NONLINEAR SYSTEM IDENTIFICATION OF HYDRAULIC ACTUATOR FRICTION DYNAMICS USING A HAMMERSTEIN MODEL

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ABSTRACT

We present two Hammerstein-type models for parametric system identification of the lip seal friction process in a hydraulic actuator. Adaptive algorithms with least squares criteria are derived, and the performances of the two models are evaluated using experimental results.

1. INTRODUCTION

Lubricated sliding lip seals are important components in many hydraulic devices, such as actuators, solenoid valves, etc. The requirements imposed by today's high precision machines motivates the precise simulation of friction between these seals and sliding components, and there has been recent efforts to model the friction process using system identification techniques [1].

In this research, we focus on the friction of a lip seal in a hydraulic actuator shown in Fig.1. During the operation of the actuator, lip seal keeps the lubricant from leaking out of the high pressured chambers while the piston separates the actuator into two pressure regions. Since the friction between the seal and sliding shaft affect the performance of the whole system, a decent model of the friction process leads to design of a superior system.

The objective of this research is to develop models which successfully simulate this friction process with the velocity of the sliding shaft $v[k]$ and lip seal friction signal $f[k]$ as the input and output signals, respectively. Friction is a highly complicated nonlinear process, which depends on the viscosity of the lubricant, characteristic of lip seal material, roughness of the sliding surfaces, hydraulic pressure, ambient temperature and relative velocity between the surfaces, etc [1, 2, 3]. It is very difficult to use all these parameters to develop a model of friction process. Even if we successfully develop such a model, the parameters of the model often don't have an immediate relationship with physical parameters of the materials [1]. This situation motivates us to use an empirical system identification techniques. The advantage of this approach is that it is much easier to develop a model since it has fewer parameters than the models based on physics, while leading to a model with quite satisfactory performance.

In this paper, we presents some Hammerstein type models for lip seal friction process. We use an adaptive algorithm with least squares criterion to fit the parameters of the models using the measurements of the velocity of the sliding shaft and lip seal friction signal at various operating conditions.

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2. HAMMERSTEIN TYPE MODELS

The Hammerstein model is a nonlinear model, which makes the assumption that the nonlinearity of the system can be separated from the system dynamics. In a single-input-single-output case, the Hammerstein model structure can be described by the equation

$$f[k] = H(q^{-1}) \cdot g(v[k]),$$

where q^{-1} is the delay operator, $g(\cdot)$ is the static nonlinearity, $H(q^{-1})$ is the transfer function of the linear part, and $v[k]$ and $f[k]$ are the input and output of the model, respectively. Fig.2 (a) is a block diagram of a Hammerstein model. Despite the simplicity of its structure, the Hammerstein model works quite well for various nonlinear system identification problems.

Since $g(\cdot)$ is an element of l^2 space, it can be parameterized by expressing as a linear combination of basis function $\{\psi_i(\cdot)\}$ of l^2 space. A Daubechies wavelet basis function is used for the representation of the nonlinear mapping function $g(\cdot)$. The orthogonality, and time and frequency localization property of Daubechies wavelet basis function enable us to represent a function with fewer parameters. $g(\cdot)$ is represented using wavelet basis functions as

$$g(v) = \sum_{m,n} w_{m,n} \psi_{m,n}(v)$$

where $\psi(\cdot)$ is the mother wavelet, and

$$\begin{aligned} w_{m,n} &= < g(v), \psi_{m,n}(v) >, \\ \psi_{m,n}(v) &= 2^{-m/2} \psi(2^{-m} v - n). \end{aligned}$$

For the linear part, a moving average (MA) filter is used:

$$H(q^{-1}) = a_0 + a_1 q^{-1} + \cdots + a_N q^{-N}.$$

As shown in Section 3, by using MA filter, along with the parameterization of the nonlinear mapping function $g(\cdot)$, we can develop an adaptive algorithm with a *unique* global minimum.

If we have a priori information about the nonlinear mapping function of the Hammerstein model, we can take advantage of the information to develop a model. A steady state model which illustrates the nonlinear relationship between the velocity and friction of a lip seal in a hydraulic actuator when the relative velocity between the seal and sliding shaft is constant has been given in [2]. Given the physical parameters of the seal, the model provides a steady state nonlinear relationship between the velocity of the sliding shaft and friction force that the seal experiences. Fig.3 shows three dimensional plots of this steady state nonlinear model of lip seal friction. This nonlinear model is used in the place of the nonlinear mapping function of the Hammerstein model to form a model similar to Hammerstein model. This model is combined with a Hammerstein model in parallel form to complement the difference between the theoretical model and the actual velocity/friction relationship. Fig.2 (b) shows a block diagram of the parallel model.

