# THEORY AND DESIGN OF SPECTRUM-EFFICIENT BANDWIDTH-ON-DEMAND MULTIPLEXER-DEMULTIPLEXER PAIRS BASED ON WAVELET PACKET TREE AND POLYPHASE FILTER BANKS

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### ABSTRACT

A set of desirable characteristics of a multicarrier spectrumefficient bandwidth-on-demand multiplexer-demultiplexer pair for use in mobile satellite and personal communication systems is identified and described. New characteristics are the use of single VSB channels, design of multiplexer channels based on wavelet packet trees which have specified stopband attenuation, overlap of the multiplexer channel magnitude frequency responses at the 3-dB points for spectral efficiency, bandwidth on demand, reasonable lengths for the overall equivalent filters for each multiplexer-demultiplexer channel from input to receiver output, and low-complexity receivers. A multirate digital transmultiplexer is proposed consisting of a wavelet packet-based synthesis filter bank tree followed by a DFT polyphase synthesis filter bank at the transmitter, and a matching demultiplexer at the receiver. A simplified receiver for reception of one channel at a time is described. BER performances when there are phase and timing errors are given.

### **1. INTRODUCTION**

This paper is concerned with transmission in the downstream direction of a mobile satellite channel. High priority is also being given to the design of spectrum-efficient schemes for frequency division multiplexing of time-division multiplexed signals for other personal communication systems. Multicarrier communication systems are of great interest for wideband communications above about 150 Mb/s, as required for multimedia wireless communications. Besides spectral efficiency there are a number of other desirable performance and implementation requirements which these systems must meet. These requirements include:

1. The orthogonality of the signals which are received at the output of the demultiplexer from each of the separate streams of signal inputs into the multiplexer. In communication terms the orthogonality means that the receiver is matched to the transmitter so there is no intersymbol interference and no interference from other channels from the inputs to the multiplexer to the outputs of the receiver. In this paper the signal inputs into the multiplexer must be real. It should be noted that the the digital Vestigial Sideband (VSB) multiplexer channels

designed in this paper are equivalent to Offset Quadrature Phase Shift Keying (OQPSK) [7].

- 2. It is clearly desirable to keep the information for each multiplexer channel in a single narrowband radio frequency channel. If two or more radio frequency bands are used the different bands exhibit different transmissions channel degradations which are hard to compensate. As well, the receiver is probably more complex.
- 3. The stopband attenuation for each multiplexer channel must be specified to keep crosstalk from other channels, and outof-band interferences from and into other communication systems below specified values. The paths taken by the input signals through the multiplexer, to the multiplexer output, which is also the input to the transmission channel, in this paper are designed such that the magnitude frequency responses along these paths have stopband attenuations which are greater or equal to a specified minimum value. These multiplexer paths are referred to herein as multiplexer channels. This is a recent result derived and explained in [8], improving greatly on OFDM.
- 4. The whole of the frequency axis around the unit circle must be utilized and the packing of the magnitude frequency responses of the multiplexer channels for both positive and negative frequencies must be as efficient as possible to conserve the spectrum. This is done with the system described here by use of magnitude frequency responses overlapping at the 3-dB points.
- 5. Because of the growth of multimedia services of widely differing bandwidths, as much bandwidth-on-demand capability as possible must be provided. For example, a multiplexer with 8 inputs would allow, for example, 2 of the 4 users at the leaves to each transmit at the lowest rate of r kb/s, one user, feeding a signal into a node at the next level down towards the root of the tree to transmit at the rate of 2r kb/s, a user at the next level towards the root of the tree to transmit at the rate of transmit at the rate of 4r kb/s, or one user only could transmit at the highest of 8r kb/s directly into the input of the wideband VSB filter following the root of the tree [7]. This cannot be obtained with M-band filter banks only.
- 6. The delay of the signal from input to output should be as small as possible consistent with obtaining sufficient levels of bandwidth on demand. It is shown here how to use a DFT polyphase synthesis and analysis filter banks with

signals from the roots of identical trees as inputs to very substantially decrease the number of multipliers, with some loss in the number of levels of bandwidth on demand. This is a new result due to the authors.

7. The receiver must be as simple as possible. A structure is described here.

This paper is organized as follows. In Section 2, an illustrative example is provided of eight identical wavelet packet synthesis filter bank trees, four of whose signal outputs, from each of their roots, feed the four inputs of one of two parallel related DFT polyphase synthesis filter banks, and four whose signal outputs feed the inputs of the other one. The outputs of the two DFT polyphase synthesis filter banks are added to produce the output sequence of the transmitter. A receiver for receiving one selected channel at a time is described. It is shown how, and to what degree, the seven requirements in the Introduction are satisfied by this transmitter-receiver pair. In Section 3 the simulations are succinctly described which give BER versus SNR performance for receiver phase and timing errors separately and together, and for rolloffs of the channel magnitude frequency responses of 25%, 50% and 100%. The results of the simulations are briefly summarized and discussed. Section 4 provides a concluding discussion and identifies some future work.

