ADAPTIVE FUZZY MORPHOLOGICAL FILTERING OF IMAGES

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ABSTRACT

In this paper we introduce a neural network implementation of fuzzy mathematical morphology operators and apply it to image denoising. Using a supervised training method and differentiable equivalent representations for the fuzzy morphological operators, we derive efficient adaptation algorithms to optimize the structuring elements. We can then design fuzzy morphological filters for processing multi-level or binary images. The convergence behavior of basic structuring elements for the opening filter and different signals, and its significance for other structuring elements of different shape is discussed. To illustrate the performance of the fuzzy opening filter we consider the removal of impulse noise in multi-level and binary images.

1. INTRODUCTION

Despite increasing interest in the application of mathematical morphology to image processing [1], the selection of appropriate structuring elements remains a difficult problem. The shape and size of structuring elements determine the geometrical features in an image that are preserved or removed, thus the importance of their selection. Although advances in optimization of structuring elements [3, 4, 5] have been achieved, problems are still encountered in the lack of differentiability of morphological operators, and the difficulty in analyzing convergence.

In this paper we consider fuzzy morphology [2], which relates the subsethood of fuzzy rather than conventional sets and as such fuzzy morphology applies simultaneously to binary and multi-level images. We extend the concepts in fuzzy [6, 7] and morphological [8] neural networks to fuzzy morphological neural networks. To apply such structures to denoising of images, we establish a training algorithm that optimizes the structuring elements used in the network. Our optimization procedure is not constrained as those in [3, 5] and permit us to analyze the convergence of the optimization of basic structuring elements and its general implications for other structuring elements. We show how to design fuzzy morphological filters that can be applied in the removal of impulse noise in binary or multi-level images.

2. FUZZY MORPHOLOGICAL NEURAL NETWORKS

Fuzzy mathematical morphology [2] has been developed using the notion of "fuzzy fitting" or subsethood of fuzzy sets. The fuzzy fitting is characterized by an inclusion indicator $\mathcal{I}(A,B) \in [0,1]$ given by

$$\mathcal{I}(A,B) = \wedge_{x\in X}[1 \wedge (1-\mu_A(x)+\mu_B(x))] \qquad (1)$$

that yields the degree of fitting of A into B.

For a signal $f(n)$ and a structuring element $k(n)$, with support regions F and K and corresponding membership functions $\mu_f(n)$ and $\mu_k(n)$, using $\mathcal{I}(A,B)$, erosion ($\ominus$), dilation ($\oplus$), opening ($\circ$) and closing ($\bullet$) are defined as follows:

$$\mu_{f\ominus k}(n) = \wedge_{m\in K, n+m\in F}[1 \wedge (1-\mu_k(m)+\mu_f(n+m))]$$
$$\mu_{f\oplus k}(n) = \vee_{m\in K, n-m\in F}[0 \vee (\mu_k(m)+\mu_f(n-m)-1)]$$
$$\mu_{f\circ k}(n) = \mu_{(f\ominus k)\oplus k}(n)$$
$$\mu_{f\bullet k}(n) = \mu_{(f\oplus k)\ominus k}(n)$$

The implementation of the above fuzzy morphological operators can be done by modifying fuzzy neural networks [6, 7]:

Definition 1 *A fuzzy morphological neuron has inputs* $\{\mu_f(n), n \in F\}$, *with a structuring element* $\{\mu_k(m), m \in K\}$ *as the synaptic weights, and a single output* $\mu_g = \psi(\mathcal{I}(k,f))$, *where* $\psi(\cdot)$ *is an activation function.*

The synaptic weighting and aggregation correspond to the inclusion indicator $\mathcal{I}$ determining the degree of fitting between the input $f(n)$ and the structuring element $k(n)$. When considering binary images, the activation function is the sigmoidal function $\psi(a) = 1/(1+exp(-\alpha(a-\frac{1}{2})))$ where $\alpha > 0$. For multi-level images, we let $\psi(a) = a$.

Fuzzy morphological neurons can thus implement the different fuzzy morphological operators. In Fig. 1, we display a fuzzy opening neural network which consists of an input layer, a hidden layer formed by fuzzy erosion neurons, and an output layer with a single fuzzy dilation neuron.

