THE PERFORMANCE OF MAXIMUM LIKELIHOOD OVER-THE-HORIZON RADAR COORDINATE REGISTRATION

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ABSTRACT

A well-known source of target localization errors in over-the-horizon radar is the uncertainty about downrange ionospheric conditions. Maximum likelihood (ML) coordinate registration, using statistical modeling of ionospheric parameters, has recently been proposed as a method which is robust to ionospheric variablity. This paper reports ML performance results for real data from a known target using estimates of ionospheric statistics derived from ionosonde measurements. Bootstrap samples derived from these statistics are then used in a hidden Markov model approximation to the ground range likelihood function. Comparison of the ML and conventional methods for over 250 radar dwells indicates the new technique achieves better than a factor of two improvement in ground range accuracy.

1. INTRODUCTION

Over-the-horizon (OTH) radars perform wide area surveillance by exploiting the refractive and multipath nature of high frequency (HF) propagation [1]. Target detection is accomplished by tracking returns in slant range, Doppler, azimuth and azimuth rate. Coordinate registration (CR) is the process of localizing the target by converting these slant coordinates to ground coordinates [2]. Target localization is achieved by raytracing which uses an ionospheric model estimated from a quasi-vertical ionogram (QVI) and wide-sweep backscatter ionogram (WSBI) measurements. Not surprisingly, errors in the estimated down-range ionospheric parameters can seriously degrade target localization accuracy. In previous work, a maximum likelihood (ML) CR method was developed with the aim of improving target localization accuracy by statistically modeling uncertainties in the ionospheric propagation conditions [3]. For greater computational efficiency, the likelihood function was approximated by a hidden Markov model (HMM) for the probability of a sequence of observed slant coordinates given a hypothesized target location.

This paper reports the performance results of ML CR for real data from a target at a known location using estimates of the HMM parameters derived from contemporaneous QVI and WSBI measurements. In particular, the plasma frequency profile is treated as a homogenous random process about the raypaths' ground range midpoint. Spatial sampling of a 3-D ionospheric model, fitted to insitu QVI and WSBI soundings, is then used to generate quasi 2-D profile realizations.

2. ML TARGET LOCALIZATION

A statistical model for radar slant data can be obtained by mapping random plasma frequency parameters through a HF raytracing model to determine the *random* slant coordinates produced by a target at a particular ground position. The objective of ML CR is to determine the ground position given the observed slant coordinates and a model for their underlying probability distribution. In addition to improving robustness to ionospheric uncertainty, statistical modeling also facilitates the use of less predictable observations, such as raymode amplitude rankings in the CR process.

Ordering the observed slant coordinates in terms of the signal-to-noise ratio (SNR) of each return, let x_n denote the observation from the nth strongest return. Associating numbers with the different raymode types, let s_1 denote the mode number corresponding to the strongest return, s_2 the mode number of the second strongest return, and so on, until s_N is the mode number of the weakest return. The complete slant coordinate observation "sequence" is then given by

$$x_n = d_{s_n}(r) + \varepsilon \quad \text{for} \quad n = 1, \ldots, N \tag{1}$$

where $d_{s_n}(r)$ is the group path length of raymode s_n to ground range r, and ε represents the delay estimation error or "jitter" and is modeled by an independent zero-mean Gaussian random variable. Note that Equation (1) depends on both the sequence of raymode numbers, s_n, and on the mapping from ground range to slant range, $d_{s_n}(r)$, given the raymode number. Because both these quantities are random due to ionospheric variability, this statistical model is doubly stochastic. Given a specific set of ordered slant coordinates, $X = [x_1, \ldots, x_N]$, the objective here is to determine the most likely target ground coordinates. Let $p_x(X|r)$ denote the probability density function of the ordered slant coordinate observations for a hypothesized target ground range, r. The ML estimate is obtained by substituting the observed slant coordinates X into $p_x(X|r)$ and maximizing with respect to hypothesized ground range r. In terms of the probability, $P(S|r)$, of a given sequence of raymodes $S = s_1, \ldots, s_N$:

$$p_x(X|r) = \sum p_{x|s}(X|S, r)P(S|r) \tag{2}$$

where $p_{x|s}(X|S, r)$ is the probability density function of the ordered slant coordinate observations given the raymode sequence and the summation is over all possible raymode sequences, which can in general be very large.

