NONLINEAR CHANNEL IDENTIFICATION AND EQUALIZATION FOR OFDM SYSTEMS[1]

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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) has become increasingly popular due to its potential applications in digital audio broadcasting, digital terrestrial TV broadcasting, and satellite communication. A notable drawback of OFDM systems is their sensitivity to nonlinear distortion. For maximum power efficiency, amplifiers and transmitters of modern communication systems often operate near their saturation regions which leads to nonlinear distortion. In this paper, we use the special property that the transmitted OFDM symbols are asymptotically white Gaussian to derive an algorithm that identifies the nonlinear channel. A nonlinear equalizer is built to compensate for the undesired nonlinearities. Simulation results show that the nonlinear equalizer outperforms its linear counterpart when nonlinear distortion is present.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has received a considerable amount of attention in the recent years. It has been proposed for use in digital audio broadcasting, digital terrestrial TV broadcasting, and satellite communication [2]. OFDM's current popularity stems from a number of nice traits that it offers. Equalization of potentially long multipath channels that typify urban environments is possible with a bank of 1 tap equalizers [8]. It is bandwidth efficient since the spectra of neighboring channels overlap, yet the channels can still be separated through the use of orthogonality. Its structure makes it easy to allocate more bits to parts of the channel with better transmission characteristics [2], and efficient hardware implementations are possible using FFTs and polyphase filtering.

In modern communication systems, a premium is often placed on power efficiency. This results in operating amplifiers such as traveling wave tubes (TWTs) in their nonlinear region. The concatenation of a linear transmitter filter, nonlinear amplifier, and linear channel, is a nonlinear channel with memory. To improve the recovery of the original OFDM symbols, nonlinear channel identification and equalization must be performed.

Past work in this area has usually centered around the use of memoryless nonlinear mappings to invert the effects of the presumably memoryless nonlinear amplifier [4]. This requires precise knowledge of the nonlinear amplifier's amplitude and phase characteristics, and the assumption that they do not change over time. Also, the nonlinearity is usually assumed to be frequency independent. For high bandwidth communication systems, these assumptions begin to break down.

In this paper we follow a different, more general approach to dealing with the nonlinear channel in the OFDM system. We model the nonlinear channel as a truncated Volterra series [10]. To identify the nonlinear channel, we exploit the property that the output of the OFDM transmitter is an asymptotically white Gaussian random process. This characteristic is often seen as a problem, since nonconstant envelope modulation formats typically perform poorly with nonlinear amplifiers. However, we use it to our advantage. It allows us to derive a formula for closed form baseband Volterra system identification when the channel input is circular complex Gaussian. (If the real and imaginary parts of a complex random variable X are mutually independent and have the same distribution, then X is said to be circular complex). This is done in a manner similar to a recent method developed for the identification of a real Volterra system when the input is real Gaussian [6]. Equalization of the nonlinear channel is done using a serial structure of Volterra filters derived from the identified channel [7], [11]. Simulations demonstrate that the nonlinear channel identification and equalization procedure improves the performance of OFDM systems in the presence of nonlinearity.

2. BACKGROUND

A simplified block diagram of an OFDM system is shown in Figure 1. The input data is encoded into a stream of symbols, often of a format such as QAM or PSK. Groups of N symbols are collected and sent to the IFFT. N is typically a power of 2 to facilitate a fast implementation. By inserting a guard interval after the IFFT (a cyclic extension that is longer than the channel), then removing it before the FFT, there is no interference between consecutive groups of N symbols. If the number of channels N is chosen such that the overall transfer function does not change significantly over an individual channel, then equalization of an individual channel is reduced to multiplication by a complex constant [8].

2.1 (Approximate) Gaussian IFFT Output

It is well known (e.g., [4] p.95) that when $X(k)$ is zero-mean, i.i.d., and with finite variance, then its IDFT values

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi n}{N}k}, \quad n = 0,1,\ldots,N-1$$

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are zero-mean, circular complex, and asymptotically white Gaussian distributed. This is true regardless of the distribution of *X(k)*. Therefore, the outputs of the IFFT block in Figure 1 are approximately Gaussian distributed, regardless of the distribution of the symbol stream (e.g., QAM, PSK) coming into the IFFT unit. The larger the block length *N*, the better the approximation. It is this special Gaussian property that allows us to perform closed form nonlinear channel identification and equalization for OFDM systems.