3. ADAPTIVE FUZZY MORPHOLOGICAL FILTER DESIGN

Optimization of the structuring elements used in the network is done using the complement of the equality index proposed in [6]. For a given input $\mu_f(n)$, corresponding to $f(n)$, let $\mu_g(n) = \psi(\mu_{f\diamond k}(n))$ be the output of a fuzzy morphological neural network implementing a morphological operator denoted by $\diamond$. Letting $\mu_{\mathbf{k}}{}^{(i)} = [\mu_k{}^{(i)}_{(-M_1)} \cdots \mu_k{}^{(i)}_{(0)} \cdots \mu_k{}^{(i)}_{(M_2)}]^T$ be the structuring element vector at iteration i, we wish to minimize the inequality index

$$J(\mu_k(m)) = | \mu_t(n) - \mu_g(n) | \tag{2}$$

where $\mu_t(n)$ denotes the target or desired function.

Using steep-descent method, the minimization yields the following training algorithm:

$$\begin{aligned} \mu_{\mathbf{k}}{}^{(i+1)} = \quad & \mu_{\mathbf{k}}{}^{(i)} + \eta \, \mathrm{sgn}(\mu_t(n) - \mu_g(n)) \psi'(\mu_{f\diamond k}(n)) \\ & \times \frac{\partial \mu_{f\diamond k}(n)}{\partial \mu_{\mathbf{k}}{}^{(i)}} \end{aligned} \tag{3}$$

where $\psi'(a) = (\alpha e^{-\alpha(a-\frac{1}{2})})/((1+e^{-\alpha(a-\frac{1}{2})})^2)$ for binary images and 1 for multi-level images, $\mathrm{sgn}(\cdot)$ denotes the sign function, and $\eta \in [0, 1]$ is the training rate.

3.1. Adaptive Fuzzy Erosion and Dilation Filters

To calculate the derivative of the output of a fuzzy erosion neuron, $\mu_g(n) = \psi(\mu_{f\ominus k}(n))$, we use the alternate representation of the minimum function given in [7]. In fact, letting $\mu_{e(f(n))}(m) \triangleq [1 \wedge (1 - \mu_k{}^{(i)}_{(m)} + \mu_f(n+m))]$ and

$$U_2[a] = \begin{cases} 1 & \text{if } a \geq 0 \\ 0 & \text{if } a < 0 \end{cases} \quad U_3[a] = \begin{cases} 1 & \text{if } a > 0 \\ \frac{1}{2} & \text{if } a = 0 \\ 0 & \text{if } a < 0 \end{cases}$$

we have that

$$\begin{aligned} \mu_{f\ominus k}(n) &= \wedge_{m\in K} \mu_{e(f(n))}(m) \\ &= \frac{2^{N_e}}{N_e} \sum_{m\in K} \{ \prod_{\tilde{m}\in K} U_3[\mu_{e(f(n))}(\tilde{m}) \\ &\qquad - \mu_{e(f(n))}(m)]\} \mu_{e(f(n))}(m). \end{aligned}$$

It can be shown that

$$\begin{aligned} \frac{\partial \mu_{f\ominus k}(n)}{\partial \mu_k{}^{(i)}_{(m)}} &= \frac{\partial \wedge_{\tilde{m}\in K} \mu_{e(f(n))}(\tilde{m})}{\partial \mu_{e(f(n))}(m)} \frac{\partial \mu_{e(f(n))}(m)}{\partial \mu_k{}^{(i)}_{(m)}} \\ &= -\frac{2^{N_e}}{N_e} \prod_{\tilde{m}\in K} U_3[\mu_{e(f(n))}(\tilde{m}) - \mu_{e(f(n))}(m)] \\ &\quad \times U_3[\mu_k{}^{(i)}_{(m)} - \mu_f(n+m)], \end{aligned}$$

and noting that

$$\prod_{\tilde{m}\in K} U_3[\mu_{e(f(n))}(\tilde{m}) - \mu_{e(f(n))}(m)] = \begin{cases} (\frac{1}{2})^{N_e} & \text{if } \mu_{e(f(n))}(\tilde{m}) \geq \mu_{e(f(n))}(m), \forall \tilde{m} \\ 0 & \text{otherwise} \end{cases}$$

we then have that

$$\frac{\partial \mu_{f\ominus k}(n)}{\partial \mu_{\mathbf{k}}{}^{(i)}} \triangleq \dot{\mathbf{e}}^{(i)}_{f(n)} = [\dot{e}^{(i)}_{f(n)}(-M_1) \cdots \dot{e}^{(i)}_{f(n)}(M_2)]^T$$

where

$$\dot{e}^{(i)}_{f(n)}(m) = \begin{cases} -\frac{1}{N_e} U_3[\mu_k{}^{(i)}_{(m)} - \mu_f(n+m)] \\ \qquad \text{if } \mu_{e(f(n))}(m) \text{ is selected as min} \\ 0 \quad \text{otherwise} \end{cases} \tag{4}$$

