SIMULATION OF THREE-DIMENSIONAL SOUND PROPAGATION WITH MULTIDIMENSIONAL WAVE DIGITAL FILTERS

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ABSTRACT

The propagation of sound waves is described by partial differential equations for the acoustic pressure and the acoustic fluid velocity. The solution depends on the shape of the enclosure and on the boundary conditions. Among various methods for the discretization of partial differential equations, the multidimensional wave digital filter approach is known to yield robust algorithms for the discrete simulation of continuous problems.

This paper describes the derivation of a discrete model for three-dimensional sound propagation according to multidimensional wave digital filtering principles. The correct treatment of boundary conditions for various wall impedances is shown. A numerical example for the sound propagation in three interconnected rooms of a building demonstrates the capabilities of the method.

1. INTRODUCTION

In many applications of spatial sound processing, like acoustic echo cancellation, active noise control, or array processing, tremendous advances have been made without explicit knowledge of the sound field. This has been mostly accomplished by intelligent use of adaptive filters. However, further progress may require evaluation of the dynamics of spatial sound fields, especially for applications within enclosures. Some suitable methods are briefly reviewed here.

Methods for the simulation of sound propagation in enclosures can be divided into geometric and computational acoustics. They differ in the propagation model for the sound waves. Geometric acoustics simplifies the propagation process by a plane wave assumption. Reflections are modelled by acoustic rays, which are reflected at boundaries according to the laws of geometry. Examples for geometrical acoustics are the mirror image method, ray tracing, and the radiosity method. The latter two methods are adapted from graphical rendering to the special problems of acoustics, e.g. by considering the finite propagation speed of sound. The methods from geometric acoustics are computationally tractable, but at the cost of simplifying assumptions. They are the state of the art for the numerical determination of room impulse reponses in enclosures like buildings or cars.

Computational acoustics, on the other hand, rely on first principles of physics, like the laws of kinetics or conservation of mass. Discrete simulation algorithms are obtained by proper discretization of the underlying partial differential equations. They are capable of exact physical modelling, provided the spatial and temporal step sizes are chosen small enough. However, the associated computational load far exceeds todays desktop capabilites and constitutes a major drawback. In computational acoustics, to date, mainly wave guide methods have been presented for the dynamic simulation of room acoustics [8, 9, 10]. But also multidimensional wave digital filters (MD-WDF) are known to yield robust algorithms for the discrete simulation of continuous MD problems [4].

This contribution presents the application of MD-WDF principles to the simulation of 3D sound propagation. It is an extension of the general approach described in [5] to three dimensions. In addition, special attention is paid to the proper treatment of boundary conditions for various cases that are important in practice.

The presentation starts with a definition of the physical problem in terms of partial differential equations (PDE). The next step is the network description of the MD problem, followed by the discretization procedure which gives the MD-WDF algorithm in form of a state description with proper treatment of the boundary conditions. Finally, the capability of the method is demonstrated by an example.

2. PROBLEM DEFINITION

The propagation of sound waves in air is governed by two basic relations (see [11]) for the acoustic pressure $p(\mathbf{x}, t)$ and the acoustic fluid velocity vector $\mathbf{v}(\mathbf{x}, t)$. Pressure p and velocity $\mathbf{v}$ denote small-amplitude acoustic signals, which depend on time t and the space vector $\mathbf{x}$. These basic relations are the equation of motion and the equation of continuity. Under reasonable simplifications, they are given by

$$\rho_0 \frac{\partial}{\partial t}\mathbf{v}(\mathbf{x},t) + \operatorname{grad} p(\mathbf{x},t) = 0 \qquad (1)$$

$$\frac{\partial}{\partial t} p(\mathbf{x},t) + \rho_0\, c^2 \operatorname{div}\mathbf{v}(\mathbf{x},t) = 0 \qquad (2)$$

where ρ_0 is the static density of the air and c is the speed of sound. Frequently, both equations are combined into the wave equation by elimination of either pressure p or velocity $\mathbf{v}$ [11]. However, the application of wave digital filtering principles requires starting from the basic equations (1,2). We rewrite them in terms of the scalar components x, y, z of $\mathbf{x}$ in cartesian coordinates

$$\rho_0 \frac{\partial}{\partial t} v_\xi + \frac{\partial}{\partial \xi} p = 0 \qquad (3)$$

$$\frac{1}{\rho_0 c^2}\frac{\partial}{\partial t} p + \sum_\xi \frac{\partial}{\partial \xi} v_\xi = 0, \qquad (4)$$

where (3) stands for any of the three equations with $\xi = x, y, z$, respectively and

$$\sum_\xi \frac{\partial}{\partial \xi} v_\xi = \operatorname{div}\mathbf{v} = \frac{\partial}{\partial x} v_x + \frac{\partial}{\partial y} v_y + \frac{\partial}{\partial z} v_z\ . \qquad (5)$$

