A NEW BLIND ZEROFORCING EQUALIZER FOR MULTICHANNEL SYSTEMS *

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ABSTRACT

Blind channel equalization has recently been a very active research topic due to its potential application in mobile communications and digital TV systems. In this paper, we present a new blind zero-forcing equalizer that utilizes second order statistics from the multi-channel configuration. The algorithm is simple and relies only on nullspace decomposition. It can actively select the desired delay of the equalizer output signal. The performance of this new algorithm is demonstrated through simulation examples.

1. INTRODUCTION

In many data communication systems digital signals are transmitted through unknown channels which introduce severe linear distortion. In order to improve the system performance, receivers must remove channel distortion through equalization. When the available input training signal is either non-existent, or too short for channel identification, blind channel identification and equalization can play a useful role.

Blind channel identification relies solely on the received channel output signal and some *a priori* statistical knowledge of the original input signal. Traditionally, blind channel identification and equalization are based on exploiting higher order statistics of baud-rate sampled channel output signals. The algorithm presented by Tong, Xu, and Kailath [1], is one of the first subspace based methods exploiting only second order statistics for fractionally sampled channel identification. Using the sub-channel representation of the fractionally sampled QAM channels, Xu *et al.* [2] derived a sub-channel matching algorithm that also relies on the subspace separability of signal and noise. Another subspace method for channel estimation similar to the well-known MUSIC algorithm in array application was presented by Moulines *et al* [3]. Since subspace separability requires the knowledge of channel model orders, subspace methods tend to be sensitive to errors in channel order estimates.

Many of the subspace methods for channel estimation tend to be very unreliable when the channel order is over-estimated. To avoid sensitivities to channel order, we adopt a direct equalization approach to find the zero forcing equalizer, without the intermediate step of channel estimation. Our goal is to avoid the sensitivity to unknown channel order suffered by many SOS methods.

This paper is organized as follows. In Section 2, we first describe the statistical model of the blind multi-channel equalization problem. Spectral diversities achieved from over-sampled channel output and multiple sensors (antennas) are considered. In Section 3 we present our new algorithm, with a proof of perfect equalization in the zero noise case. Implementation issues and simulation results are presented in Sections 4 and 5, respectively.

2. PROBLEM FORMULATION

Multi-user quadrature amplitude modulation (QAM) data communication systems can be described using a baseband representation. Assume that there are N user channels, all linear and causal with impulse responses $\{h_u(t), \quad u = 1, 2, \ldots N\ \}$. The received output signal can be written as

$$x(t) = \sum_{u=1}^{N} \sum_{k=-\infty}^{\infty} s_{k,u} h_u(t - kT) + w(t), \quad s_{k,u} \in \mathcal{A}_u, \quad (1)$$

where T is the symbol baud period and $\mathcal{A}_u$ is the input signal set of user u. The channel input sequences $\{s_{k,u}\}$ are typically independent for different users and are also i.i.d. The noise $w(t)$ is stationary, white, and independent of channel input sequences $s_{k,u}$, but not necessarily Gaussian.

Note that $h_u(t)$ is a "composite" channel impulse response that includes transmitter and receiver filters as well as the physical channel response. In a typical multi-user system, multiple channels of observations will be available from multiple sensors. If J sub-channels (sensors or antennas) exist, then $x(t)$, $h_u(t)$, and $w(t)$ are all $J \times 1$ vectors.

In blind *channel identification*, the objective is to identify the unknown channel responses $h_u(t)$ based solely on the channel output $x(t)$. Only the statistical knowledge of the channel input sequences is known, not their actual values. In blind *equalization*, the desired objective is to recover each channel input, without necessarily needing to estimate the channel responses. The problem of single user ($N = 1$) and single channel ($J = 1$) blind identification and equalization requires the application of higher order statistics. It has been studied in works such as [6, 7, 8] and references therein.

It has been shown by Tong, et al.[1] that blind channel identification benefits from oversampling the channel outputs. In fact, single channel identification based on second order statistics is possible only for oversampled chan-

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nel outputs. This essentially arises from the spectral diversity available when the channel has bandwidth higher than $1/2T$.

