

# JOINT COMPRESSION AND RESTORATION OF IMAGES USING WAVELETS AND NON-LINEAR INTERPOLATIVE VECTOR QUANTIZATION

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## ABSTRACT

In this paper, we present a wavelet based non-linear interpolative vector quantization scheme for joint compression and restoration of images; two tasks which are traditionally regarded as having conflicting goals. Vector quantizer codebook training is done using a training set consisting of pairs of the original image and its diffraction-limited counterpart. The designed VQ is then used to compress and simultaneously restore diffraction-limited images. Results from simulations indicate that the image produced at the output of the decoder is quantitatively and visually superior to the diffraction-limited image at the input to the encoder. We also compare the performance of several wavelet filters in our algorithm.

## 1. INTRODUCTION

Vector quantization(VQ) is a very widely used compression technique in image coding [1]. In recent years, it has also been used to do various kinds of image processing while concurrently achieving compression [2]. VQ has been used to do such diverse image processing as edge detection [3], histogram modification [4], volume rendering [5], classification [6], and restoration [7]. In this paper, we present a non-linear interpolative vector quantization (NLIVQ) based restoration technique using wavelets. We train a vector quantizer on pairs of original and diffraction-limited images, while using the wavelet transform to scale down the complexity of the training procedure.

## 2. IMAGE RESTORATION AND SUPER RESOLUTION

The problem of image restoration deals with estimating an image given its degraded version. In this paper, the problem of blur caused by diffraction-limited, unaberrated optics with incoherent illumination is dealt with. Optical systems with these characteristics can be modeled as linear, shift-invariant operators with well defined optical transfer functions derived from the scalar diffraction theory of light [8]. These optical transfer functions have a cutoff frequency such that all information in the object above it is suppressed.

The diffraction-limited image,  $g(x, y)$ , can be expressed as a 2-D convolution of the form

$$g(x, y) = \int \int f(\alpha, \beta) h(x - \alpha, y - \beta) d\alpha d\beta \quad (1)$$

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where  $f(x, y)$  is the original object and  $h(x, y)$  is the point spread function of the optical system. The problem of estimating  $f(x, y)$ , given  $g(x, y)$ ,  $h(x, y)$ , and (possibly) some *a priori* knowledge of  $f(x, y)$ , is referred to as the inverse problem. In general, this problem may be of an ill-posed nature due to the presence of noise in the acquired image and/or singularities in the imaging system.

Image restoration techniques can be broadly classified as being linear or non-linear. While linear techniques such as Inverse and Wiener filtering do a good job of restoring information below the optical cutoff frequency, they are incapable of extracting information above it [9]. However, there exist non-linear techniques capable of doing passband restoration as well as extracting information above the optical cutoff. Such a restoration process is referred to as *super resolution*.

Algorithms based on Bayesian estimation [9] and convex set projections [10] have been shown to perform super resolution. For a detailed review of super resolution, the reader is referred to [11].

## 3. NON-LINEAR INTERPOLATIVE VECTOR QUANTIZATION

Vector quantization is a mapping of vectors from one domain to another, in order to better achieve a desired purpose such as compression. Mathematically, a vector quantizer,  $Q$ , of dimension  $k$  and size  $K$  is a mapping from a vector in  $k$ -dimensional Euclidean space,  $R^k$ , into a finite set  $C$  containing  $K$  reproduction points called codewords. Thus

$$Q : R^k \rightarrow C \quad (2)$$

where the set  $C$  is called the codebook of size  $K$ . The rate of such a VQ is said to be  $r = \frac{\log_2 K}{k}$  bits/dimension.

NLIVQ was introduced by Gersho, as a technique that can be used to mitigate the complexity barrier posed by direct VQ at higher dimensions, even at moderately high rates [12]. The basic idea of NLIVQ is as follows. Let  $X$  be a random vector of dimension  $k$ . Now, if  $k$  is relatively large, then ordinary full search VQ is not feasible at high rates. But if it is possible to extract a suitable "feature vector",  $U$ , of dimension  $n < k$ , then  $X$  can be estimated from the vector quantized version,  $\hat{U}$ , of  $U$ . This estimation process of  $X$  from  $\hat{U}$  can be accomplished in one step by designing the interpolative decoder such that it is optimal for a given encoder. It may be noted here that the encoder and decoder codebooks would be of the same size but of dimensions  $n$  and  $k$  respectively. It is also important to realize that the choice of a suitable feature vector depends entirely on the application and that there is no restriction on its dimensionality.