# 2. DESCRIPTION OF MULTIPLEXER-DEMULTIPLEXER PAIR

Fig. 1 shows the basic building block for the wavelet packetbased synthesis full tree filter bank into which the real data sequences are inputs into the leaves at the left. The magnitude frequency responses of each of the multiplexer channels from the leaves to the root of the wavelet packet-based filter bank tree [9] used here as the first stage of the multiplexer, consist of two channels, symmetrically located with respect to the origin of frequency, one at positive frequencies and one at negative frequencies. When these channels are modulated to radio frequencies the positive and negative frequencies around the unit circle are modulated to opposite sides of the carrier frequency, so the signals in the two parts of each multiplexer channel will generally experience different channel degradations. If  $G_{10}(-jz)$  or  $G_{10}(jz)$  is used as the transfer function of a wideband VSB filter after the root of the wavelet packet-based multiplexer tree in Fig. 1, the channel magnitude frequency responses will be, respectively, at positive or negative frequencies only. The networks outputs with only positive frequency channels and only negative frequency channels can be multiplexed to effectively double the number of channels. Here, DFT polyphase synthesis filter banks will be used to replace  $G_{10}(-jz)$  and  $G_{10}(jz)$  and frequency compress the multiplexer channels by four. The output signal from the root of the tree,  $X_0(z)$ , is used as the top one of the four input signals into a DFT polyphase synthesis filter bank shown in Fig. 2 (Fig. 8.21 of [1]). The multiplexer channel numbers correspond to monotonically increasing centre frequencies of the magnitude frequency responses along the paths from the leaves



Fig. 1. The first basic 4-input wavelet packet-based synthesis filter bank repeated 3 times, with different channel numbering in each time with successive output signals at the roots as the input signals  $X_1(z)$ ,  $X_2(z)$  and  $X_3(z)$  to a DFT polyphase synthesis filter bank shown in Fig. 2.



Fig. 2: Arrangement of DFT polyphase synthesis filter banks.

to the root of the tree, and through the DFT polyphase synthesis filter bank to its output. The QMF filter bank pairs of which each tree is composed are exactly the same for all eight trees which are used to provide a total of up to 32 multiplexer channels. For the design example the equations derived in [8], for the case in which the QMF pairs are different at the two levels of the filter bank, were used to guarantee 40 dB attenuation in the stopband. The lowpass (with second subscript labeled 0) and highpass (with second subscript labeled 1) filters in the QMF pair at level 1, next to the root of the tree, have length 22. Those at level 2, at the leaves of the tree, have length 10. This resulted in channel magnitude frequency responses with 100% rolloff. Filters resulting in 50% and 25% rolloff were also designed to study the effects of multiplexer transition bandwidths. The design resulted in the lowpass filters being minimum phase and the highpass filters being maximum phase. Thus, channel 1 is minimum phase, and has the lowest average group delay of the 4 shown in Fig.1. Channel 3 has a path through the tree which traverses two maximum phase filters, and so has the highest average group delay of the four channels in Fig. 2. For the filter bank tree with output at the root of the tree which is the input  $X_1(z)$  to the DFT

polyphase synthesis filter bank the numbering of the leaves from top to bottom corresponding to the multiplexer channels which begin there is 9, 10, 12 and 11. Corresponding to  $X_2(z)$  the numbering of the leaves is 17, 18, 20 and 19. Finally, corresponding to  $X_3(z)$  the numbering is 25, 26, 28 and 27. The remaining 16 channels will be accounted for shortly. The prototype filter from which the four polyphase subfilters are computed, with transfer function denoted by A(z), is an 8<sup>th</sup> band filter of length 88 with equal ripple in the stopband and minimum stopband loss of about 43 dB with magnitude frequency response shown in Fig. 3 [5]. The composite output magnitude frequency responses of the 16 multiplexer channels, at the output,  $\hat{X}_1(z)$ , of the polyphase filter bank with numbers given above are shown

polyphase filter bank, with numbers given above are shown in Fig. 4 for the case of 100% rolloff.



Fig. 3: The magnitude frequency response of the 8<sup>th</sup>-band equiripple stopband VSB filter used for calculating the polyphase components for the DFT polyphase filter banks in the example.

