MULTI-DWELL MATCHED-FIELD ALTITUDE ESTIMATION FOR OVER-THE-HORIZON RADAR

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ABSTRACT

In previous work, electromagnetic matched-field processing was proposed for estimating aircraft altitude with over-the-horizon radar using a single radar dwell. Although this approach exploits the altitude dependence of unresolved multipath returns in complex delay-Doppler space, its performance suffers in situations where the coherent integration time (CIT) of the radar is short. To overcome this limitation, this paper presents a matched-field estimation approach which uses multiple consecutive dwells on the target. The technique exploits the altitude dependence of dwell-to-dwell shape changes in the complex delay-Doppler multipath return. Monte Carlo simulations results indicate that using short CIT's, moderate signal bandwidth, and a 30 second revisit rate, multi-dwell matched-field estimation can achieve better than 5,000 ft. accuracy after as few as four radar dwells. The results of processing actual radar data for a high flying commercial aircraft of known altitude are presented which serve to validate the technique.

1. INTRODUCTION

Over-the-horizon (OTH) radar provides a means for detecting and tracking long range targets which are beyond the range of conventional line-of-sight radars. Although OTH radars are currently capable of localizing targets in ground range and azimuth, [1, 2, 3], altitude estimation has thus-far not been reliably achieved. In this paper, a matched-field estimate of altitude which uses multiple radar dwells is presented. Although a large literature on matched-field processing (MFP) techniques exists for underwater source localization [4], the approach is relatively new in radar applications. In a general sense, MFP consists of correlating the received data with predictions from multipath propagation models for a set of hypothesized target locations. In previous work, electromagnetic MFP has been applied to low angle line-of-sight radar for height finding in the presence of specular multipath reflections from the ground surface [5] and the estimation of tropospheric refractivity parameters using point-to-point microwave transmissions [6]. It has also recently been proposed for single-dwell target altitude estimation with OTH radar [7]. MFP for altitude estimation exploits the altitude dependence of differential delays and Dopplers between the micro-multipath returns from a bistatic skywave radar illustrated in Figure 1. Although this approach exploits the altitude dependence of unresolved multipath returns in complex delay-Doppler space, its performance suffers in situations where the coherent integration time (CIT) of the radar is short. Alternatively, super-resolution methods for altitude estimation have been proposed in [8] in order to resolve the delays of these micro-multipath returns but is limited by available signal bandwidth. And a third approach which exploits the fading characteristics of the target's log-amplitude delay-Doppler peak over a series of dwells has been proposed in [9] but requires unacceptably long observation times.

The multiple dwell matched-field altitude estimation technique proposed here exploits the altitude dependence of dwell-to-dwell shape changes in the complex delay-Doppler multipath return. To handle slowly changing amplitude variations due to Faraday rotation and aspect-dependent target backscatter characteristics which are not target altitude dependent, a Markov model for the multipath reflection coefficients is used. This leads to a time-evolving maximum likelihood (ML) estimate of altitude which is derived in Section 3. Monte Carlo simulations and a result with real data presented in Section 4 suggests that using using short CIT's, moderate signal bandwidth, and a 30 second revisit rate, multi-dwell matched-field estimation can achieve better than 5,000 ft. accuracy after as few as four radar dwells.

2. MODELING MICRO-MULTIPATH RADAR RETURNS

The signal model described here is based on an FM/CW radar system which is able to extract both time delay and Doppler information from a target. For the k^{th} revisit, the radar transmits a coherent series of linear FM chirp waveforms with waveform repetition interval $T(k)$ and sweep rate $b(k)$. The reflected signal contains contributions from the $L = 4$ micro-multipath ray combinations, shown in figure 1. Slant range is estimated by performing a DFT over a pulse repetition period. A second DFT is performed over several pulses to obtain Doppler shift.

Consider the radar return due to a single bistatic ray path, l. This component of the return will have a time delay $\tau_l(k)$ and a Doppler shift $\omega_l(k)$ which are a function of aircraft altitude, z, as well as the slant range, $g_0(k)$, and Doppler, $\omega_0(k)$, of a ray which intersects the ground directly beneath the aircraft. For slant range index n, Doppler index m, and time $t(k)$, let the post-DFT slant range waveform for the l^{th} component be $R_l[n,k]$ and the post-DFT Doppler waveform be $P_l[m,k]$. The model for the complex range-

Doppler return is

$$x[n,m,k] = \tag{1}$$
$$e^{\jmath\theta_k} \sum_{l=1}^{L} c_l(k) R_l[n,k] P_l[m,k] e^{\jmath\omega_l t(k)} + \eta[n,m,k]$$

where $c_l(k)$ denotes the complex amplitude of the l^{th} micro-multipath ray, θ_k is the unknown starting phase of the dwell, and $\eta[n,m,k]$ represents additive noise.