$N_e = \sum_{m\in K} \prod_{\tilde{m}\in K} U_2[\mu_{e(f(n))}(\tilde{m}) - \mu_{e(f(n))}(m)]$ is the number of inputs $\mu_{e(f(n))}(m)$ equal to the minimum output of the aggregator. Replacing this derivative in (3) the training for the fuzzy erosion filter is obtained.

Similarly, the derivative for the adaptive fuzzy dilation filter is given by

$$\frac{\partial \mu_{f\oplus k}(n)}{\partial \mu_{\mathbf{k}}{}^{(i)}} \triangleq \dot{\mathbf{d}}^{(i)}_{f(n)} = [\dot{d}^{(i)}_{f(n)}(-M_1) \cdots \dot{d}^{(i)}_{f(n)}(M_2)]^T$$

$$\dot{d}^{(i)}_{f(n)}(m) = \begin{cases} \frac{1}{N_d} U_3[\mu_k{}^{(i)}_{(m)} + \mu_f(n-m) - 1] \\ \qquad \text{if } \mu_{d(f(n))}(m) \text{ is selected as max} \\ 0 \quad \text{otherwise} \end{cases} \tag{5}$$

where $\mu_{d(f(n))}(m) \triangleq [0 \vee (\mu_k{}^{(i)}_{(m)} + \mu_f(n-m) - 1)]$ and $N_d = \sum_{m\in K} \prod_{\tilde{m}\in K} U_2[\mu_{d(f(n))}(m) - \mu_{d(f(n))}(\tilde{m})]$ is the number of inputs $\mu_{d(f(n))}(m)$ equal to the maximum output of the aggregator. Replacing this derivative in (3) the training for the fuzzy dilation filter is obtained.

3.2. Adaptive Fuzzy Opening (AFO) Filter

Combining the adaptive fuzzy erosion and dilation filters, we obtain the opening filter. The derivative for this filter is given by

$$\frac{\partial \mu_{f\circ k}(n)}{\partial \mu_{\mathbf{k}}{}^{(i)}} = (\dot{\mathbf{E}}^{(i)}_f \mathbf{R} + \mathbf{I}) \dot{\mathbf{d}}^{(i)}_{f\ominus k(n)} \tag{6}$$

where $\dot{\mathbf{E}}^{(i)}_f = [\dot{\mathbf{e}}^{(i)}_{f(n-M_2)} \cdots \dot{\mathbf{e}}^{(i)}_{f(n)} \cdots \dot{\mathbf{e}}^{(i)}_{f(n+M_1)}]$, and $\mathbf{R}$ and $\mathbf{I}$ are $(M_1+M_2+1)\times(M_1+M_2+1)$ reflection and identity matrix, respectively. Clearly, the derivative for AFO filter indicates that it consists of multiple derivative erosion vectors with input $\mu_f(n)$, and a derivative dilation vector with input $\mu_{f\ominus k}(n)$. Replacing this derivative in (3) the training for the fuzzy opening filter is obtained.