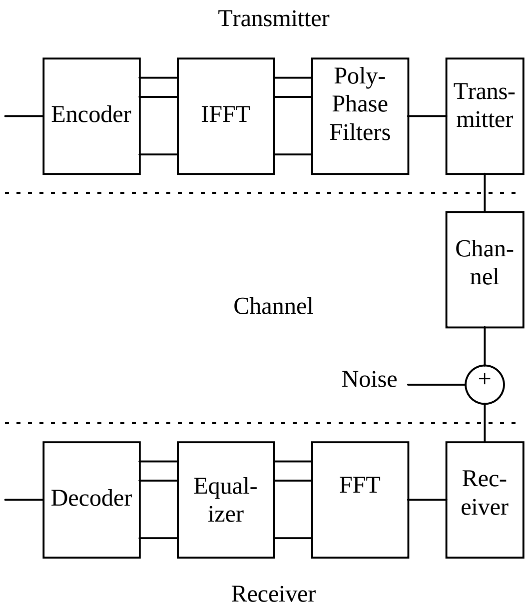

Figure 1: OFDM block diagram.

2.2 Nonlinear Channel

Nonlinearity enters into the OFDM system in a number of places. The most obvious is at the transmitter, where a nonlinear amplifier such as a TWT is often used. The output of the IFFT is a signal with varying amplitude which usually has a high peak-to-average signal power ratio. Therefore, the concatenation of the polyphase filters, TWT, channel, and receiver filters is a nonlinear system with memory. The Volterra series provides a general approach to modeling nonlinearities of this type.

3. CHANNEL IDENTIFICATION

3.1 The Baseband Volterra Channel

Previous work has been done on using the Volterra series to model nonlinear channels with memory. It has been shown in [1] that for a baseband equivalent model of a bandpass communication system, only the odd-order kernels of the Volterra series contribute to the output. For a *(2P+1)*th-order baseband Volterra series, the input-output relationship can be written as

$$y(n)=\sum_{p=0}^{P} y_{2p+1}(n)$$
$$y_{2p+1}(n)=\sum_{\tau_1}\cdots\sum_{\tau_{2p+1}} h_{2p+1}(\tau_1,\ldots,\tau_{2p+1})\times x^*(n-\tau_1)\cdots x^*(n-\tau_p)\,x(n-\tau_{p+1})\cdots x(n-\tau_{2p+1}) .$$

Note that the *(2p+1)*th-order term has *p* conjugated copies and *p+1* unconjugated copies of the input.

3.2 Baseband Volterra Kernel Identification

In this subsection we derive a closed form expression for the baseband Volterra kernels in terms of input-output data assuming that the input *x(n)* is zero mean, i.i.d., Gaussian distributed with variance *A* (c.f. Section 2.1). The derivation mimics that of [6], but is different in the sense that we have conjugated entries in the baseband Volterra series and that *x(n)* is circular complex. To estimate the highest (i.e. *2P+1*) order kernel, we consider the cross cumulant (for definitions and properties of cumulants, see [4] p.19) between *y(n)* and *2P+1* delayed copies of the input, *P* of which are unconjugated and *P+1* of which are conjugated. Using the Leonov-Shiryaev formula [4, p.21], we find that

$$cum\{y(n),x(n-u_1),\ldots,x(n-u_P),x^*(n-u_{P+1}),\ldots,x^*(n-u_{2P+1})\}$$
$$=s(u_1,\ldots,u_{2P+1})A^{2P+1}h_{2P+1}(u_1,\ldots,u_{2P+1})$$

where $s(u_1, \ldots, u_{2P+1})$ is the number of ways that conjugated (unconjugated) delayed copies of the inputs can be paired with unconjugated (conjugated) inputs in $y_{2P+1}(n)$ having the same delay. As an example, for the cubic kernel we have: $s(0, 0, 0) = 2$, $s(0, 0, 1) = 1$, $s(1, 0, 1) = 1$, and $s(1, 1, 1) = 2$.

Therefore, given the input *x(n)* and the output *y(n)*, we can estimate the highest order nonlinear kernel once the above cross cumulant is estimated. Afterwards, we remove $y_{2P+1}(n)$ from *y(n)* to reduce the highest order nonlinearity to *2P-1*. We then repeat the same procedure, using the residual series and *2P-1* copies of the input, *P-1* of which are unconjugated and *P* of which are conjugated, to estimate the *(2P-1)*th-order baseband Volterra kernel. This process is repeated until all of the kernels have been estimated.

