CONSTRAINED EQUALIZERS AND PRECODING FOR MAGNETIC STORAGE CHANNELS

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ABSTRACT

The nonlinear write process of magnetic recording allows to write the symbols ± 1 only. The magnetic channel is a differentiating channel. The locations of the transitions from +1 to -1 and vice versa in the input signal to the magnetic channel are important for the received waveform.
This paper defines a noise enhancement constrained, finite dimensional equalizer. This equalizer trades some misequalization of the data signal for less noise enhancement after the equalizer. In a second step the misequalization is decreased by the precoding. Precoding shapes the communication signal before entering the communication channel. Since precoding works on the noise free signal, there is no noise enhancement. Since precoding in magnetic channels is limited to shifting the positions of the transitions around, precoding does not allow for full equalization at the receiver. Therefore the equalizer in the receiver and the precoder are optimized. In order to find the optimal transition positions a linearized representation of the transition shift is produced. This representation leads to a constrained optimization problem.

1. INTRODUCTION

Maximum likelihood detection in the presence of long channel impulse responses is expensive when built in hardware at the speeds magnetic recording demands. Thus most schemes use an equalizer to shape the channel impulse response into a partial response target. The maximum likelihood detector uses this shorter target. Since the target impulse response is shorter than the channel impulses response the equalizer slims the pulse by boosting the higher frequency components of the signal. This boosts also the noise for the higher frequencies. Boosting the noise results in a suboptimal behavior of the ML detector.

Equalization using finite dimensional equalizers is considered in [1]. Here we consider a data source $d(k)$, an equivalent digital magnetic channel $h_C(k)$, an AWGN noise source $n(k)$, an equalizer $h_E(k)$ and a target $h_T(k)$ for a partial response maximum likelihood algorithm. The equalizer is constrained by its noise enhancement. A linearization of the shift of the position of a transition is the differentiation of the analog magnetic channel model. Since we model the analog magnetic channel as a superposition of sincs, the differentiation is superposition of $\frac{\cos \pi k}{k} - \frac{\sin \pi k}{\pi k^2}$. The latter term vanishes except for the $k = 0$ term, where the sum of the two terms vanishes. Thus there are two filters we can adapt, the equalizer and the transition shift filter $h_A(k)$.

2. CONSTRAINED EQUALIZER

2.1. Finite Dimensional Equalizer

Given the equivalent digital channel impulse response $h_C(k)$, the finite dimensional zero forcing equalizer is found by solving the following problem

$$\min_{h_E} |h_C * h_E - h_T|^2,$$

where $*$ denotes convolution. In matrix notation this minimization problem can be expressed with the help of the Toeplitz matrix:

$$T_h = \begin{pmatrix} h(0) & 0 & \dots & 0 \\ h(1) & h(0) & & 0 \\ \vdots & \vdots & & \vdots \\ h(N-1) & h(N-2) & & \vdots \\ 0 & h(N-1) & & \vdots \\ \vdots & & & \vdots \\ 0 & 0 & & h(N-1) \end{pmatrix}$$

$$\min_{h_E} (T_{h_C} h_E - h_T)^T (T_{h_C} h_E - h_T)$$

and its solution is:

$$h_E = (T_{h_C}^T T_{h_C})^{-1} T_{h_C}^T h_T.$$

2.2. Maximal Boost Constrained Equalizer

If the maximal boost of the equalizer is to be constrained by a constant, the following constrained optimization problem can be set up:

$$\min_{h_E} \max_{v(\varphi)} (T_{h_C} h_E - h_T)^T (T_{h_C} h_E - h_T)$$

$$\text{subj.to} \quad \mathrm{h_E^T v(\varphi) v(\varphi)^* h_E \le C h_T^T v(\varphi) v(\varphi)^* h_T},$$

$$\varphi_{\min} \le \varphi \le \varphi_{\max}.$$

The noise enhancement of the equalizer within the frequency band $\varphi_{\min} \le \varphi \le \varphi_{\max}$ is bounded by the spectral shaping of the target within the signal band. Some code rate loss is accepted in magnetic recording, to code out sequences that would rely on transmission of energy close to dc or Nyquist frequency.

Constraining the equalizer will raise the misequalization. This leads to misequalization data noise at the detector. To lessen this effect, precoding is considered.

3. PRECODING

Since the precoder works on the data signal, the channel impulse response can be equalized by the precoder without noise boost. One major draw-back in the application of a precoder is that the sender has to have knowledge about the channel. In transmission channels this involves a feedback channel from the receiver to the sender. The magnetic channel does not change much over the lifetime of a magnetic disk. Thus during calibration of the disk, one is able to measure the magnetic channel impulse response and compute the optimal precoder for this channel.