For the parallel model, the output of the model can be described by the equation

$$f[k] = H_1(q^{-1}) \cdot d(v[k]) + H_2(q^{-1}) \cdot g(v[k]).$$

where $d(\cdot)$ is the theoretical nonlinear model of lip seal friction, $g(\cdot)$ is the nonlinear mapping function, and

$$\begin{aligned} H_1(q^{-1}) &= a_{10} + a_{11}q^{-1} + \cdots + a_{1N_1}q^{-N_1}, \\ H_2(q^{-1}) &= a_{20} + a_{21}q^{-1} + \cdots + a_{2N_2}q^{-N_2}. \end{aligned}$$

3. LMS ADAPTIVE ALGORITHM

In this section stochastic adaptive algorithms are derived for the Hammerstein and Parallel model with least squares criterion.

Let $v[k]$ be the velocity signal of the sliding shaft, and $\tilde{f}[k]$ be the output of $g(\cdot)$ in Fig.2 (a). Then $\tilde{f}[k]$ can be represented by wavelet basis function as

$$\tilde{f}[k] = g(v[k]) = \sum_{m,n} w_{m,n}\psi_{m,n}(v[k]) = W^T\Psi(v[k]),$$

where W and $\Psi(v[k])$ are column vectors whose elements are $\{w_{m,n}\}$ and $\{\psi_{m,n}(v[k])\}$, respectively. Then the lip seal friction signal $f[k]$ can be written as

$$f[k] = H(q^{-1})\tilde{f}[k] = A^T\tilde{F}[k],$$

where
$$\begin{aligned} A &= [a_0\ a_1\ \cdots\ a_N]^T \\ \tilde{F}[k] &= [\tilde{f}[k]\ \tilde{f}[k-1]\ \cdots\ \tilde{f}[k-N]]^T. \end{aligned}$$

Define

$$\Theta = [A^T\ W^T]^T, \quad f_\Theta[k] = H(q^{-1},\Theta) \cdot \tilde{f}[k],$$

where $H(q^{-1},\Theta)$ is the transfer function with parameter vector Θ. Then, the cost function to be minimized is

$$\begin{aligned} C(\Theta) &= \frac{1}{2}E\{f[k] - f_\Theta[k]\}^2 = \frac{1}{2}E\{e^2[k]\} \\ &= \frac{1}{2}(\sigma_f^2 + A^TR_{\tilde{F}\tilde{F}}A - 2A^TR_{\tilde{F}y}) \end{aligned}$$

where $f[k]$ is the measurement of friction signal, $e[k]$ is the error signal, σ_f^2 is the variance of $f[k]$, and

$$\begin{aligned} R_{\tilde{F}\tilde{F}} &= \begin{bmatrix} W^TR(0)W & \cdots & W^TR(-N)W \\ \vdots & \ddots & \vdots \\ W^TR(N)T & \cdots & W^TR(0)W \end{bmatrix}, \\ R_{\tilde{F}y} &= \begin{bmatrix} W^TR_{\psi y}(0) \\ \vdots \\ W^TR_{\psi y}(N) \end{bmatrix}, \\ R(i) &= E\{\Psi[k-i]\Psi^T[k]\}, \\ R_{\psi y}(i) &= E\{\Psi[k-i]y[k]\}. \end{aligned}$$

$C(\Theta)$ has a unique global minimum, since it is a quadratic function of A and W, or equivalently of the parameter vector Θ. By parameterizing $g(\cdot)$ and using an MA filter, the nonlinear system identification problem becomes a linear least squares problem with unique global minimum.

To find the parameter vector Θ, the following stochastic gradient search algorithm is used.

$$\Theta_{k+1} = \Theta_k - \mu\hat{\nabla}[k],$$

where μ is the convergence parameter, $\hat{\nabla}[k]$ is the estimation of the gradient at time k, and Θ_k is the parameter vector at time k. We use

$$\hat{\nabla}[k] = \frac{\partial}{\partial\Theta}\frac{1}{2}e^2[k]$$

as the estimation of the gradient. Then, the resulting adaptive algorithm is

$$\begin{aligned} A_{k+1} &= A^*_{k+1}/\parallel A^*_{k+1}\parallel, \quad A^*_{k+1} = A_k + \mu e[k]\tilde{F}[k], \\ W_{k+1} &= W_k + \mu e[k]A(q^{-1},\Theta_k)\Psi[k], \end{aligned}$$

where A_k and W_k are A and W vectors at time k.

