega)\psi_{2\ l_1}(\omega)\neq 0 \qquad (10)$$

It leads to Z_1 and Z_2 reconstruction. In fact (9) corresponds for l_1 and l_2 to:

$$\forall\omega\in[-\omega_0/2,\omega_0/2[,$$
$$\begin{cases} \psi_{1\ l_1}(\omega)d\Theta_{Z_1}(\omega)+\psi_{2\ l_1}(\omega)d\Theta_{Z_2}(\omega)=d\Theta_X(\omega+l_1\omega_0) \\ \psi_{1\ l_2}(\omega)d\Theta_{Z_1}(\omega)+\psi_{2\ l_2}(\omega)d\Theta_{Z_2}(\omega)=d\Theta_X(\omega+l_2\omega_0) \end{cases} \qquad (11)$$

From (10), this system admits the following solution:

$$\forall\omega\in[-\omega_0/2,\omega_0/2[,$$
$$\begin{cases} d\Theta_{Z_1}(\omega)=\dfrac{\psi_{2\ l_2}(\omega)d\Theta_X(\omega+l_1\omega_0)-\psi_{2\ l_1}(\omega)d\Theta_X(\omega+l_2\omega_0)}{\psi_{1\ l_1}(\omega)\psi_{2\ l_2}(\omega)-\psi_{1\ l_2}(\omega)\psi_{2\ l_1}(\omega)} \\ d\Theta_{Z_2}(\omega)=\dfrac{\psi_{1\ l_2}(\omega)d\Theta_X(\omega+l_1\omega_0)-\psi_{1\ l_1}(\omega)d\Theta_X(\omega+l_2\omega_0)}{\psi_{1\ l_2}(\omega)\psi_{2\ l_1}(\omega)-\psi_{1\ l_1}(\omega)\psi_{2\ l_2}(\omega)} \end{cases} \qquad (12)$$

Z_1 and Z_2 are then the response of X through linear periodic time-varying filters. In general, the pair (l_1, l_2), verifying (10), is not unique. Thus, we can obtain several redundant solutions for (9). Our multi-user system can also realise error correction as it had be seen previously for the transmission of an unique information [2].

3.2. Use of particular clock changes

We suppose now that $\tilde{h}_1$ and $\tilde{h}_2$ are two periodic clock changes, that are particular cases of linear periodic time-varying filters [4]. They are defined by the two frequency responses $H_{1\ t}(\omega)$ and $H_{2\ t}(\omega)$ such that:

$$\forall n \in \{1, 2\},\ H_{n\ t}(\omega) = g_n(t) e^{-i\omega f_n(t)} \tag{13}$$

where $f_n(t)$ is a real mesurable function and $g_n(t)$ is a complex integrable function, $f_n(t)$ and $g_n(t)$ being $T = 2\pi/\omega_0$ periodic. For this application, we take:

$$f_1(t) = -f_2(t) = f(t) \text{ and } g_1(t) = g_2(t) = 1 \tag{14}$$

Let $\{\psi_k(\omega)\}_{k\in\mathbf{Z}}$ be:

$$\psi_k(\omega) = \frac{1}{T} \int_{-T/2}^{T/2} e^{-i\omega f(t) - ik\omega_0 t} dt \tag{15}$$

The frequency responses of $\tilde{h}_1$ and $\tilde{h}_2$ become:

$$\begin{cases} H_{1\ t}(\omega) = \sum_{k=-\infty}^{+\infty} \psi_k(\omega) e^{ik\omega_0 t} \\ H_{2\ t}(\omega) = \sum_{k=-\infty}^{+\infty} \psi_k(-\omega) e^{ik\omega_0 t} \end{cases} \tag{16}$$

Moreover, we consider that $f(t)$ has the following property:

$$f(-t) = -f(t) = f(t + T/2) \tag{17}$$

Then we have:

$$\begin{aligned} \psi_k(\omega) &= \frac{1}{T} \int_{-T/2}^{T/2} [\cos(\omega f(t))\cos(k\omega_0 t) - \sin(\omega f(t))\sin(k\omega_0 t)]\, dt \\ &= \frac{1}{T} \int_{0}^{T/2} [\cos(\omega f(t))\cos(k\omega_0 t) + (-1)^k \cos(\omega f(t - T/2))\cos(k\omega_0 t) \\ &\quad - \sin(\omega f(t))\sin(k\omega_0 t) - (-1)^k \sin(\omega f(t - T/2))\sin(k\omega_0 t)]\, dt \\ &= \frac{1}{T} \int_{0}^{T/2} [\cos(\omega f(t))\cos(k\omega_0 t) + (-1)^k \cos(\omega f(t - T/2))\cos(k\omega_0 t) \\ &\quad - (1 - (-1)^k)\sin(\omega f(t))\sin(k\omega_0 t)]\, dt \end{aligned} \tag{18}$$

It means that:

$$\psi_k(-\omega) = (-1)^k \psi_k(\omega) \tag{19}$$

We submitted Z_1 and Z_2 to the periodic clock changes $\tilde{h}_1$ and $\tilde{h}_2$ of frequency responses defined in (16) and verifying (19). We observe X, sum of the outputs of the filters, and we attempt to reconstruct the initial signals. As previously seen, the reconstruction is possible if the condition (10) is verified, in other words if it exists two integers l_1 and l_2, with $l_1 - l_2$ odd, such that:

$$\forall \omega \in [-\omega_0/2, \omega_0/2[, \quad \begin{cases} \psi_{l_1}(\omega) \neq 0 \\ \psi_{l_2}(\omega) \neq 0 \end{cases} \tag{20}$$