The representation of these equations by a discrete MD-WDF algorithm is shown in the next sections. The presentation is rather concise, since it follows the space-time-domain analysis approach outlined in [5].

3. NETWORK DESCRIPTION

The first step is a network description of (3,4). By representing these equations in graphical form, we can express mathematical manipulations as network operations familiar from circuit theory. Note however, that the network impedances are not idealizations of real world components, rather they constitute a graphical description of the differential operators of the PDEs (3,4).

The network description is obtained by four basic steps:

1. Since ρ_0 and c are constant with respect to time and space, we express pressure and velocity in exponential form as

$$v_\xi(\mathbf{x},t) = V_\xi e^{\mathbf{s}^T\mathbf{t}}, \quad p(\mathbf{x},t) = P e^{\mathbf{s}^T\mathbf{t}}, \qquad (6)$$

   with the complex wave numbers s_x, s_y, s_z and the complex frequency s_t (T denotes transposition)

$$\mathbf{s}^T = [s_x, s_y, s_z, s_t], \quad \mathbf{t}^T = [x, y, z, t]. \qquad (7)$$

   and obtain

$$\rho_0 s_t V_\xi + s_\xi P = 0 \qquad (8)$$

$$\frac{1}{\rho_0 c^2} s_t P + \sum_\xi s_\xi V_\xi = 0 \qquad (9)$$

2. We express the complex pressure amplitude P as velocity amplitude P/r_0 with an arbitrary real impedance r_0. The choice of r_0 determines the properties of the resulting algorithm and is limited only by stability considerations [5]. The most simple arrangement is obtained for $r_0^2 = 3\rho_0^2 c^2$, resulting in

$$\rho_0 s_t V_\xi + r_0 s_\xi \frac{P}{r_0} = 0, \qquad (10)$$

$$3\rho_0 s_t \frac{P}{r_0} + \sum_\xi r_0 s_\xi V_\xi = 0. \qquad (11)$$

3. In order to express each of the four equations (10,11) as a mesh equation of a network, we introduce the additional terms $\pm r_0 s_\xi V_\xi$ and $\pm r_0 s_\xi (P/r_0)$. The resulting set of equations

$$\left(\rho_0 s_t - r_0 s_\xi\right) V_\xi + r_0 s_\xi \left(v_\xi + \frac{P}{r_0}\right) = 0, \quad (12)$$

$$\sum_\xi \left(\rho_0 s_t - r_0 s_\xi\right)\left(\frac{P}{r_0}\right) + + \sum_\xi r_0 s_\xi \left(v_\xi + \frac{P}{r_0}\right) = 0 \quad (13)$$

   is equivalent to the network description of Fig. 1 with the "mesh currents" V_x, V_y, V_z, and P/r_0.

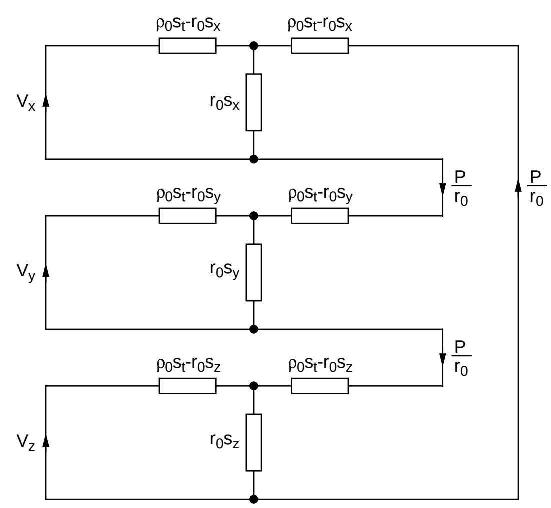

Figure 1: Network description: T-Circuits

4. The final form of the network description of the continuous system results, when the T-circuits in Fig. 1 are replaced by the corresponding lattice structures as shown in Fig. 2. The important property of this structure is that the impedances

$$Z_\xi = \rho_0 s_t - r_0 s_\xi, \quad Z'_\xi = \rho_0 s_t + r_0 s_\xi \qquad (14)$$