Let the $NM \times 1$ vector $\mathbf{x}_k$ represent an $N \times M$ block of the complex range-Doppler map in the neighborhood around the slant range and Doppler of the target of interest. The data model in (1) can then be written as

$$\mathbf{x}_k = e^{\jmath\theta_k} \mathbf{H}_k \mathbf{D}_k \mathbf{c}_k + \mathbf{n}_k \tag{2}$$

where

$$(\mathbf{H}_k)_{nN+m,l} = R_l[n,k] P_l[m,k] \tag{3}$$

and $\mathbf{D}_k$ is a diagonal matrix with $(\mathbf{D}_k)_{l,l} = e^{\jmath\omega_l(k)t(k)}$. The complex micro-multipath ray amplitudes are treated here as zero mean Gaussian random variables with covariance

$$E\left[\mathbf{c}_k \mathbf{c}_k^H\right] = \mathbf{\Lambda}_c = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_L \end{bmatrix}, \tag{4}$$

and the noise is assumed to be zero mean Gaussian with covariance $\sigma_n^2 \mathbf{I}$. Thus $p(\mathbf{x}_k|z)$ is Gaussian with mean zero and covariance $\mathbf{R}_{k,k} = \mathbf{H}_k \mathbf{\Lambda}_c \mathbf{H}_k^H + \sigma_n^2 \mathbf{I}$.

3. MAXIMUM LIKELIHOOD ESTIMATION WITH MULTIPLE DWELLS

Let the set of data snapshots for the $K+1$ revisits, $k = 0 \ldots K$ be denoted by $\mathbf{X}_K$, and the associated phase differences by $\Delta\mathbf{\Theta}_K = (\Delta\theta_1, \Delta\theta_2, \ldots, \Delta\theta_K)$, where $\Delta\theta_k = \theta_k - \theta_{k-1}$. In order to obtain a multiple dwell ML estimate of altitude, the joint distribution of $\mathbf{X}_K$ conditioned on $\Delta\mathbf{\Theta}_K$ and z is needed. In the absence of Faraday rotation and aspect dependent target backscatter characteristics, the complex micro-multipath ray amplitudes should be perfectly correlated so that $E\left[\mathbf{c}_k \mathbf{c}_{k-\mu}^H\right] = \mathbf{\Lambda}_c$ for arbitrary μ. In practice, slow random fluctuations can be handled by modeling $\mathbf{c}_k$ as a first-order Markov process, in which case

$$p(\mathbf{X}_K|z, \Delta\Theta_K) = p(\mathbf{x}_0|z) \prod_{k=1}^{K} p(\mathbf{x}_k|\mathbf{x}_{k-1}, \Delta\theta_k, z). \tag{5}$$

In general the term $p(\mathbf{x}_k|\mathbf{x}_{k-1}, \Delta\theta_k, z)$ describes the variation of the signal from one dwell to the next.

An ML estimate of altitude is obtained by maximizing $p(\mathbf{X}_K|z, \Delta\mathbf{\Theta}_K)$ with respect to $\Delta\mathbf{\Theta}_K$ and z. Assuming that consecutive micro-multipath amplitudes $\mathbf{c}_{k-1}$ and $\mathbf{c}_k$ are perfectly coherent, then $p(\mathbf{x}_k|\mathbf{x}_{k-1}, \Delta\theta_k, z)$ is also Gaussian with mean and covariance

$$\begin{aligned} \mathbf{m}_k &= \mathbf{R}_{k,k-1} \mathbf{R}_{k-1,k-1}^{-1} \mathbf{x}_{k-1} \\ \mathbf{Q}_k &= \mathbf{R}_{k,k} - \mathbf{R}_{k,k-1} \mathbf{R}_{k-1,k-1}^{-1} \mathbf{R}_{k,k-1}^H \end{aligned} \tag{6}$$

where $\mathbf{R}_{k,k-1} = \mathbf{H}_k \mathbf{D}_k \mathbf{\Lambda}_c \mathbf{D}_{k-1}^H \mathbf{H}_{k-1}^H e^{\jmath\Delta\theta_k}$. Maximizing the log of the likelihood in (5) over $\Delta\mathbf{\Theta}_k$ yields

$$L(z) = \log p(\mathbf{x}_0|z) + \sum_{k=1}^{K} L_k(z) \tag{7}$$

where $L_k(z) = \max_{\Delta\theta_k} \log p(\mathbf{x}_k|\mathbf{x}_{k-1}, z, \Delta\theta_k)$, which, from (6), can be written

$$\begin{aligned} L_k(z) &= -\log \pi^N |\mathbf{Q}_k| - \mathbf{x}_k^H \mathbf{Q}_k^{-1} \mathbf{x}_k \\ &\quad + 2|\mathbf{x}_k^H \mathbf{Q}_k^{-1} \mathbf{P}_k \mathbf{x}_{k-1}| - \mathbf{x}_{k-1}^H \mathbf{P}_k^H \mathbf{Q}_k^{-1} \mathbf{P}_k \mathbf{x}_{k-1} \end{aligned} \tag{8}$$

where $\mathbf{P}_k = \mathbf{H}_k \mathbf{D}_k \mathbf{\Lambda}_c \mathbf{D}_{k-1}^H \mathbf{H}_{k-1}^H \mathbf{R}_{k-1,k-1}^{-1}$. The ML estimate of altitude can now be obtained by a one dimensional numerical maximization of (7) with respect to z, which is computationally efficient.