In [7], Sheppard *et al.* introduced a novel use of NLIVQ for image restoration. Mathematically, the NLIVQ for image restoration can be described as follows. Let  $\{F^i, G^i\}_{i=1}^N$  be a sequence of image pairs, where  $F^i$  and  $G^i$  are the original and diffraction-limited images, respectively. Decompose each image into  $M \times M$  non-overlapping blocks. Let  $f^{ij}$  and  $g^{ij}$  be blocks  $j$  from  $F^i$  and  $G^i$  respectively. Assume that the encoder  $\mathbf{E}$ , decoder  $\mathbf{D}$ , and the associated codebook  $C$ , are given for a VQ that minimizes the mean-squared error

$$MSE = E \| g^{ij} - \hat{g}^{ij} \|^2. \quad (3)$$

The process of choosing the quantized block  $\hat{g}^{ij}$  can be written as

$$\hat{g}^{ij} = \mathbf{D}(\mathbf{E}(g^{ij})) = \arg \min_{c_l \in C} \| g^{ij} - c_l \|^2, \quad (4)$$

where  $c_l$  refers to the  $l$ th entry of  $C$ .

Next, a new decoder  $\mathbf{D}^*$  and its associated codebook  $C^*$  is derived by minimizing the conditional expectation

$$E[\| f^{ij} - \hat{f}^{ij} \|^2 | \mathbf{E}(g^{ij}) = l], \quad (5)$$

where encoder  $\mathbf{E}$  returns the index of the optimal codebook entry. For a given set of training data, let  $R_l = \{f^{ij} : \mathbf{E}(g^{ij}) = l\}$ . Define entry  $l$  of  $C^*$  as the centroid of  $R_l$ , or

$$c_l^* = \left( \frac{1}{|R_l|} \right) \sum_{f^{ij} \in R_l} f^{ij} \quad (6)$$

where  $|R_l|$  denotes the cardinality of  $R_l$ . Finally, the NLIVQ restoration algorithm is given by

$$\hat{f}^{ij} = \mathbf{D}^*(\mathbf{E}(g^{ij})) = c_{\mathbf{E}(g^{ij})}^*, \quad (7)$$

where  $\hat{f}^{ij}$  is the restored image block. The NLIVQ based restoration strategy, described above, is shown in Figure 1.

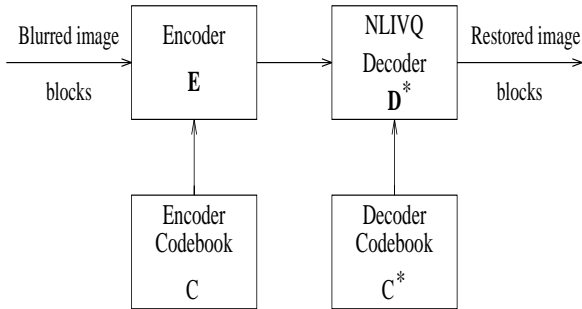


Figure 1: NLIVQ based restoration strategy.

#### 4. CODEBOOK DESIGN ALGORITHMS

The crux of the design procedure is the design of the encoder codebook  $C$ . The decoder codebook,  $C^*$ , can be derived easily once  $C$  has been determined. A technique commonly used in VQ codebook design is the Lloyd algorithm [13]. Although the Lloyd algorithm is conceptually simple, it has heavy computational requirements and its use limits the NLIVQ to low encoding rates and small block sizes.

In [7], a non-iterative, discrete cosine transform (DCT) based approach was proposed to overcome this problem. Although this approach allowed the use of larger block sizes and resulted in significant improvements quantitatively and visually, the restored/compressed images had blocking artifacts. Later, an improved version of the algorithm was presented in [14], wherein, overlapped blocks were employed. Many of the blocking artifacts of the earlier algorithm were suppressed, and the restored images were qualitatively and quantitatively better. Unfortunately, with the use of overlapping of blocks, no compression was achieved.

In this work, we improve the DCT-based design procedure of [7] using a wavelet-based approach as described in this section. The proposed algorithm does not have the blocking artifacts of [7], and, unlike [14], performs joint compression and restoration.

Given a training image, we perform a  $m$ -level uniform sub-band decomposition using the discrete wavelet transform, resulting in  $4^m$  equal-sized subbands. Spatially corresponding coefficients from each band are combined to form blocks of size  $2^m \times 2^m$ . These wavelet coefficient blocks are then treated as VQ training vectors. This procedure is illustrated in Figure 2 for a 2-level uniform decomposition.