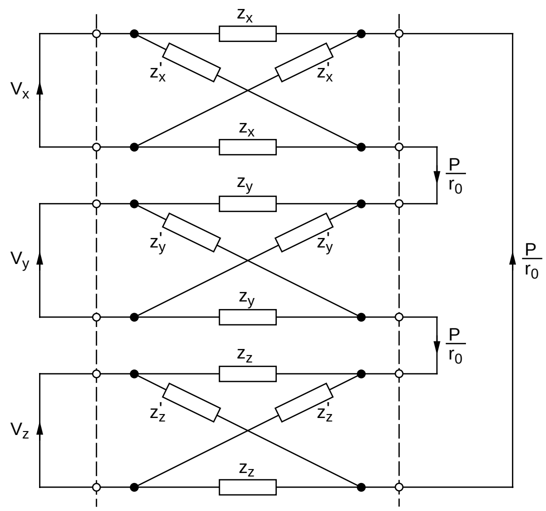

Figure 2: Network description: Lattice Structure

which characterize the lattices for each spatial dimension, contain both space and time derivatives. This will guarantee an explicit algorithm after discretization.

4. DISCRETIZATION

Once a suitable network description of the PDEs (3,4) has been obtained, a discrete algorithm follows by standard wave digital filtering principles [3, 5]. The two basic steps are

1. Introduction of wave quantities (pressure waves) by

$$A_i = P + R_0 V_\xi, \quad B_i = P - R_0 V_\xi \tag{15}$$

   for $i = 1 \ldots 6$ at each port of the three lattices in Fig. 2 with the port resistance R_0 .

2. Integration of the resulting differential operators with the so called *generalized trapezoidal rule* [5]. For the special choice of r_0 in the last section, this means that the integration path in the space-time-domain follows the direction of wave propagation. With the choice of the time step size T ($t = kT, \ k \in \mathbb{N}$), the spatial step size $h_\xi = h$ is identical in all three dimensions ($[x, y, z] = [lh, mh, nh], \ l, m, n \in \mathbb{N}$) and is linked to the time step size by $h = \sqrt{3}cT$. The port resistances R_0 are determined by the continuous impedances Z_ξ and Z'_ξ and by the step sizes T and h

$$R_0 = 2\frac{\rho_0}{T} = 2\frac{r_0}{h} \ . \tag{16}$$

Fig. 3 shows the resulting MD-WDF structure. Note, that spatial shifts by h occur only in connection with a delay by T. This follows from the lattice structure of the continuous network in Fig 2 and guarantees the explicit nature of the algorithm, as noted earlier. The adaptor at the right hand side is a three port adaptor as given in [3]. The wave quantities a_i and b_i, $i = 1 \ldots 6$ are discrete versions of the wave amplitudes A_i and B_i in (14).

The MD-WDF structure from Fig. 3 is implemented by setting up a state description in terms of the states x_i, $i = 1 \ldots 6$ at the outputs of the combined shift and delay elements. In each time step k, the new states $x_i(l, m, n, k+1)$ are computed from shifted versions of the current states $x_i(l, m, n, k)$. Since the states x_i are given in terms of wave quantities a_i and b_i, they have to be converted back to pressure and velocity components. By inversion of (15) at the ports a_2/b_2 and a_1/b_1 follows (indices (l, m, n, k) are omitted)

$$p = \frac{r_0}{2R_0}(a_2 - b_2), \quad v_x = \frac{1}{R_0}a_1 \tag{17}$$

and similarly for v_y and v_z. Finally, p and v_ξ are stated in terms of the state variables by expressing the wave quantities a_i, b_i by the states x_i, using the structure from Fig. 3 and the adaptor equations.