As an example, consider a linear-cubic system. The third-order kernel is estimated from:

$$h_3(u_1,u_2,u_3)=\frac{cum\{y(n),x(n-u_1),x^*(n-u_2),x^*(n-u_3)\}}{s(u_1,u_2,u_3)A^3} .$$

Next, we remove

$$y_3(n)=\sum_{\tau_1}\sum_{\tau_2}\sum_{\tau_3} h_3(\tau_1,\tau_2,\tau_3)x^*(n-\tau_1)x(n-\tau_2)x(n-\tau_3)$$

from *y(n)* to form $y_1(n) = y(n) - y_3(n)$. The linear kernel is then estimated from

$$h_1(u_1)=\frac{cum\{y_1(n),x^*(n-u_1)\}}{A} .$$

Formulas for the second and fourth-order cumulants (needed for the first and third-order kernel estimates, respectively) of zero-mean random variables are:

$$cum\{x_1, x_2\} = E[x_1 x_2]$$

$$\begin{aligned} cum\{x_1, x_2, x_3, x_4\} &= E[x_1 x_2 x_3 x_4] - E[x_1 x_2]E[x_3 x_4] \\ &\quad - E[x_1 x_3]E[x_2 x_4] - E[x_1 x_4]E[x_2 x_3] \end{aligned}.$$

In practice, we substitute the expected value $E[\bullet]$ by the time average to form the sample moment estimate. We then use the above formulas to find the sample cumulant.

4. CHANNEL EQUALIZATION

After the Volterra kernels are estimated, the next step is to design an equalizer to remove the nonlinear intersymbol interference. We used an approach based on the contraction mapping theorem (CMT) that gives some implementation flexibility while providing a theoretical basis for convergence [7], [11].

We would like to design an equalizer G, for the $(2P+1)$th-order system H with input x_0 and output y, such that the output of G, x, approximates x_0 (see Figure 2). Following the derivation in [7], the formula for the equalizer is found as:

$$\begin{aligned} Hx_0 &= H(Gy) = H_1(Gy) + \sum_{p=1}^{P} H_{2p+1}(Gy) \\ H_1^{-1}Hx_0 &= Gy + H_1^{-1}\sum_{p=1}^{P} H_{2p+1}(Gy) \end{aligned}.$$

Let $x = Gy$, $B = H_1^{-1}H$, and $C = H_1^{-1}\sum_{p=1}^{P} H_{2p+1}$. Then

$$Bx_0 - Cx = x \text{ or } Bx_0 - Cx_{i-1} = x_i.$$

The resulting equalizer takes the form of Figure 3. If the mapping $T(x) = Bx_0 - Cx$ satisfies the requirements of the CMT [5], then $x_i \rightarrow x$ and the system is equalized. Simulations have shown that the good results are typically obtained after a few iterations [7].

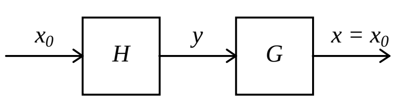

Figure 2: Nonlinear system H with equalizer G.

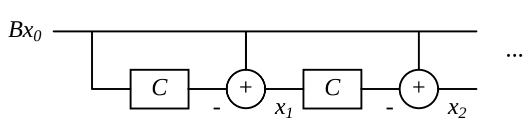

Figure 3: Serial equalizer structure.

5. SIMULATIONS

5.1 Channel Identification

We provide here a numerical example to illustrate the proposed identification algorithm for an OFDM system. The nonlinear channel was linear-cubic and 16384 16-QAM symbols were used as the input. The symbol stream was split into 16 frames of length 1024 each. 100 Monte Carlo trials were performed to determine the mean and standard deviation of the kernel estimates, with results shown in Tables 1 and 2.

Table 1: First-Order Kernel

Lag	0	1	2
Value	1.0000	0.5000	-0.2400
Mean	1.0006	0.5009	-0.2401
Std. Dev.	0.0141	0.0225	0.0036

Table 2: Third-Order Kernel

Lag	(0,0,0)	(0,0,1)	(1,0,1)	(1,1,1)
Value	-0.5000	0.2000	-0.3000	0.4000
Mean	-0.4987	0.1935	-0.3058	0.4002
Std. Dev.	0.0196	0.0363	0.0370	0.0425

Since the cubic kernel values are comparable to those of the linear kernel, the nonlinearity here is rather severe. In general, higher order kernel estimates tend to have larger variance due to the higher-order cumulant used. But lower order kernels may also suffer from high variance because of error propagation. (Keep in mind that the linear kernel is estimated last). The bias and standard deviation shown in Tables 1 and 2 seem to be reasonable.

5.2 Channel Equalization

Once the nonlinear channel was identified as illustrated in Section 5.1, we implemented a 5 stage nonlinear equalizer based on the CMT principal explained in Section 4. Figure 4 shows the scatter diagram of the unequalized output symbols at SNR = 30dB. SNR is defined here as

$$SNR = 20\log_{10}\left(\frac{var\{y(n)\}}{var\{v(n)\}}\right)$$

where $v(n)$ is the additive white Gaussian noise in Figure 1. Figure 5 shows the equalized symbols after 5 iterations of the nonlinear equalizer. We varied the SNR level and calculated the error probability in the equalized symbols. As a comparison, we also implemented the 1 tap linear equalizer and computed its error probability. The result is shown in Figure 6. The nonlinear equalizer outperforms the linear equalizer, especially at higher SNR.