Let the sampling interval be $\Delta = T/p$ where p is an integer. The oversampled discrete signals and responses are

$$x_i \triangleq x(i\Delta), \quad h_u[i] \triangleq h_u(i\Delta) \quad \text{and} \quad w_i \triangleq w(i\Delta), \qquad (2)$$

each of which is a $J \times 1$ vector for integer i. The channel output samples are hence

$$x_n = \sum_{u=1}^{N} \sum_{k=-\infty}^{\infty} s_{k,u} h_u[n-kp] + w_n.$$

Suppose $\{h_u(t)\}$ has joint finite support which spans m_0+1 integer baud periods. Let Mp be the number of sampled channel outputs to be collected in a block and let the superscript $(.)'$ represent the matrix transpose. By defining the following notation

$$\begin{aligned}
s_k &\triangleq [s_{k,1} \;\; s_{k,2} \;\; \dots \;\; s_{k,N}\,]; \\
\mathbf{s}[k] &\triangleq [s_k \;\; s_{k-1} \;\; \dots \;\; s_{k-m_0-M+1}]' \\
\mathbf{w}[k] &\triangleq [w'_{kp} \;\; w'_{kp+1} \;\; \dots \;\; w'_{kp-Mp+1}]' \\
\mathbf{h}_u[i] &\triangleq \begin{bmatrix} h_u[ip] \\ h_u[ip+1] \\ \vdots \\ h_u[ip+p-1] \end{bmatrix}, \\
\mathbf{H}_i &\triangleq [\mathbf{h}_1[i] \; \mathbf{h}_2[i] \; \dots \; \mathbf{h}_N[i]],
\end{aligned}$$

it is evident that

$$\begin{bmatrix} x_{kp} \\ x_{kp+1} \\ \vdots \\ x_{kp+p-1} \end{bmatrix} = \sum_{i=0}^{m_0} \mathbf{H}_i s'_{k-i} + \begin{bmatrix} w_{kp} \\ w_{kp+1} \\ \vdots \\ w_{kp+p-1} \end{bmatrix}.$$

Now form an $MpJ \times (m_0+M)N$ block Toeplitz matrix with $(M-1)N$ trailing zeros in the first pJ rows

$$H \triangleq \begin{bmatrix} \mathbf{H}_0 & \mathbf{H}_1 & \dots & \mathbf{H}_{m_0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \dots & \mathbf{H}_{m_0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \dots & \mathbf{H}_{m_0} \end{bmatrix}. \qquad (3)$$

Clearly, m_0 is the order of the N dynamic FIR channels. Were we to need them, there would be a total of $(m_0+1)NpJ$ unknown parameters to identify in the blind identification problem. Instead, our algorithm will bypass channel estimation, and will provide channel equalizers directly.

Let $Q = Jp$ be the total number of stationary outputs (p from oversampling and J from multiple antennas). With this notation, a sampled channel output signal vector of length MQ can be written as

$$\vec{x}[k] \triangleq \begin{bmatrix} x_{kp} \\ x_{kp+1} \\ \vdots \\ x_{kp+p-1} \\ x_{(k-1)p} \\ x_{(k-1)p+1} \\ \vdots \\ x_{kp-Mp+1} \end{bmatrix} = H\mathbf{s}[k] + \mathbf{w}[k]. \qquad (4)$$

3. ZERO-FORCING CHANNEL EQUALIZATION

The additional channel zero condition for $\mathbf{H}$ to be full rank has been characterized in [12] and is not the focus of our work. We shall assume, from here on, that $\mathbf{H}$ has full column-rank and is identifiable.

Assume that both the channel input signal and channel noise are white with zero mean. Let their respective covariance matrices be

$$R_s = E\{\mathbf{s}[k]\mathbf{s}[k]^H\} = \sigma_s^2 I$$

and

$$R_w = E\{\mathbf{w}[k]\mathbf{w}[k]^H\} = \sigma_w^2 I.$$

Using the block Toeplitz channel convolution matrix H, we have

$$R_i \triangleq E\{\vec{x}_k \vec{x}_{k+i}^H\} = \sigma_s^2 H J^{-iN} H^H + \sigma_n^2 J^{-iQ}. \qquad (5)$$

J is the Jordan matrix which has unit entries on its first subdiagonal and zero everywhere else, and J^{-1} is the transpose of J. If the channel is noiseless, then $\sigma_n^2 = 0$.

Denote the parameter vector of the equalizer as $\vec{g}$. Zeroforcing equalization requires that no inter-symbol interference remains after equalization. Define $\vec{e}_i$ as the $i-$th coordinate unit vector of dimension $N(M+m_0) \times 1$. For N users, zero ISI requires that

$$H^H \vec{g} = \sum_{k=1}^{N} \beta_{iN+k} \vec{e}_{iN+k}, \qquad i = 0,\ 1,\ 2,\ \dots\ (m_0+M-1). \qquad (6)$$

where β_k is a constant for each k. Equation (6) effectively states that only N elements (one for each user) from the input vector $\mathbf{s}[k]$ will appear at the equalized output. And since the channel inputs s_k shift through $\mathbf{s}[k]$, each input will be observed (without ISI) at the equalized output with delay iT.

Our algorithm is derived under noiseless conditions.