The system (12) becomes for l_1 even and l_2 odd:

$$\forall \omega \in [-\omega_0/2, \omega_0/2[, \quad \begin{cases} d\Theta_{Z_1}(\omega) = \dfrac{d\Theta_X(\omega + l_1\omega_0)}{2\psi_{l_1}(\omega)} + \dfrac{d\Theta_X(\omega + l_2\omega_0)}{2\psi_{l_2}(\omega)} \\ d\Theta_{Z_2}(\omega) = \dfrac{d\Theta_X(\omega + l_1\omega_0)}{2\psi_{l_1}(\omega)} - \dfrac{d\Theta_X(\omega + l_2\omega_0)}{2\psi_{l_2}(\omega)} \end{cases} \tag{21}$$

In this particular case, Z_1 and Z_2 reconstruction is then very simple to obtain.

4. EXAMPLE

We applied the previous result for multi-user transmission of binary signals. Thus, for this example, $Z_1(t)$ and $Z_2(t)$ are two stationary N.R.Z. signals wich spectrum is neglectible out of $[-\omega_0/2, \omega_0/2[$, it means that 99% of the spectrum is in $[-\omega_0/2, \omega_0/2[$. $f(t)$ is chosen equal to:

$$f(t) = -\alpha \sin(\omega_0 t) \tag{22}$$

$f(t)$ verifies (17) and the $\{\psi_k(\omega)\}_{k\in\mathbf{Z}}$ are given by:

$$\psi_k(\omega) = \frac{1}{T} \int_{-T/2}^{T/2} e^{i\omega\alpha\sin(\omega_0 t) - ik\omega_0 t} dt = J_k(\alpha\omega) \tag{23}$$

where $J_k(\alpha\omega)$ is the k'th order Bessel function. We take $\alpha = 1$ and $\omega_0 = 16\pi$. The transmitted signal is given by Figure 3.

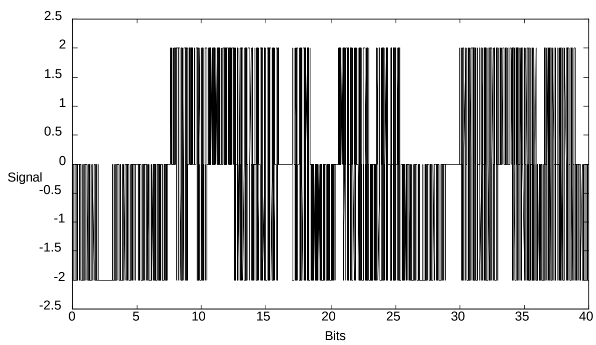

Fig. 3, Observed signal

It is impossible to recognize Z_1 and Z_2 directly observing X. We reconstruct then $Z_1(t)$ and $Z_2(t)$ thanks to (21) for $l_1 = 0$ and $l_2 = 1$. Figures 4 and 5 show the initial signal and the reconstructed signal respectively for $Z_1(t)$ and $Z_2(t)$.

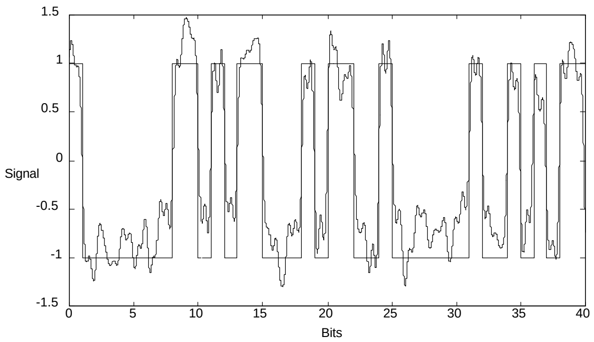

Fig. 4, Reconstruction of $Z_1(t)$

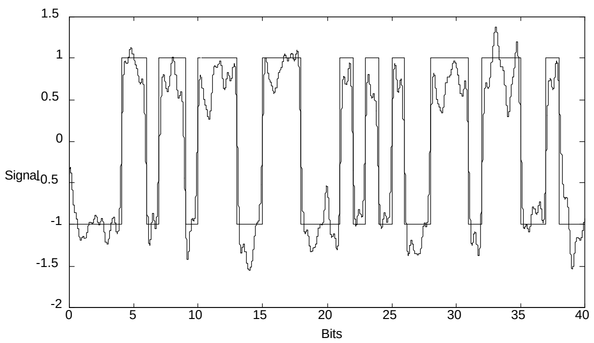

Fig. 5, Reconstruction of $Z_2(t)$

For binary signals perfect reconstruction is then possible after a bit detection.

5. CONCLUSION

In this article we have presented an original method of multi-user communication system. We have used linear periodic time-varying filters. In particular, it was shown that perfect reconstruction was simple to obtain with two users by means of periodic clock changes. Simulations have been done with two signals. They can yet be generalised with any number of stationary signals.

6. REFERENCES

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