Due to the nonlinear characteristics of the magnetic read channel, the write waveform consists of two symbols ± 1. Since the magnetic channel is differentiating, only the transition positions are relevant for the received signal from the disk. The precoder shifts these transitions inside the bit window. This puts a limitation on the available precoding equalization. Thus precoding as a stand-alone equalizer seems unable to handle all the equalization needs of a high-density magnetic channel. It is useful in conjunction with a constrained, finite dimensional equalizer.

3.1. Linearization of the Transition Shift

At the receiver the signal passes a matched filter $\hat{h}_C(t)$, which is the time reverse of the analog channel impulse response $h_C(t)$. Following sampling the noise of the signal is equalized by the noise whitening filter $h_W(k)$. This noise whitening filter can be pulled back into the analog domain as $h_W(t) = \sum h_W(k)\delta(t - kT)$. Here $\delta(t)$ is the Dirac function and T is the sampling period. Thus there is an equivalent analog transfer function $h_A(t) = h_C(t) * \hat{h}_C(t) * h_W(t)$ for the whitened matched filter.

As an approximation of the equivalent analog transfer function $h_A(t)$, one can consider the interpolation of the equivalent discrete transfer function of the channel consisting of the magnetic channel, the matched filter and a discrete noise whitening filter

$$\tilde{h}_A(t) = \sum h_D(k) \frac{\sin \pi(k - t/T)}{\pi(k - t/kT)}.$$

A transition shift will result in an impulse that can be approximated by the Taylor series development for the shift

$$h(t + \Delta) = h(t) + \frac{\Delta}{1!} \frac{dh(t)}{dt} + \ldots$$

If the approximation to the equivalent analog impulse response $\tilde{h}_A(t)$ is used, the derivative of the impulse response can be computed in terms of derivatives of the sinc function $\frac{\sin(\pi t)}{\pi t}$. The first derivative is

$$\frac{d}{dt} \frac{\sin(\pi t)}{\pi t} = \frac{\cos(\pi t)}{t} - \frac{\sin(\pi t)}{\pi t^2}.$$

Evaluating the derivative for the integer values $k = \ldots, -1, 0, 1, \ldots$ results in

$$g(k) = \frac{d}{dt} \frac{\sin(\pi t)}{\pi t}(t = k) = \begin{cases} \frac{\cos \pi k}{k}, & \text{if } k \neq 0; \\ 0, & \text{if } k = 0. \end{cases}$$

The second derivative

$$\frac{d^2}{dt^2} \frac{\sin(\pi t)}{\pi t} = -\frac{\sin(\pi t)\pi}{t} - \frac{2\cos(\pi t)}{t^2} + \frac{2\sin(\pi t)}{\pi t^3}$$

evaluates 0 at 0. Thus the rest of the Taylor series will be dominated by the third derivative of the channel impulse response with respect to time.

$$h(k + \Delta) \approx h(k) + \Delta g(k) * h(k)$$

Using the linearization of the transition shift, the precoder can be modeled by a direct path and the convolution of the precoding filter and the first derivative of the sinc.

3.2. Precoding for the Constrained Equalizer

Given the impulse response of the constrained equalizer $h_E(k)$, one can compute the error $E_x(k)$ between the target impulse response $h_T(k)$ and the convolution of the magnetic channel $h_C(k)$ and the constrained equalizer $h_E(k)$.

$$E_x(k) = h_T(k) - h_C(k) * h_E(k)$$

A precoder impulse response $h_A(k)$ is convolved with the linear approximation of a transition shift $h_P(k) = g(k) * h_C(k) * h_E(k)$ to yield the influence of the precoder. This would yield the equation

$$T_{h_P} h_A = E_x,$$

which should be evaluated in the mean square sense. This formulation is still incorrect, since there are not always edges to move around at the input of the precoder. If one considers the statistical occurance of the edges, then the equation is

$$T_{h_P} P_d D_{dA} h_A = D_{dx} E_x.$$

Here P_d is a function depending on a data pattern d and representing whether an edge is present or not and can be moved or not. The matrices D_{dA} and D_{dx} are the influences of the data pattern. This equation has to be solved in a mean square sense:

$$D_{dA}^T P^T T_{h_P}^T T_{h_P} P D_{dA} h_A = D_{dA}^T P^T T_{h_P}^T D_{dx} E_x.$$