Theorem 1 *Consider noiseless systems with $\sigma_w^2 = 0$. Let $K_i \triangleq Nullspace(R_i)$, where R_i is defined in (5). If the equalizer parameter vector $\vec{g}_{i+1}$ satisfies*

$$\vec{g}_{i+1} \in Nullspace \begin{bmatrix} R_{i+1} \\ K_i^H R_0 \end{bmatrix}, \quad \textit{subject to } R_0 \vec{g}_{i+1} \neq 0, \qquad (7)$$

then $\quad H^H \vec{g}_{i+1} = \sum_{k=1}^{N} \beta_{iN+k} \vec{e}_{iN+k}$.

Proof: For noiseless output

$$R_i = HJ^{-iN}H^H,$$

in which H has full column rank. As a result,

$$J^{-iN}H^H K_i = 0.$$

Hence,

$$K_i \subset K_{i+1}$$

and

$$dim(K_{i+1}) = dim(K_i) + N.$$

Also, from the up-shifting property of J^{-iN}, it can be shown

$$H^H K_i = [\vec{e}_1 \quad \vec{e}_2 \quad \cdots \quad \vec{e}_{iN}]A$$

where A is an $iN \times (i+d_0)N$ matrix with column rank iN and full-row rank of iN.

If we find a $\vec{g}_{i+1} \in K_{i+1}$, then

$$J^{-(i+1)N}H^H\vec{g}_{i+1} = 0,$$

which implies that

$$H^H\vec{g}_{i+1} = \sum_{k=1}^{(i+1)N} \beta_k \vec{e}_k = [\vec{e}_1 \quad \vec{e}_2 \quad \cdots \quad \vec{e}_{(i+1)N}]\vec{\beta}. \tag{8}$$

for suitable β_k. Because of the additional requirement that $\vec{g}_{i+1}$ is orthogonal to R_0K_i, then for noiseless channels

$$K_i^H HH^H \vec{g}_{i+1} = 0. \tag{9}$$

This implies that

$$A^H \begin{bmatrix} \vec{e}_1 \\ \vec{e}_2 \\ \vdots \\ \vec{e}_{iN} \end{bmatrix} [\vec{e}_1 \quad \vec{e}_2 \quad \cdots \quad \vec{e}_{(i+1)N}]\vec{\beta} = 0. \tag{10}$$

$$[A^H \quad \vec{0}]\vec{\beta} = 0. \tag{11}$$

When A^H has full column rank, it is simple that

$$\beta_k = 0, \quad k = 1, 2, \ldots, iN.$$

Hence

$$H^H\vec{g}_{i+1} = \sum_{k=iN+1}^{(i+1)N} \beta_k\vec{e}_k = \sum_{k=1}^{N} \beta_{iN+k}\vec{e}_{iN+k}$$

which holds for each $i = 0,\ 1,\ 2,\ \ldots\ (m_0 + M - 1)$.

Q.E.D.

Theorem 1 therefore describes the steps needed to find a zeroforcing equalizer with delay i for noiseless channels. It is clear that ideally

$$dim\left(Nullspace \begin{bmatrix} R_{i+1} \\ K_i^H R_0 \end{bmatrix}\right) = d_0 + N. \tag{12}$$

However,
only N vectors satisfy the constraint $\vec{g}_{i+1}^H R_0 \vec{g}_{i+1} \neq 0$. For noisy channels with higher SNR, this approach is expected to remain effective in canceling ISI. Define

$$d_i \triangleq dim(K_i) = d_0 + iN.$$

The major obstacle in noisy systems is to accurately estimate d_0. In addition, one must find N vectors among d_0+N nullspace vectors that result in larger output signal power $\vec{g}_{i+1}^H R_0 \vec{g}_{i+1} \neq 0$.

4. IMPLEMENTATION ISSUES

There are two difficulties in implementing our new algorithm when there is noise in the measurements. First it is necessary to estimate d_0, and second it is necessary to pick the N best equalizers from the set of $\vec{g}$'s obtained (there are $d_0 + N$ of them for each delay i).

The first problem arises because the nullspace of the R_i's cannot be accurately obtained in noise. An algorithm to overcome this is to estimate K_0 (for example, using the Akaike information criterion (AIC) or Schwartz and Rissanen's MDL criterion) and set d_0 accordingly. Then K_i can be estimated by setting $d_i = d_0 + iN$. The estimation of d_0 requires an eigen decomposition of R_0.

The second problem is critical. The difference between achieving an outstanding BER and a terrible BER is in the choice of which $\vec{g}$ to use. To choose N equalizer vectors among $d_0 + N$ potential solutions, we propose two methods for selecting the desired equalizer vector(s). The first method is to select the $\vec{g}$ that maximizes the equalizer output power $\vec{g}^H R_0 \vec{g}$. The second method involves maximizing the ratio between the output power and a measure of the ISI power $(\vec{g}_{i+1}^H R_0 \vec{g}_{i+1})/(||R_{i+1}\vec{g}_{i+1}||^2)$.