Note that the coherence assumption of $\mathbf{c}_k$ is limited by effects such as Faraday rotation. Over long target tracks, Faraday rotation will have the effect of decorrelating the complex ray amplitudes. If the Faraday decorrelation is known, it can be incorporated into $p(\mathbf{x}_k|\mathbf{x}_{k-1}, \Delta\theta_k, z)$. It should also be noted that the likelihood functions in this section were derived assuming that $g_0(k)$ and $\omega_0(k)$ are known. In practice, these can be estimated by finding the peak correlation of $\mathbf{x}_k$ with the signal model in (3) for the strongest radar returns and interpolating at the weaker revisit times along the radar track.

4. SIMULATION AND REAL DATA RESULTS

To evaluate the expected performance of the ML altitude estimation approach presented here, the probability of correct localization (PCL) within a 5,000 ft. altitude band is estimated over 200 Monte Carlo simulations as a function of SNR and target altitude. The CIT is nominally 2.5 seconds, the radar bandwidth is 17 kHz, and the operating frequency is 10 MHz. The target ground range is 1200 km and the radial velocity is -190 m/s. The revisit interval between dwells is 30 seconds. The ionosphere was modeled using a single quasi-parabolic E-layer [10] with critical frequency 3.5 MHz, height 110 km, and thickness 32 km.

Figure 2 illustrates an example of the log likelihood function in (7) evolving over five minutes for a target altitude of 30,000 ft. The bandwidth and CIT are clearly not sufficient to estimate altitude with a single dwell, but after five dwells the ML estimate converges to the correct altitude. Figure 3 shows the PCL for a fixed altitude of 26,000 ft. as a function of time for varying SNR over 200 random trials. After ten minutes, the PCL reaches 0.8 for a 15 dB target. However, for SNR's of 25 dB and above, this level of performance is achieved after only four minutes. These results indicate that the performance threshold for this estimator is near 20 dB.

Figure 4 shows the PCL results as a function of target altitude for a fixed SNR of 25 dB. In general, estimation performance improves with increased target altitude. Note that the range-Doppler map of the received radar signal is composed of L returns due to the ray combinations described in section 2. As the target altitude increases, the separation of these returns in the range-Doppler map also increases. This increased separation might be expected to lead to increased accuracy in the altitude estimate.

Figure 5 shows the result from processing real data from an OTH radar track of a strong, high altitude commercial aircraft. The resulting altitude estimate in Figure 5 converges to within 200 ft. of the true aircraft altitude after four minutes. Furthermore, the evolving likelihood function shows very similar behavior to the simulation likelihood function in Figure 2, which serves to validate the simulation performance described above.

5. CONCLUSIONS

In this paper, a matched-field maximum likelihood estimate of target altitude for OTH radar was extended for multiple radar dwells. Through simulation it was shown that, for typical radar operating parameters, a PCL of 0.8 could be achieved at moderate SNR after relatively few radar revisits. This is an improvement over previous attempts at altitude estimation which require high radar CIT and bandwidth or many radar revisits. A real data result was presented with a strong, high altitude target, and the resulting altitude estimate for was within 200 ft. of the true altitude, which is within the 5,000 ft. accuracy predicted by simulation.

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6. REFERENCES

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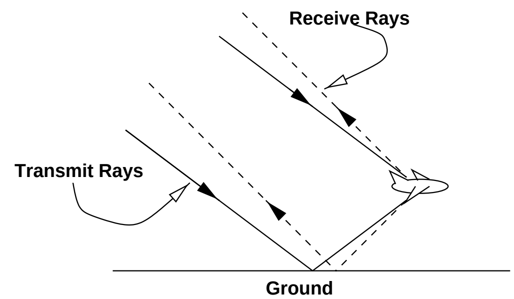

Figure 1: Multipath raypaths local to the aircraft target for transmission and reflection with a bistatic radar.