To reduce the computation associated with performing the summation in Equation (2), a HMM is used here to characterize X defined by Equation (1) [3]. In the terminology of HMM's, the ordered slant coordinates define the *observation sequence* and their associated unobservable raymodes define the *hidden state sequence*. The use of a HMM to compute the likelihood surface requires two approximations. First, given that the mode sequence is known *a priori*, the slant returns of different raymodes are assumed to be statistically independent based on the fact that the raymodes traverse different parts of the ionosphere exhibiting largely independent variations. Second, the raymode sequence is described by a first-order nonstationary Markov model which implies that given the nth strongest raymode type, s_{n-1} and s_{n+1} are statistically independent. While clearly an approximation for $n > 2$, results presented below suggest that this adequately models the structure of S. In terms of the output probabilities, $p_{x_n|s_n}(x_n|s_n, r)$, and the transition probabilities, $P(s_n|s_{n-1}, r)$, the likelihood function can then be written as

$$p_x(X|r) \cong \sum \left(\prod_{n=1}^{N} p_{x_n|s_n}(x_n|s_n, r) P(s_n|s_{n-1}, r) \right) \qquad (3)$$

where now the summation over all possible state sequences can be performed recursively by computing the set of forward variables,

$$\alpha_n(s_{n,k}) = \sum_{k=1}^{K} \alpha_{n-1}(s_{n-1,k}) P(s_{n,k}|s_{n-1,k}) p(x_n|s_{n,k}) \qquad (4)$$

where $s_{n,k}$ denotes the kth value of s_n and all quantities are conditioned on r. At the Nth iteration of Equation (4), the likelihood function is found by taking

$$p_x(X|r) = \sum_{k=1}^{K} \alpha_N(s_{N,k}) \qquad (5)$$

The maximum likelihood estimate (MLE) of target location is found by maximizing Equation (5) over a discrete grid of hypothesized ground ranges.

3. ESTIMATION OF HMM PARAMETERS

Estimating the parameters of a probabilistic model invariably requires a "training set" of measurements derived from data statistically similar to the observations of interest. In the current application, this means obtaining measurements of the ionosphere which encompass the variability exhibited in the region roughly half-way between the radar and the target [4]. The instruments typically available to obtain this data are the quasi-vertical ionosonde (QVI) and wide-sweep backscatter ionosonde (WSBI). The QVI is a measurement of group delay versus frequency for HF skywave propagation between a transmitter and receiver less than about 150 km apart. QVI's can be inverted to estimate the plasma frequency profile near the radar. To simplify raytracing, this profile is often represented by analytic layers (e.g. Chapman) which are parameterized by their heights, critical frequencies, and thicknesses. To determine the downrange ionospheric layer parameters, the overhead profile parameters estimated from the QVI can be used as starting points for fitting the WSBI leading edge. The WSBI measures the ground backscatter intensity as a function of time delay and frequency. The WSBI leading edge is an estimate of the minimum group delay of a return from the ground as a function of frequency [2]. Precise fitting of a set of WSBI leading edges at different azimuths requires raytracing through a 3-D ionospheric model. However, due to the limited nature of the ionospheric data, estimates of the 3-D spatially-varying ionospheric parameters are not unique. Thus in addition to ionosonde data, estimation of the down-range ionospheric model relies heavily on the use of empirical ionospheric models [5, 6].

Several approaches could be taken to model the uncertainty in the down-range ionosphere estimated from QVI and WSBI measurements. First, computational models for the second-order statistics of ionospheric fluctuations due to trans-ionospheric disturbances could used to represent the variability about the estimated profile. Alternatively, a database of historical ionospheric measurements could be used to derive empirical models of variability which would extend the first-order description given by empirical models. Different realizations of the ionosphere could then be obtained by fitting the ionosonde measurements to different realizations of the historical data. The difficulty with these approaches, however, is that they rely on either computational or empirical models of ionospheric variability which have not yet been available. In light of this, the approach taken here is to treat the plasma plasma frequency profile as a spatially homogeneous random process in latitude and longitude around the midpoint between the radar and the dwell illumination region (DIR). Samples of the 3-D ionospheric estimate in azimuth and ground range can then be treated as different 1-D realizations of the midpoint profile.