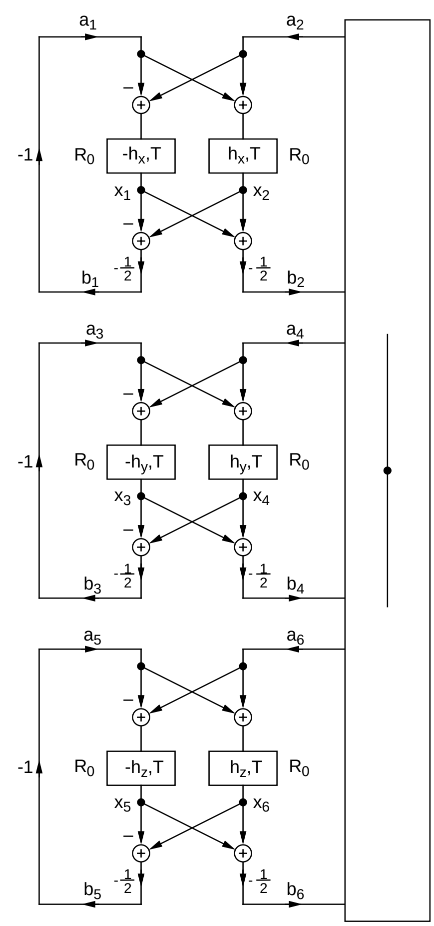

Figure 3: Multidimensional wave digital filter for the simulation of three-dimensional sound propagation

5. BOUNDARY CONDITIONS

Hard and reflection-free boundary conditions for two spatial dimensions have been treated in [5], while the extension to more general boundary conditions for one spatial dimension is shown in [1, 2, 6, 7]. We will briefly discuss the 1D approach and show its extension to higher dimensions.

Consider the calculation of the state variable x_1 at the boundary. Its determination from Fig. 3 would require the knowledge of values located beyond the boundary by h. Instead we use the description of the reflection properties of the boundary as given in terms of the wall impedance Z by $p = Zv_x$. Expressing at first p and v_x by (17) and then the wave quantities a_1, a_2 and b_2 by the states x_i gives a relation between all six state variables at the boundary. It serves to determine the value of x_1 which is in accordance with the boundary conditions. The cases of hard and soft reflection or absorbtion are included for special values of Z.

In two or three dimensions, the wall impedance may depend on the angle of incidence. In this case, the determination of states at a boundary involves also neighbouring points. Frequency dependent wall impedances can be modelled in the same way, when the unkown state is calculated from previous states in a time recursive fashion.

6. NUMERICAL RESULTS

The algorithm described above has been used to simulate the propagation of a sound wave in a building with three interconnected rooms (compare [8]). The resulting pressure field for a fixed point in time in a plane parallel to the floor is displayed in Fig 4. The outer walls (not shown) and the inner walls are highly reflecting, whereas floor and ceiling (not shown) are absorbing. A sound pulse emanates from the corner in front and propagates through all three rooms. Note the diffraction effects at the openings.

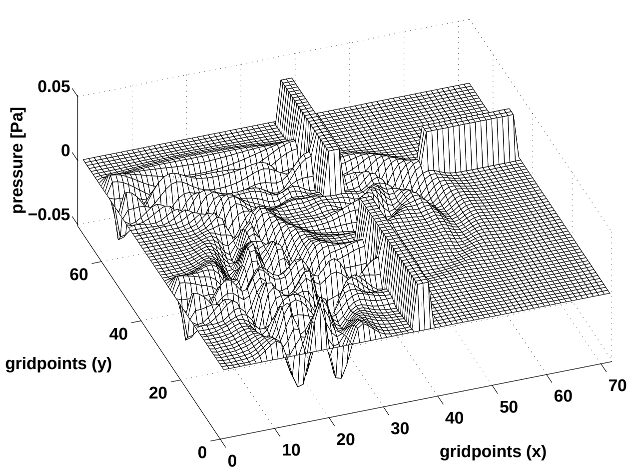

Figure 4: Sound propagation in a three-room building

7. CONCLUSIONS

It has been demonstrated, that multidimensional wave digital filters are a suitable tool for the simulation of room acoustics. The simulation algorithm including the boundary conditions are derived from first principles of physics. This allows the correct adaptation to boundary value conditions stated in terms of the wall impedances of the enclosure. The derivation shown here considered only the most simple case for the construction of the algorithm. More degrees of freedom in the choice of spatial and temporal step sizes can be gained by a more general layout of the network description.

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