Since this equation has to hold for all data patterns, this equation can be solved using expected values. The left hand side involves the fourth order expected values of the data, the right hand side third order expected values. The data consist of the transitions and has the symbol set $-1, 0, 1$. No transition is represented by 0 and has probability $\frac{1}{2}$, positive and negative transitions are represented by ± 1 and have probability $\frac{1}{4}$. Disregarding the fact that positive and negative transitions have to interlace each other for a valid magnetic recording signal, the expected values can be expressed as:

$$\mathrm{E}\left(|d_k|\ d_l\ d_m\right) = \begin{cases} 0, & \text{if } l \neq m; \\ \frac{1}{4}, & \text{if } l = m,\ k \neq l; \\ \frac{1}{2}, & \text{if } k = l = m. \end{cases}$$

$$\mathrm{E}\left(|d_k|\ |d_l|\ d_m\ d_n\right) = \begin{cases} 0, & \text{if } m \neq n; \\ \frac{1}{8}, & \text{if } m = n,\ k \neq l \neq m; \\ \frac{1}{4}, & \text{if } m = n,\ k = l,\ l \neq m; \\ \frac{1}{4}, & \text{if } m = n = k,\ l \neq m; \\ \frac{1}{4}, & \text{if } m = n = l,\ k \neq l; \\ \frac{1}{2}, & \text{if } k = l = m = n. \end{cases}$$

The matrix R is defined as $R = T_{h_P}^T T_{h_P}$. Let e_S be the Sth unit vector and $Diag(R)$ be the diagonal matrix that just has the diagonal values of the matrix R. Then the equation for the precoder coefficients can be written:

$$\left(R + Diag(R) + e_S e_S^T R + R e_S e_S^T\right) h_A =$$

$$2\left(T_{h_P} + T_{h_P} e_S e_S^T\right)^T E_x.$$

If the precoder could take influence at all times, the solution would zero out the subspaces with the K_A largest singular values. K_A is the dimension of the precoding filter h_A. Since there are only the transitions to be shifted, when the write current changes, the subspaces are not zeroed out, but diminished by a factor. Thus there is only a modest performance gain achievable by precoding which shifts the transition times of the write current.

4. NUMERICAL RESULTS

4.1. Modeling of the magnetic storage channel

The channel impulse response of the magnetic storage channel is modeled by a blended Lorentzian-Gaussian pulse. A Lorentzian isolated pulse is:

$$h_L(w,t) = \frac{\frac{1}{w}}{1 + \left(\frac{(2t)}{w}\right)^2}.$$

A Gaussian isolated pulse is

$$h_G(w,t) = \frac{1}{w}\exp\left(-kt^2\right), \quad k = \frac{4\ \ln(.5)}{w^2}.$$

A 50:50 pulse blend is chosen to model the magnetic head response. It rolls off faster at high frequencies than the Lorentzian model. The parameter w is the width of the pulse at half the peak amplitude. To model a user channel rate of 3 and a code rate of 16/17, the parameter w is $3\frac{3}{16}$. The channel autocorrelation function was computed at an oversampling factor of 4. A spectral factorization gives the minimal phase digital impulse response of the channel. The desired target is EEPR4, $h_T = (1 \quad 2 \quad 0 \quad -2 \quad -1)$.

4.2. Constrained Equalizer

The finite dimensional (length 8) equalizer is constrained to have not more than 15dB noise enhancement in the band $0.0234 \leq \varphi \leq 0.4512$. Here φ=0.5 corresponds to the Nyquist frequency. In Figure 1, the impulse response of the constrained equalizer is shown as a solid line, the impulse response of the unconstrained equalizer as a dash dotted line. The slightly higher high frequency content of the unconstrained equalizer is apparent from the time domain signals.

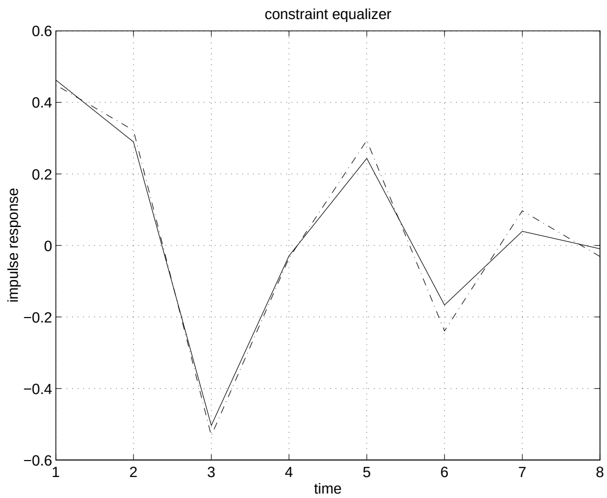

Figure 1: Constrained Equalizer Impulse Response

4.3. Precoded Signal

The precoder shift impulse response is shown in Figure 2. The transition the output of this filter influences is the fourth one. Note the relatively small scale of the y-axis, indicating only small overall shifts in the transitions. On one hand for small shifts the linear model for the shift holds best. On the other hand small shifts imply only a small overall influence on the data noise. The