The remaining 16 channels to densely pack the remaining frequency spectrum in Fig. 4 are obtained from a set of four filter bank trees with four input leaves each identical to the ones used

for the first four filter bank trees. However, the prototype filter from which the four polyphase subfilters are computed is given

by  $jz^{-D}A(1/z)$  [4]. Ordered from the top input to the second DFT polyphase synthesis filter bank, and from the top leaf in the trees as before, the four sets of ordered channel numbers are (32, 31, 29 and 30), (8, 7, 5 and 6), (16, 15, 13 and 14), and (24, 23, 21 and 22). The composite output magnitude frequency responses of these 16 multiplexer channels, at the output,  $\hat{X}_{2}(z)$ , of the second polyphase filter bank, numbered as above,



Fig. 4: The composite magnitude frequency response  $\hat{X}_{1}(z)$ .

fill the gaps in Fig. 4 completely. Multiplexing  $\hat{X}_{1}(z)$  and  $\hat{X}_{2}(z)$  thus yields a composite response which densely fills the complete frequency axis with VSB channels overlapping at the 3-dB points. The lengths of the pulse responses from the inputs at the leaves to the output of one or the other of the polyphase filter banks is 116 compared to 476 for two 16-input synthesis filter bank trees, each followed by a wideband VSB filter [1]. Note that the requirements 1 through 6 in the Introduction are satisfied.

The reasons for using two different, DFT polyphase synthesis filter banks at the transmitter, and two matching DFT polyphase analysis filter banks at the receiver, are discussed next.

A prototype filter with an equiripple stopband meeting a specified minimum attenuation is often considered to be a desirable choice because this approximation of the stopband attenuation results in the shortest filter lengths. We design a linear phase, real coefficient, 8-band Nyquist filter with double zeros on the unit circle, and factor it into minimum and maximum phase factors. As noted, the factorization is into a minimum phase polynomial, A(z), at the transmitter and into a

maximum phase polynomial,  $z^{-D}A^*(1/z)$ , at the receiver, as shown in Fig. 5, for the first polyphase network, and into a maximum phase polynomial,  $jz^{-D}A(1/z)$ , at the transmitter,

and into a minimum phase polynomial,  $jA^*(z)$ , for the

second polyphase network. As explained in [8] this choice of prototype polynomials and phasing results in imaginary values of crosstalk between adjacent channels, so the real part of the crosstalk vanishes at all times. If the factorization were into linear phase polynomials then the prototype polynomials at the transmitter and receiver would be the same, but would require more coefficients, and two polyphase networks would not be necessary [6]. A method involving factorization into conjugate symmetric polynomials with complex coefficients due to Lawton [2] also yields one network and nearly linear phase.



Note 1: The  $G_{10}(z)$  and  $G_{11}(z)$  are the lowpass and highpass transfer functions of the QMF pair next to the root of the synthesis filter bank trees, and the  $G_{20}(z)$ and  $G_{21}(z)$  are the transfer functions of the QMF pairs at the leaves of the trees.

Fig. 5: Block diagram of the receiver architecture, and relative symbol rates at various points.

# 3. SIMULATION OF PERFORMANCE

Using the theory derived in [3] extensive simulations have been run to evaluate the sensitivities of the BER vs. SNR to errors in phase alone, timing error alone, and both phase and timing, for multiplexer channel rolloffs of 100%, 50% and 25%, thus obtaining the sensitivities to the rolloff as well. The system is relatively insensitive to phase errors at the receiver. For 100% rolloff, or excess bandwidth, 15 degrees phase shift, a SNR,  $E_h/N_0$ , of 5 dB (for which the ideal BER is .0055) and all channels fully loaded, the error rate is .021, while for the 50% and 25% cases it is .020 and .022, whereas the respective numbers for a single channel are .015, .018 and .020. For a timing error of 4 out of the 16 samples per symbol, the corresponding error rates are .021, .017 and .016 for the fully loaded case and .0098, .013 and .015 for the single channel cases. Indeed, it appears that a choice of 100% rolloff, considering all the simulation results, is a good compromise.

# 4. CONCLUDING REMARKS AND FUTURE RESEARCH

A new multiplexer-demultiplexer pair for multicarrier downstream transmission which appears to be promising for personal and mobile communications, and especially for wideband communications, composed of wavelet packet-based filter bank trees concatenated with DFT polyphase filter banks, with a simplified receiver for receiving one channel at a time, satisfies the seven requirements identified in the Introduction, and exhibits BER performance which is in the range correctable with error coding and control schemes and equalizers. The next steps are to design an efficient and effective synchronization schemes, add error coding and control and equalization for the downstream direction, and to design a system incorporating multi-user joint detection for upstream transmission.

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