It turns out there are two additional implementation modifications for noisy conditions. Simulations show that setting rows of the estimates of R_i's to zero for large i is greatly beneficial. This effectively ignores those equations in the nullspace evaluation of (7) that involve autocorrelation functions of large delays (which are typically small and sensitive to noise). Also, when the channel have small pre-cursor and post-cursor ISI, the $\vec{g}_1$s generated from the R_1 tend to be unreliable. It can be advantageous to not use $\vec{g}_1$.

5. SIMULATION RESULTS

We now present simulation results to illustrate the performance of the proposed algorithm. We use a multi-path channel model with a single sensor, i.e., $J = 1$. We consider a raised-cosine pulse $P(t)$ limited in $6T$ with roll-off factor 0.1 and a two ray multi-path channel. The overall channel impulse response is

$$h(t) = c(t) * P(t) = P(t) - 0.7P(t - T/3)$$

A single user is assumed. The data input signal is i.i.d. BPSK and the oversampling factor is $p = 2$. We also choose M to be as long as $P(t)$.

Figure 1 shows the benefit of setting rows of the estimates of R_i's to zero for large i. For this plot, d_0 is assumed known, and in this case equals 1. This implies that there are 12 elements in the measurement vector $\vec{x}[k]$. Also, the equalizer selection from the $\vec{g}$'s is done on the basis of maximum output power. The figure shows that excellent equalization is achieved for SNRs bigger than about 15dB. This figure compares favourably with other linear equalization algorithms.

Figure 2 shows the small sensitivity to incorrect d_0 estimates. For this simulation, there are 14 elements in the measurement vector $\vec{x}[k]$ (2 more than the minimum required for the channel we are using), resulting a $d_0 = 2$. But because of the small samples of $h(t)$ at both ends, leading and trailing columns of H matrix is small, making $d_0 \geq 2$ in estimation. We also show how the output power criterion can be used to select the equalizer vector without much loss in performance. The output power (o/p power curve) differs from the best $\vec{g}$ (i.e. if you were able to check BER for each one) only when SNR is very large. For this simulation, we have neglected the $\vec{g}_1$ generated from R_1 since it was observed that R_1 produces unreliable equalizers for $d_0 > 1$.

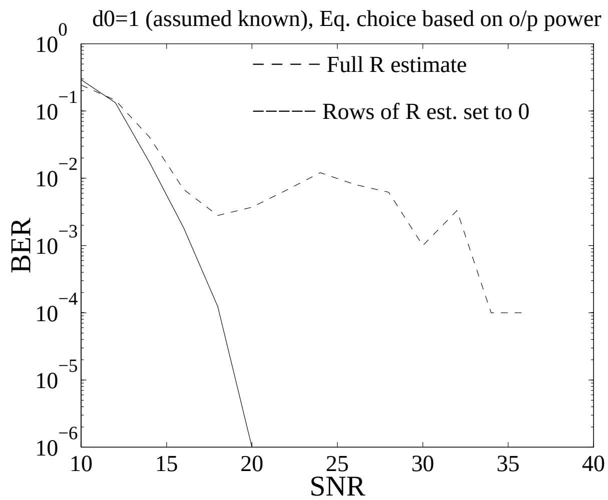

Figure 1. Advantage of setting rows of R to zero

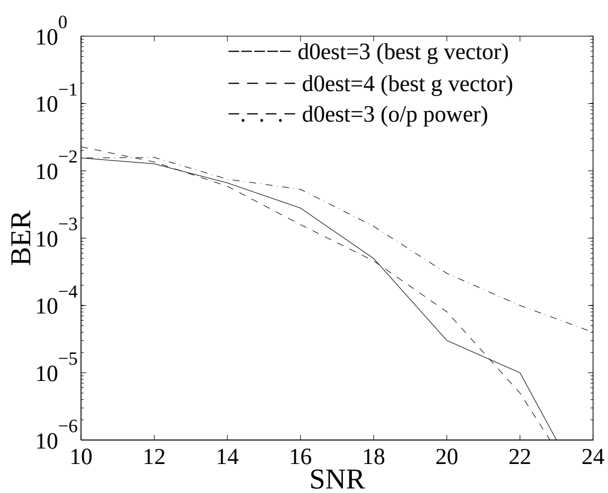

Figure 2. Wrong d_0 and selecting $\vec{g}$

6. CONCLUSIONS

We present a new blind equalizer that is derived from the zeroforcing principle. The algorithm relies only on second order statistics of the unknown channel input and output. The algorithm is simple and relies only on nullspace decomposition. It allows the receiver to actively select desired delay of the equalizer output signal. This method is general for multiple user systems.

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