3.3. Convergence of Structuring Elements

Consider the following basic structuring elements:
Definition 2 *A structuring element, $\mu_k(m)$ is said to be a delta structuring element if $\mu_k(m) \ll 1, \forall m(\neq 0) \in K$, $\mu_k(0) = 1$ and it is such that $\mu_{f\diamond k}(n) = \mu_f(n)$,* for any fuzzy morphological operation $\diamond$.
Definition 3 *A structuring element, $\mu_k(m)$ is said to be a flat structuring element if $\mu_k(m) = 1, \forall m \in K$.*
For these structuring elements we have the following properties:
Proposition 1 $(\dot{\mathbf{E}}_f^{(i)}\mathbf{R} + \mathbf{I})\dot{\mathbf{d}}_{f\ominus k(n)}^{(i)} = \mathbf{0}$ for the delta structuring element.
Proposition 2 Let $\mu_f(n)$ correspond to a flat, or a monotonically increasing or decreasing signal. Then $(\dot{\mathbf{E}}_f^{(i)}\mathbf{R} + \mathbf{I})\dot{\mathbf{d}}_{f\ominus k(n)}^{(i)} = \mathbf{0}$ for the flat structuring element.
As expected, proposition 1 indicates the delta structuring element for opening is optimal for any input signal. Likewise, the flat structural element according to proposition 2 is optimal for flat or increasing or decreasing signals.
Proposition 3 Let $\Delta_{\mathbf{o}}^{(i)}$ be the opening update vector at iteration i.

$$\Delta_{\mathbf{o}}^{(i)} = (\dot{\mathbf{E}}_f^{(i)}\mathbf{R} + \mathbf{I})\dot{\mathbf{d}}_{f\ominus k(n)}^{(i)} = [\Delta_o^{(i)}(-M_1)\cdots\Delta_o^{(i)}(0)\cdots\Delta_o^{(i)}(M_2)]^T$$

where $\Delta_o^{(i)}(m) = \frac{\partial \mu_{f\circ k}(n)}{\partial \mu_{k^{(i)}_{(m)}}}$. Then for an initial flat structuring element we have that $\sum_{m\in K}\Delta_o^{(0)}(m) = 0$.
This proposition indicates that for an initial flat structuring element, if there are some positive update terms $\Delta_o^{(0)}(m) > 0$ for some $m \in K$, then there exist some negative update terms $\Delta_o^{(0)}(\tilde{m}) < 0$ for some $\tilde{m} \in K$. Therefore, the shape of resulting structuring element from the initial flat structuring element lies between the shape of delta and flat structuring elements at next iteration.

4. SIMULATION RESULTS

In this simulation, multilevel images are linearly normalized into the range $[0, 1]$ to get the membership functions, 3×3 flat structuring element is used for training, and positive impulse is set to 1. Also each image is divided into equal size blocks of pixels, which are then concatenated to create the input vector. To get locally optimized structuring elements, the image is partitioned into regions. Each optimal structuring element obtained from the training process is then used to filter the noisy image. Figure 2 illustrates AFO filtering for "Circular Zone Plate" (CZP). As shown in Fig. 2, each optimal structuring element adapts its shape to the region. Thus the noise component which does not fit the optimal structuring element is effectively removed, while the detail signal is well preserved. Figure 3 shows filtering of "Lena" image corrupted by 20% positive impulse noise. The peak-to-peak SNR values of restored images by two-state filter[9] and AFO filter are 28.08 and 32.61 dB, respectively. (In [9], it is indicated that the performance of multistate filter using supervised training is 1-2 dB better than that of two-state filter.)

We can develop unsupervised training algorithm based on propositions 1-3. In this case, the desired signal is the input (noisy) signal itself. Using 4 directional flat structuring elements with 1×3 size, the restored "FINGERPRINT" image by an unsupervised AFO filter is shown in Fig. 4.

5. CONCLUSIONS

In this paper we introduced the neural network implememtation of fuzzy morphological operators. An algorithm to optimize the structuring elements was proposed. We then showed a way to design fuzzy morphological filters for the removal of noise from binary and multi-level images. We presented some preliminary results in the convergence behavior of the optimization of the structuring elements. This is an open area of research where more work needs to be done. The simulation results indicate that the method compares well to existing methods.