Similar adaptive algorithm can be derived for the Parallel model. Let $\tilde{f}_1[k]$ and $\tilde{f}_2[k]$ be the output of $d(\cdot)$ and $g(\cdot)$, respectively. Then $\tilde{f}_2[k] = W^T\Psi[k]$. The output of the model can be written as

$$\begin{aligned} f[k] &= H_1(q^{-1}) \cdot \tilde{f}_1[k] + H_2(q^{-1}) \cdot \tilde{f}_2[k] \\ &= A_1^T\tilde{F}_1[k] + A_2^T\tilde{F}_2[k], \end{aligned}$$

where
$$\begin{aligned} A_1 &= [a_{10}\ a_{11}\ \cdots\ a_{1N_1}]^T, \\ A_2 &= [a_{20}\ a_{21}\ \cdots\ a_{2N_2}]^T, \\ \tilde{F}_1[k] &= [\tilde{f}_1[k]\ \tilde{f}_1[k]\ \cdots\ \tilde{f}_1[k-N_1]]^T, \\ \tilde{F}_2[k] &= [\tilde{f}_2[k]\ \tilde{f}_2[k]\ \cdots\ \tilde{f}_2[k-N_2]]^T. \end{aligned}$$

Define

$$\begin{aligned} \Theta &= [A_1^T\ A_2^T\ W^T]^T, \\ f_\Theta[k] &= H_1(q^{-1},\Theta) \cdot \tilde{f}_1[k] + H_2(q^{-1},\Theta) \cdot \tilde{f}_2[k]. \end{aligned}$$

Then, the cost function to be minimized is

$$C(\Theta) = \frac{1}{2}E\{f[k] - f_\Theta[k]\}^2 = \frac{1}{2}E\{e[k]\}^2,$$

which is also a quadratic function of parameter vector Θ. Following is the stochastic adaptive algorithm for the Parallel model.

$$
\begin{aligned}
A_{1,k+1} &= A_{1,k} + \mu e[k]\tilde{F}_1[k], \\
A_{2,k+1} &= A^*_{2,k+1} / \parallel A^*_{2,k+1} \parallel, \\
&\quad \text{where } A^*_{2,k+1} = A_{2,k} + \mu e[k]\tilde{F}_2[k], \\
W_{k+1} &= W_k + \mu e[k] H_2(q^{-1}, \Theta_k)\Psi[k].
\end{aligned}
$$

4. EXPERIMENTAL RESULTS

The experimental data were obtained from a data acquisition system for an actual hydraulic actuator. The system was driven by an eccentric drive system to generate periodic velocity and friction signals. The details of the data acquisition system are explained in [2]. In the simulation, the nonlinear mapping function $g(\cdot)$ was represented by Daubechies wavelet basis function of order 3, ${}_3\psi(\cdot)$ [6], and the highest resolution used was $m = -1$. For all cases, the order of $H(q^{-1})$, $H_1(q^{-1})$, and $H_2(q^{-1})$ was 8. Using higher order for the MA filters slightly improved the performance of the models, but the difference were not significant.

Fig.4 shows velocity and friction signals at various conditions; (a) 130^oF, 50 psi, $T = 5$ sec, (b) 130^oF, 50 psi, $T = 1$ sec and (c) 130^oF, 200 psi, $T = 5$ sec, where T represents the period of the essentric drive system.

Fig.5 and 6 show the experimental results of the Hammerstein model and the Parallel model, respectively. The results show that, for both models, the estimated lip seal frictions converge to the actual friction signal quite closely, but the Parallel model works better than the Hammerstein model. Note that the Parallel model can give a good estimate of the friction signal even before the convergence. That is because the theoretical nonlinear model provides a very good initial condition for the Parallel model. Also note that the estimated nonlinear mapping function $g(\cdot)$ of the Hammerstein model in Fig.5 have similar shape to the theoretical nonlinear model except for the magnitude and sign. The difference of the magnitude is caused by the normalization process of the adaptive algorithm.