Let F denote the distribution of the down-range profile parameter samples $\mathbf{g}(\mathbf{r}_i)$ taken at ground locations $\mathbf{r}_i$ for $i = 1, \ldots, L$. In previous work, F was assumed known, but in practice F must be estimated from the QVI and WSBI measurements. Smoothed bootstrap resampling is a means of generating realizations of $\widehat{F}$ without the need to explicitly estimate a complicated joint distribution function [7, 4]. Once a large set of random realizations from $\widehat{F}$ are available, the parameters of the HMM for each hypothesized target location can be determined by Monte Carlo evaluation of the raytrace propagation model. In particular, the raymodes types take on discrete values and their corresponding probabilities must be represented by discrete distributions. For example, an estimate of the transition probability, $P(s_n|s_{n-1}, r)$ can be computed by using the proportion of realizations such that mode s_n is the nth strongest mode given that s_{n-1} is the $(n-1)$th strongest mode.

The slant coordinate observations, however, have continuous values and so the HMM output probabilities must be represented by continuous proability density functions. The output probability density, $p_{x|s}(x_n|s_n, r)$, can be estimated given random realizations from F using either a parametric or a nonparametric method [4]. For example, a parametric approximation to $p_{x|s}(x_n|s_n, r)$ is the Gaussian PDF with sample mean $\widehat{\mu}_{s_n}(r) = 1/N \sum_{i=1}^{N} d_{s_n}(r, \gamma_i)$ and variance

$$\hat{\sigma}^2_{s_n}(r) = \frac{1}{N}\sum_{i=1}^{N}[d_{s_n}(r,\gamma_i) - \hat{\mu}_{s_n}(r)]^2 + \sigma^2_\varepsilon, \qquad (6)$$

where $d_{s_n}(r,\gamma_i)$ is the slant range for a target at ground range r using the ith ionospheric realization. Note that in Equation (6), the first term depends on the ionospheric variability and the second term contains the slant range jitter variance, σ^2_ε, determined by radar parameters such as SNR and bandwidth.

4. LOCALIZATION PERFORMANCE RESULTS

The random ionospheric model in the both simulation results and real data results was based on QVI and WSBI measurements, taken December 7 and 8, 1994. First, overhead and down-range ionospheric parameters of a 3-D ionospheric model were determined using software routines developed by Nickisch and Hausman [5]. The 3-D model profile parameters were sampled at eight ranges along the eight bearings to generate 64 ionospheric samples over roughly an 800x800 km down-range sample region. To reduce the computational burden of Monte Carlo evaluation of the raytracing model, an analytic multi-quasi-parabolic (MQP) raytracing model was employed. Detailed descriptions of the MQP model for a stratified ionosphere are given in [8] and for a quasi 2-D tilted ionosphere in [9]. For the results here, the MQP layer heights and critical frequencies were taken equal to the 3-D model's parameters, and the semi-thickness parameters were computed from the layer heights to approximate the vertical profiles of the 3-D model. After obtaining samples from the 3-D model, the quasi 2-D MQP zenith tilt angle parameters were computed for each sample by a least-squares fit of the MQP synthetic leading edges to the corresponding WSBI leading edges. Comparisons of CR curves suggest that the random realizations of the quasi 2-D MQP statistical model closely approximate the results of numerical raytracing through the 3-D ionospheric model.

200 ionospheric realizations, generated from the down-range samples via smoothed bootstrap resampling, were used in Monte Carlo evaluation of the MQP raytracing model to compute the slant ranges and associated raymode amplitudes for the bistatic extraordinary raymodes. The random propagation model was run at 22 MHz for a scenario with a minimal number of possible paths between the radar and target and also was run at 12 MHz for a case with a large number of raypaths. HMM parameters for the 12 strongest raymodes were retained for ground ranges between 2000 and 2500 km in 1 km intervals. The *a priori* statistical information contained in the raymode amplitudes can be appreciated by considering the probability that a particular raymode corresponds to the strongest return. The probabilities for the raymode types, are plotted as a function of ground range in Figure 1. Note that at 22 MHz the probabilities have little variation in ground range. In contrast, the probabilities for 12 MHz indicate that more raymodes are probable for the strongest observation. The changes in these probabilities correspond to the ground ranges over which the F1 layer caustic exists in the transmit or receive paths.