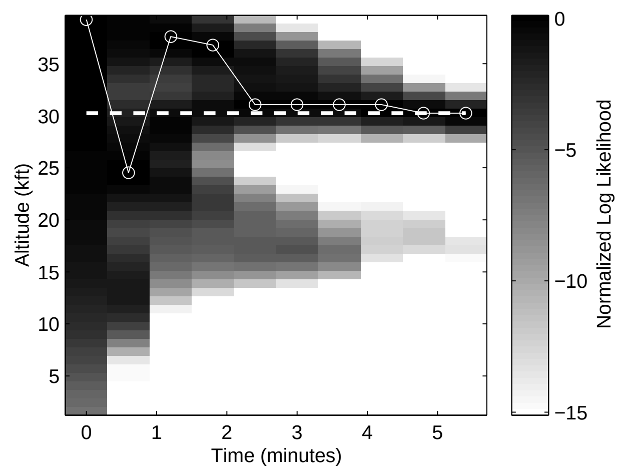

Figure 2: Simulation log-likelihood function evolving over 30 second revisits. True altitude is 30,000 ft., and estimated altitude is 30,000 ft. after 5 minutes (9 dwells). The SNR is 25 dB, the nominal CIT is 2.5 seconds, and the bandwidth is 15 kHz.

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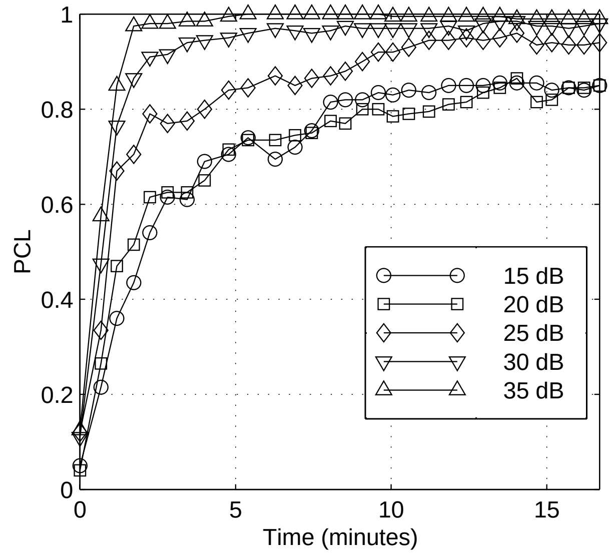

Figure 3: Probability of correct localization (PCL) within 5,000 ft. bands vs. time for a 26,000 ft. target for varying SNR (200 realizations).

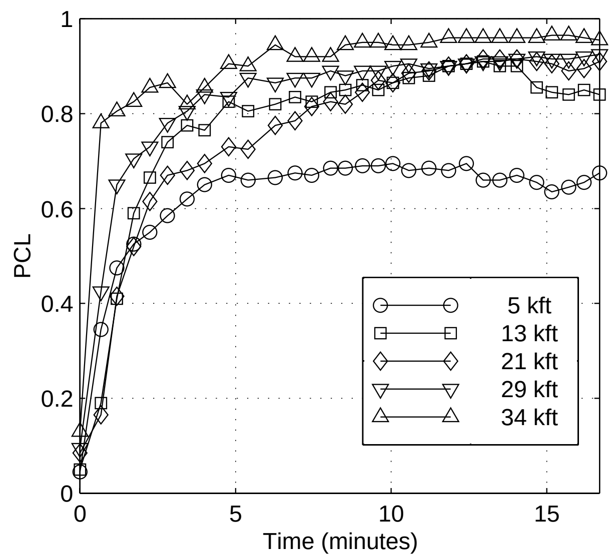

Figure 4: Probability of correct localization (PCL) within 5,000 ft. bands vs. time for a 25 dB target at different altitudes (200 realizations).

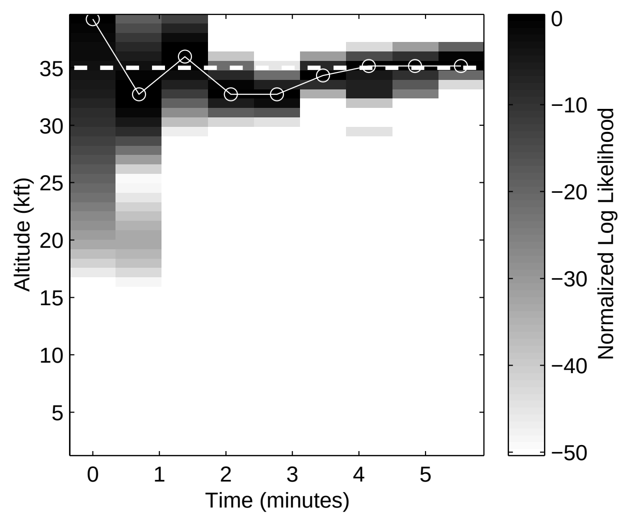

Figure 5: Real data log-likelihood vs. time, true altitude is 35,000 ft., estimated altitude is 35,200 ft.