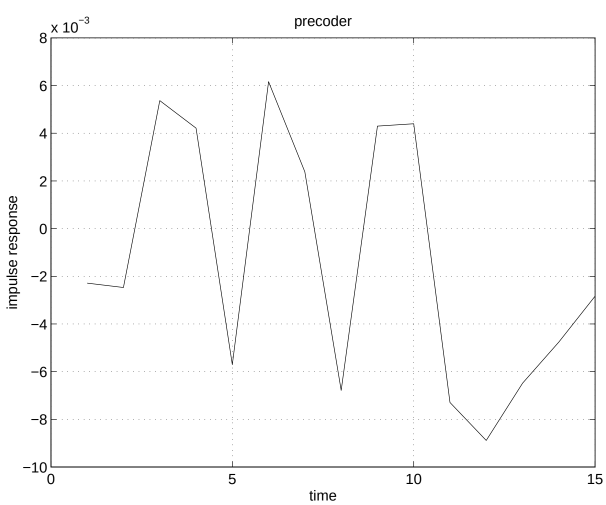

Figure 2: Precoder Shift Impulse Response

data noise is graphed in Figure 3 for the constrained equalizer (dash dotted line) and for the precoded constrained equalizer (solid line). Precoding allows one to drop the misequalization by about 0.5dB. The maximal

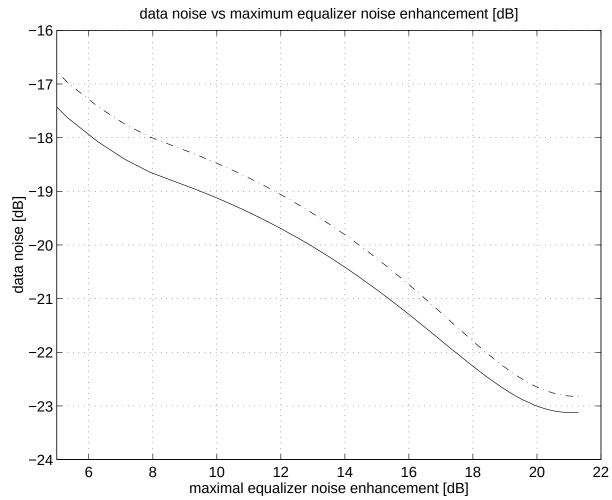

Figure 3: Data Noise vs. Equalizer Noise Enhancement

shift $S_{\max}$ of the precoder was computed as

$$S_{\max} = \sum_{k=1}^{K} |h_A(k)|.$$

The maximal shift $S_{\max}$ is shown vs. the maximal equalizer noise enhancement in Figure 4. Even the worst case transition shift for a highly constrained equalizer of 5dB does not get higher than approx. 11% of a bit window.

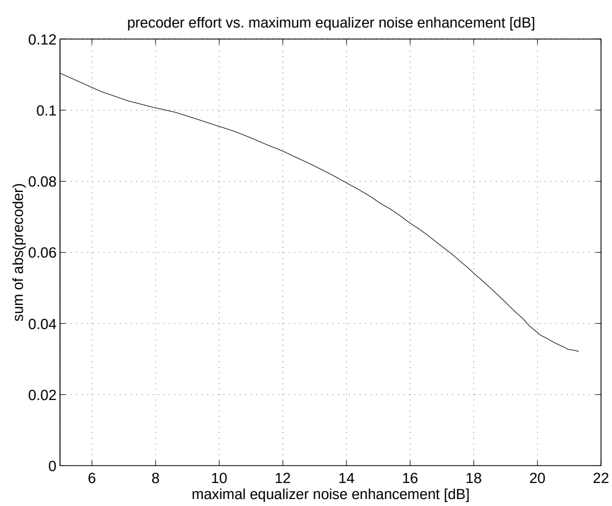

Figure 4: Precoder Shift Effort

5. BIBLIOGRAPHY

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