5. CONCLUSION

In this paper, two models for the nonlinear system of the friction process of lip seal in hydraulic actuator have been developed using the Hammerstein model structure. By parameterizing the nonlinear mapping function and choosing MA filter for the linear block, adaptive algorithms with a unique global minimum was also derived. By incoperating a theoretical nonlinear model of the lip seal in the Parallel model, we were able to use a priori knowledge of the lip seal to improve the system identification model. The improvement came in two folds. First, the estimated friction signal converged more closely to the actual friction signal. Secondly, the theoretical model increased the convergence speed of the model by providing a good initial condition.

6. REFERENCES

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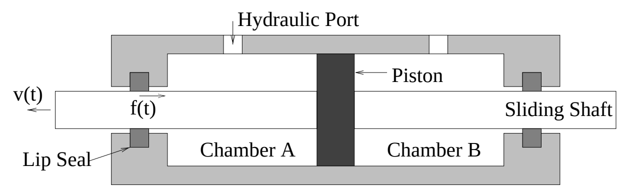


Figure 1: Hydraulic actuator in a power steering system. $f(t)$: the lip seal friction, $v(t)$: velocity of the shaft.

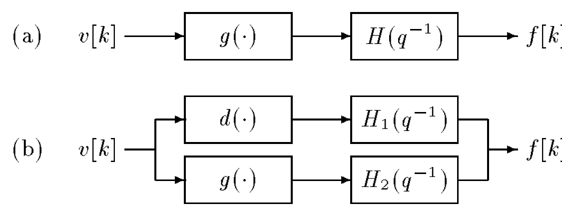


Figure 2: (a) Hammerstein model. (b) Parallel model.

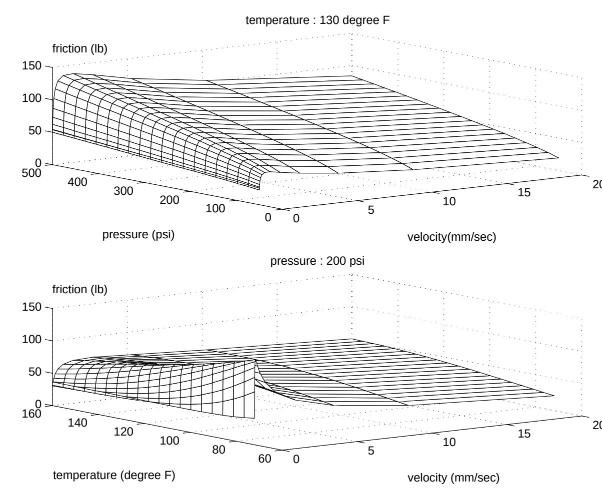
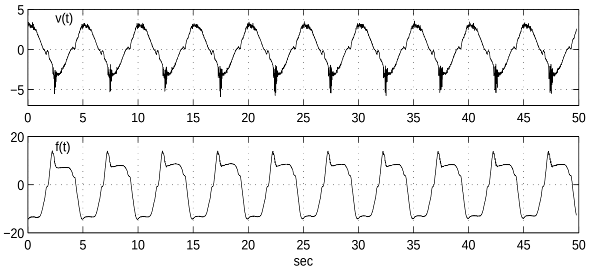
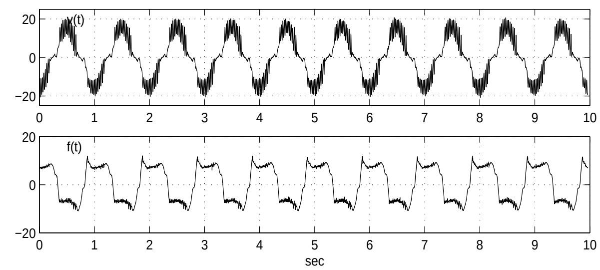


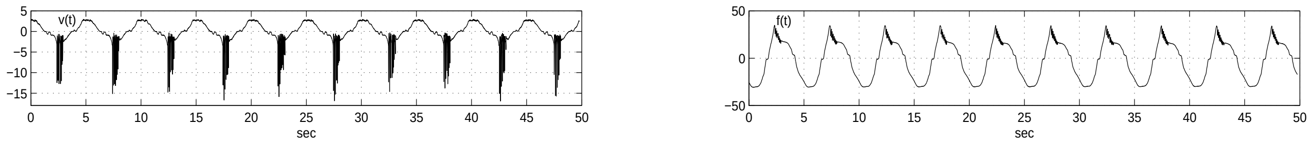
Figure 3: The theoretical steady state nonlinear model of lip seal. Velocity/pressure vs. friction at 130 ^oF, and velocity/temperature vs. friction at 200 psi.