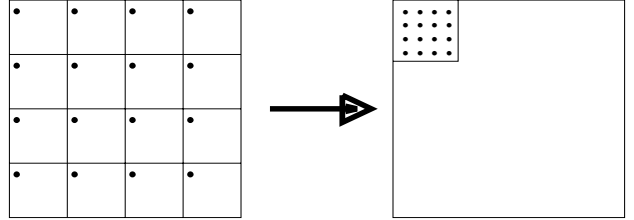


Figure 2: Formation of wavelet training blocks.

#### Wavelet-Based Encoder Design

- Given  $N$  original training images  $F^i$ , create their diffraction-limited counterparts  $G^i$ .
- Create the  $2^m \times 2^m$  wavelet coefficient blocks,  $f^{ij}$  and their counterparts,  $g^{ij}$ , as described above.
- For a given bit rate of  $R$  bits/pixel, the  $L = R \times 2^m \times 2^m$  bits available for each block are allocated among the wavelet coefficients so that the quantization error is minimized.
- If  $r_p$  is the number of bits allocated to the wavelet coefficient in the  $p$ th subband, a Lloyd-Max scalar quantizer is designed for these coefficients assuming a Laplacian distribution. This procedure is repeated for each subband.
- An encoder codeword index is defined to be the decimal equivalent of the concatenation of binary codes for the scalar quantized wavelet coefficients.

This process implicitly defines a codebook  $C$  which is never explicitly computed or stored.

#### NLIVQ Decoder Design

- For each input wavelet coefficient block,  $g^{ij}$ , compute the index produced by the encoder  $\mathbf{E}(g^{ij}) = q$ .
- Add the corresponding wavelet coefficient block,  $f^{ij}$ , from the original image to the accumulator  $a_q^*$  and increment the counter  $s_q^*$ .

- Once all training blocks have been exhausted, compute each codeword in  $C^*$  as the average

$$c_q^* = \frac{1}{s_q^*} a_q^*. \quad (8)$$

It is worthwhile to note here that the algorithms described above are non-iterative and therefore much less computationally intensive than the traditional Lloyd training algorithm.

## 5. SIMULATION RESULTS

We have performed simulations using the algorithms described in the previous section with several different wavelet transforms. Our training set consisted of 53 grayscale “urban” images, each of size  $512 \times 512$ . The blurred images were generated using a diffraction-limited optical transfer function with a cut-off frequency equal to half the folding frequency. We used a 2-level uniform subband decomposition resulting in  $4 \times 4$  wavelet blocks.

In order to achieve relatively high rates that enable super resolution, we treat the low-pass coefficients separately as follows. We first apply a Wiener filter on the diffraction-limited image. Then we perform a 2-level uniform subband decomposition of this Wiener-restored image and quantize the low-pass coefficients using  $r_0 = 8$  bits per coefficient. We allocate 20 bits to the remaining 15 coefficients of each block. This results in an overall rate of 1.75 bits/pixel.

In Table 1, we present the PSNR results on a test image that was not in the training set. In the table, the PSNR between the original and the blurred image is given, followed by the PSNR’s between the original and the compressed/restored images at 1.75 bits/pixel using different wavelets. We have used biorthogonal wavelet transforms from [15], [16], and [17]. The notation  $(r, s)$  in the table denotes a wavelet transform with  $r$  and  $s$  filter taps for the analysis and synthesis high pass filters, respectively. The PSNR improvements for the restored images range roughly from 0.2 to 1.75 dB.

Figures 3 and 4 show the original and blurred images, respectively. The restored image using the (2,2) filters of [15] is shown in Figure 5. The visual improvement of Figure 5 over Figure 4 is significant, especially considering that the restored image of Figure 5 is compressed at 1.75 bits/pixel. The restored image also shows signs of modest super resolution, which is very encouraging.

Table 1: PSNR results

PSNR (dB)						
Blurred	CDF [15]		VBL [16]			TVC [17]
	(2,2)	(3,9)	(3,9)	(6,2)	(11,13)	(18,10)
22.55	24.28	22.74	23.00	24.00	23.51	23.27

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Figure 3: The original image.



Figure 4: The blurred image.



Figure 5: The compressed/restored image at 1.75 bits/pixel.