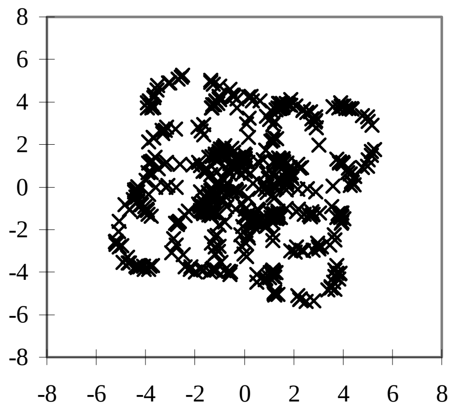

Figure 4: Unequalized output constellation.

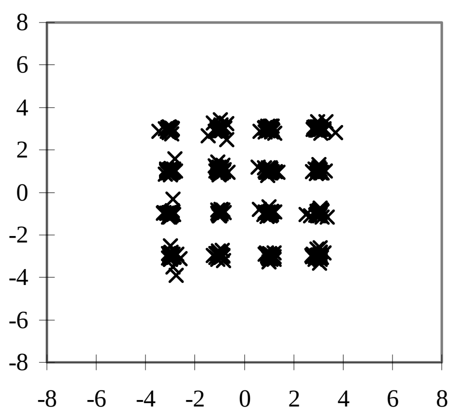

Figure 5: Equalized output constellation using 5 iterations of the equalizer designed from the estimated kernels.

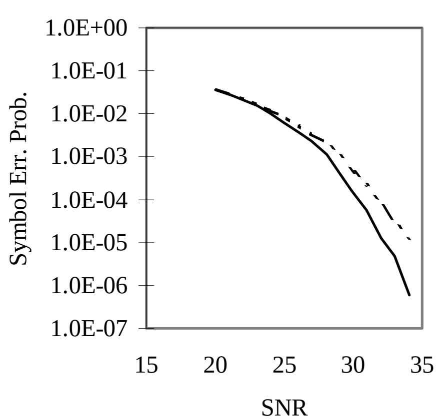

Figure 6: Symbol error probability vs. SNR. The linear equalizer corresponds to the dashed line, and the nonlinear equalizer to the solid line.

6. CONCLUSIONS

We proposed a new method for dealing with possible nonlinearities in OFDM systems, taking advantage of the approximately white Gaussian property of the IFFT output in OFDM systems. The channel was modeled as a baseband Volterra series and identified by using a new formula derived for baseband Volterra system identification. Equalization was achieved via a serial structure of Volterra filters created from the identified channel. Simulations demonstrated the effectiveness of the proposed methods.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

[1] S. Benedetto, E. Biglieri, and R. Daffara, “Modeling and performance evaluation of nonlinear satellite links - A Volterra series approach,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 15, no. 4, pp. 494-507, 1979.

[2] J. Bingham, “Multicarrier modulation for data transmission: An idea whose time has come,” *IEEE Communications Magazine*, pp. 5-14, May 1990.

[3] A. Brajal and A. Chouly, “Compensation of nonlinear distortions for orthogonal multicarrier schemes using predistortion,” *IEEE Global Tele. Conf.*, vol. 3, pp. 1909-1914, 1994.

[4] D. Brillinger, *Time Series: Data Analysis & Theory*. San Francisco: Holden-Day, 1981.

[5] H. Khalil, *Nonlinear Systems*. Upper Saddle River: Prentice Hall, 1996.

[6] P. Koukoulas and N. Kalouptsidis, “Nonlinear system identification using Gaussian inputs,” *IEEE Transactions on Signal Processing*, vol. 43, no. 8, pp. 1831-1841, 1995.

[7] R. Nowak and B. Van Veen, “Volterra filter equalization: A fixed point approach,” *IEEE Transactions on Signal Processing*, vol. 45, no. 2, pp. 377-388, 1997.

[8] A. Peled and A. Ruiz, “Frequency domain data transmission using reduced computational complexity algorithms,” *IEEE ICASSP*, pp. 964-967, 1980.

[9] A. Saleh, “Frequency independent and frequency-dependent models of TWT amplifiers,” *IEEE Transactions on Communications*, vol. 29, no. 11, pp. 1715-1720, 1981.

[10] M. Schetzen, *The Volterra and Wiener Theories of Nonlinear Systems*. Malabar: Robert E. Krieger, 1989.

[11] C. Tseng and E. Powers, “Nonlinear channel equalization in digital satellite systems,” *IEEE Global Telecommunications Conference*, vol.3, pp. 1639-1643, 1993.