(a) 130^oF, 50 psi, $T = 5$ sec

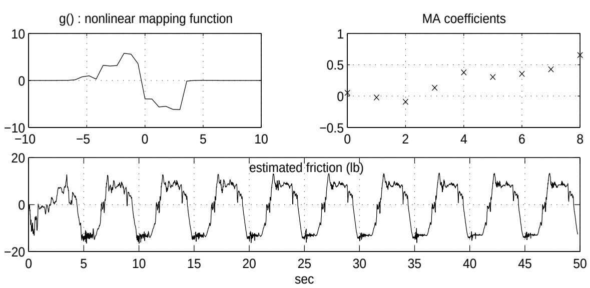


(b) 130^oF, 50 psi, $T = 1$ sec

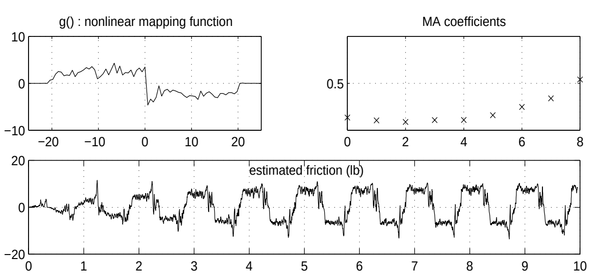


(c) 130^oF, 200 psi, $T = 5$ sec

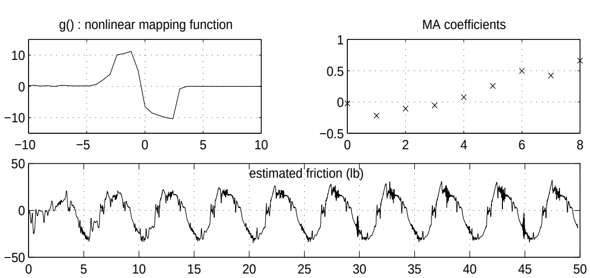
Figure 4: The velocity and friction signals measured at two different conditions.



(a) 130^oF, 50 psi, $T = 5$ sec

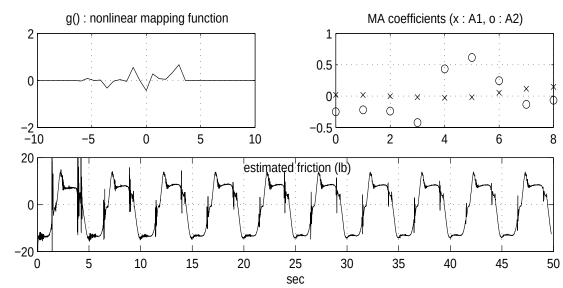


(b) 130^oF, 50 psi, $T = 1$ sec

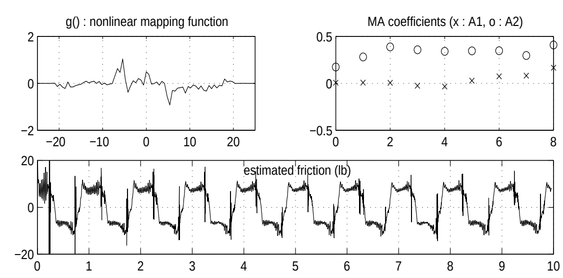


(c) 130^oF, 200 psi, $T = 5$ sec

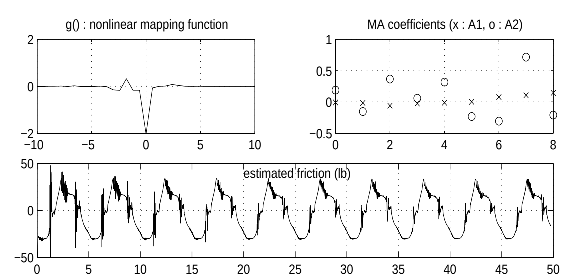
Figure 5: Estimated friction signal: Hammerstein Model.



(a) 130^oF, 50 psi, $T = 5$ sec



(b) 130^oF, 50 psi, $T = 1$ sec



(c) 130^oF, 200 psi, $T = 5$ sec

Figure 6: Estimated friction signal: Parallel Model.