A conventional CR method used for comparing localization accuracy was defined by the average of the ground ranges corresponding to raymodes identified as having the minimum ground range variance. This method is referred to here as the minimum variance (MV) CR method. The ionospheric model assumed by the conventional model was that obtained using the mean MQP parameters from the ionosonde data. For 50 Monte Carlo trials, ground range estimation was performed for a target at each ground range in 10 km intervals between 2000 and 2500 km using a maximum of 3 observations. A zero-mean Gaussian slant range jitter component, with $\sigma_\varepsilon = 3.0$ km, was added to model delay estimation error. A comparison of the MLE performance to the conventional MVE method is the variation in average absolute miss distance versus ground range, shown in Figure 2. Observe that the average miss distance achieved by the ML method is consistently less than that of the MV method. The performance improvement of the ML method over the MV method varies with true ground range but can be as much as 5 times more accurate.

To determine the achievable ML CR accuracy on real data, the method was tested on radar returns from a beacon at a ground range of 2192 km. The likelihood function, given in Equation (5) using the calculation of the forward variables in Equation (4) on a grid of hypothesized ground ranges, is shown in Figure 3 for 5 beacon dwells. Note that each likelihood function realization has multiple peaks indicating that there were several ambiguities in the ground range estimates. The ambiguities arise from the raymode uncertainty and the secondary peaks in the likelihood function correspond to less probable raymodes for a single return. To compare the ML method with conventional methods, histograms of the range errors, shown in Figure 4, were computed for each method using over 250 dwells recorded during 14 different tracking periods. The miss distances are reported in normalized units. The top two histograms correspond to the conventional methods of the existing radar system and the 3-D model respectively. The average absolute miss distances (AVMD) and average ground range biases for the three methods are shown at the left sides of the histograms. The ratios of the average absolute miss distances indicate that the ML method offers nearly a 2 to 1 improvement over the deterministic 3-D model and nearly a 3 to 1 improvement over the CR method in the existing radar system.

5. CONCLUSIONS

A ML CR method is designed to obtain the most accurate target localization performance for a given level of uncertainty about the down-range ionospheric profile, modeled here as a homogeneous random process. A HMM, with parameters estimated from down-range samples derived from ionosonde data, incorporates previously ignored relative raymode amplitude information. Beacon data results indicate that the ML method can provide nearly a 2 to 1 improvement over conventional methods.

The authors would like to thank L. J. Nickisch and M. Hausman of Mission Research Corporation for providing radar beacon data, radar ionosonde data and the CREDO software package. This work was sponsored by ONR under