6. REFERENCES

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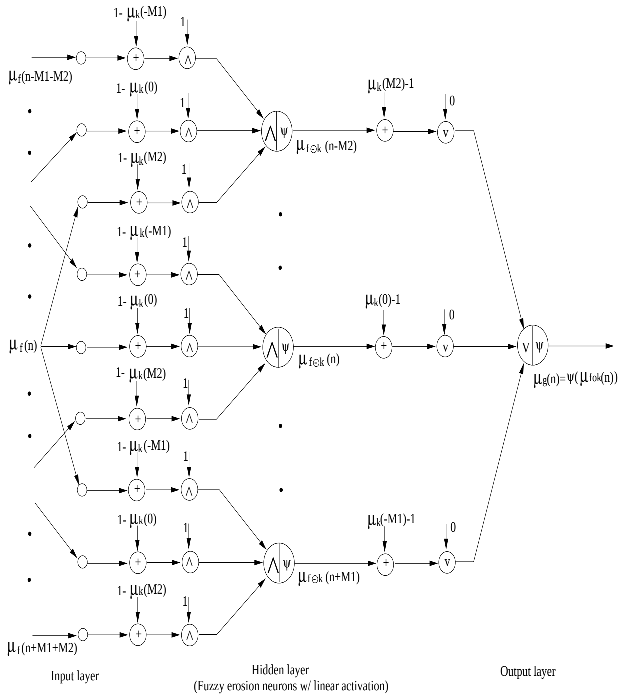

Figure 1: A fuzzy opening neural network

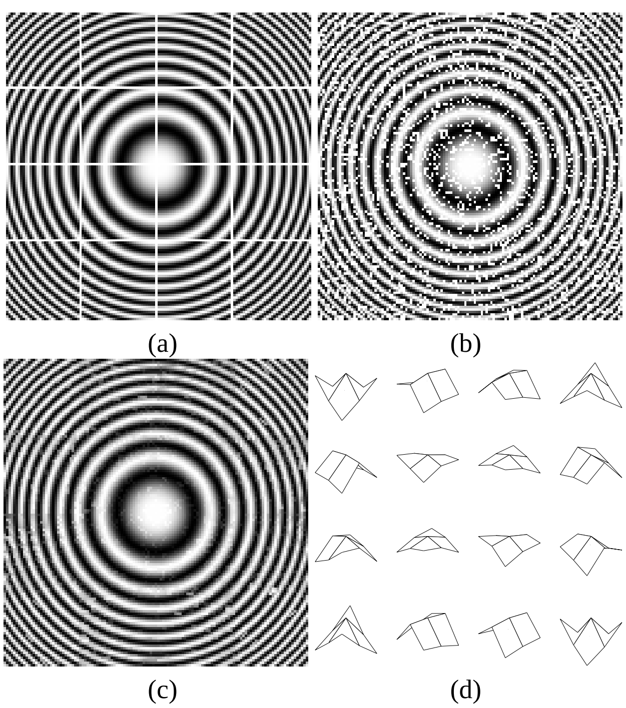

Figure 2: Noise removal of corrupted CZP by supervised AFO filter (4×4 regions): (a) Desired CZP, (b) Noisy CZP, (c) Restored CZP, (d) Optimal S.E.

Figure 3: Noise removal of corrupted "Lena" image: (a) Desired image, (b) Noisy image, (c) Restored image using method in[10], (d) Restored image by supervised AFO filter (32×32 regions)

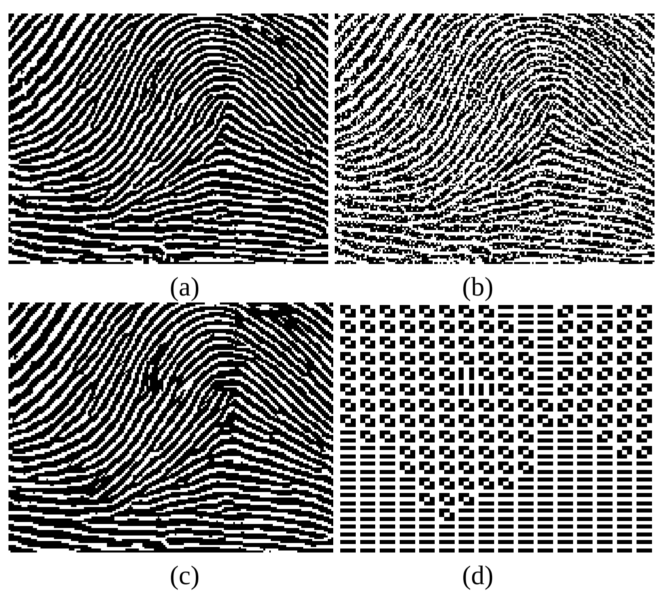

Figure 4: Noise removal of corrupted "FINGERPRINT" by unsupervised AFO filter (16×16 regions): (a) Original image, (b) Noisy image, (c) Restored image, (d) Optimal S.E.