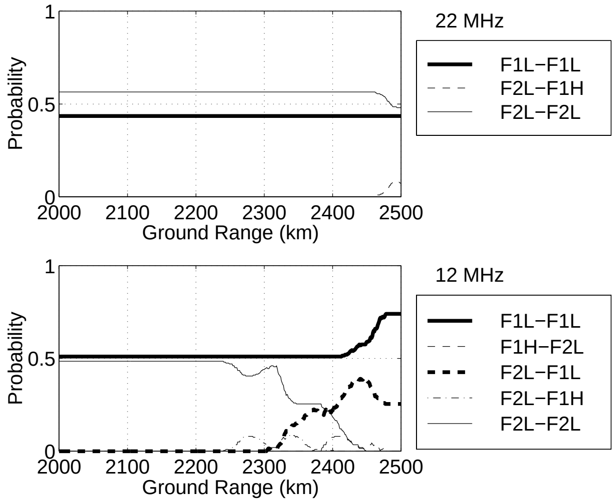

Figure 1: Probability of Different Raymode Types for Strongest Return

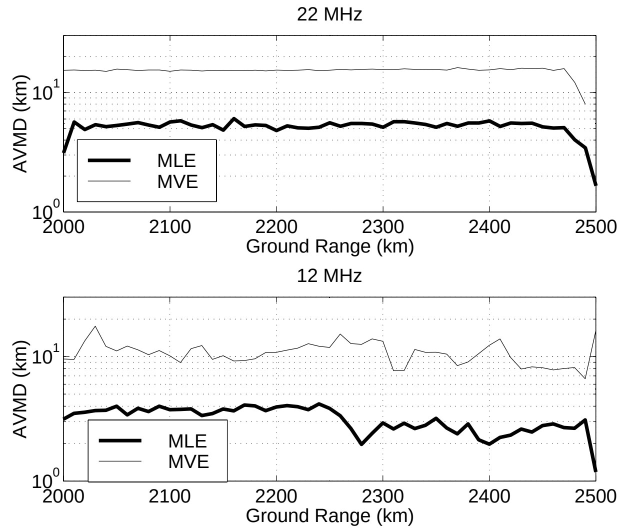

Figure 2: Average Absolute Miss Distance vs. Ground Range at 22 MHz and 12 MHz

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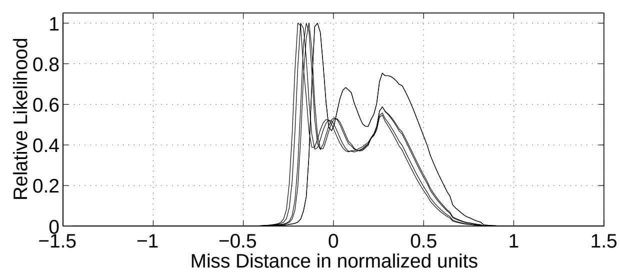

Figure 3: Likelihood Functions for 5 Beacon Dwells

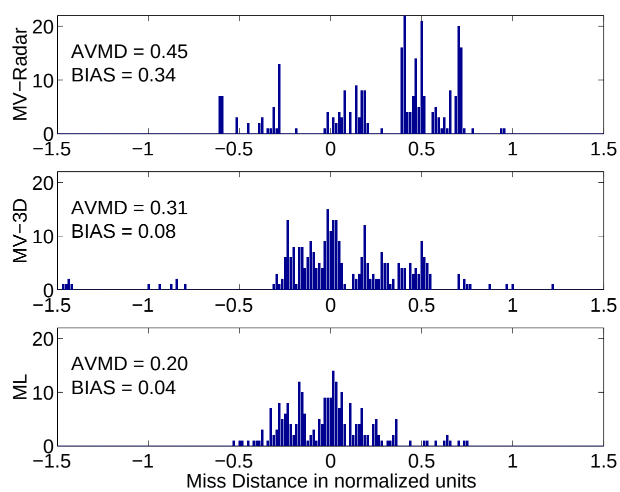

Figure 4: Beacon Ground Range Error Histograms for Dec 7-8, 1994 with over 270 dwells

6. REFERENCES

[1] J. M. Headrick and M. I. Skolnik, "Over-the-horizon radar in the HF band.," *Proc. of the IEEE*, vol. 62, pp. 664–673, June 1974.

[2] L. F. McNamara, *The Ionosphere: Communications, Surveillance, and Direction Finding.* Krieger Publishing, 1991.

[3] J. L. Krolik and R. H. Anderson, "Maximum likelihood coordinate registration for over-the-horizon radar.," *IEEE Trans. on Signal Processing*, vol. 45, pp. 945–959, April 1997.

[4] R. H. Anderson and J. L. Krolik, "Over-the-horizon radar target localization using a hidden markov model estimated from ionosonde data." Submitted to *Radio Science*, October 1997.

[5] L. J. Nickisch and M. Hausman, "CREDO: Coordinate registration enhancement by dynamic optimization," in *Proc. Ionospheric Effects Symposium (IES-96)*, pp. (1B2):1–9, May 1996.

[6] C. J. Coleman, "A propagation model for HF radiowave systems.," in *MILCOM*, pp. 875–879, 1994.

[7] B. Efron, *The Jackknife, the Bootstrap and other Resampling Plans.* Philadelphia, PA: SIAM, 1982.

[8] P. L. Dyson and J. A. Bennett, "A model of the vertical distribution of the electron concentration in the ionosphere and its application to oblique propagation studies.," *J. Atmos. Terr. Physics*, vol. 50, no. 3, pp. 251–262, 1988.

[9] P. L. Dyson and J. A. Bennett, "Exact ray path calculations using realistic ionospheres.," *IEE Proceedings*, vol. 139, pp. 407–413, October 1992.