ADAPTIVE SEPARATION OF UNKNOWN NARROWBAND AND BROADBAND TIME SERIES*

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ABSTRACT

Motivated by Thomson's multiple taper spectral estimation technique, we derive a new, robust procedure for automatically separating time series data into its constituent narrowband and broadband time series components. The new procedure avoids the pitfalls of adaptive notch filters, PCI method, or other similar algorithms, of mistaking and filtering local spectral peaks of the broadband component as narrowband components by decomposing the data vector into local subbands by a bank of matrix filters. Then in piecewise fashion, the narrowband components are estimated and then filtered from each subband using the Principal Component Inverse (PCI) method. Finally, the filtered components are coherently recombined to obtain the narrowband and broadband time series estimates. Computer simulation results show that the new procedure works well and can have performance close to the clairvoyant Wiener filter.

1. INTRODUCTION

Underwater acoustic signals are often complicated, consisting of a superposition

$$x(t) = \sum_{k=1}^{P} n_k(t) + b(t) \quad , \; t_0 \leq t \leq t_0 + T \qquad (1)$$

of non-stationary narrowband (NB) $n_k(t)$ and broadband (BB) $b(t)$ components. Such signals arise from a variety of sources, such as ship machinery, marine mammals, drilling platforms, and active sonars. In figure 1 we illustrate what a typical underwater acoustic spectrum might look like, consisting of a superposition of narrowband line-like components plus a broadband component. We point out that the BB component itself can be colored, with many local spectral peaks and valleys.

Our objective is the separation of $x(t)$ into the constituent NB $n(t) = \sum_{k=1}^{P} n_k(t)$ and BB $b(t)$ time series components when little or nothing is known about the NB and BB components and only a short data record is available. This type of problem is of great importance in sonar, especially passive sonar, where the desirable signal (for detection, classification, and localization) is often either the NB or BB component and the other is regarded as interference.

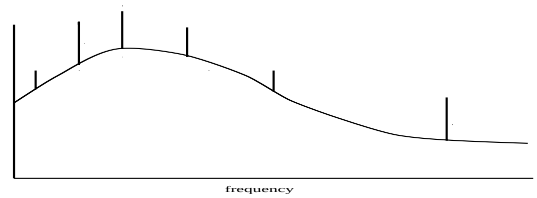

Figure 1. Typical signal spectrum.

Existing approaches primarily deal with power spectrum estimation of the composite data, rather than recovering the constituent narrowband and broadband time series. Having the separated narrowband and broadband time series available is very useful since it allows many additional forms of processing (processing which is impossible or difficult to do using only the power spectrum), such as extraction of time series statistics, improved wavelet and Wigner analysis, pattern recognition, and parametric modeling.

The separation of the time series data into the NB and BB time series components is difficult. Wiener filtering [1] is not practical since we do not know the covariance or spectral densities of the NB and BB components. Parametric methods (e.g. MA, AR, ARMA modeling [4]) require that we choose a model type for the underlying broadband component and for each of the NB components present. This is difficult since we have assumed that we know nothing at all about the NB and BB components. Adaptive methods applied directly to the time series, such adaptive notch filters or line enhancers [1] or Principal Component Inverse (PCI) method [5, 7], tend to perform poorly when the broadband spectrum has a large dynamic range. That is, if a weak NB component is present in a "valley" of the BB spectrum, the notch filter tracker or PCI method might lock onto a nearby peak of the BB spectrum and filter it as the NB component, rather than the true NB component.

We propose a new method, based on Thomson's multiple taper spectral estimation (MTSE) technique [2, 3, 4] and PCI method [5, 7], which alleviates the problems of false tracking inherent in adaptive notch filters and adaptation to the background BB and NB components.

We start with a review of MTSE, followed in section 3 by the derivation of our new procedure. In section 4 we present some experimental results showing the removal of NB components from a colored transient-like signal and then a com-

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parison of our new method against the Wiener filter and PCI method for estimating 3 tones in colored BB noise. Finally in section 5, we present some concluding remarks.

1.1. Multiple Taper Spectral Estimation

Thomson proposed MTSE [2, 3, 4] for the spectral analysis of complicated non-stationary data consisting of lines plus a background component with continuous spectrum in which the physical processes generating the data were poorly understood. A multiple taper spectral estimate [2, 3, 4] (see figure 2) is

$$|\tilde{X}(f_k)|^2 = \frac{1}{K}\,\|U^H D(f_k)\mathbf{x}\|_F^2 \qquad (2)$$

where U is a $N \times K$ matrix whose K columns are the principal *Discrete Prolate Spheroidal Sequence's* (DPSS's) $v_n^k(N, W)$, which are the eigenvectors of the $N \times N$ matrix

$$[R]_{m,n} = \frac{sin 2\pi W(n-m)}{\pi(n-m)} \quad , \qquad (3)$$

arranged to correspond to the eigenvalues in descending order, $D(f) = diag(1, e^{-i2\pi f}, \ldots, e^{-i(N-1)2\pi f})$, W is the analysis bandwidth parameter, and $K \approx 2NW$.

The matrix U in (2) acts as a lowpass filter with bandwidth W [4]. Thus, the projection of the data vector $\mathbf{x}$ onto $V_k = D(f_k)U^H$ becomes a bandpass filtering of the data to $[f_k - W, f_k + W]$. We now see that $\tilde{X}(f)$ is approximately the average energy in the band $[f_k - W, f_k + W]$. In other words, for a given set of frequency points, say $f_1, \ldots, f_M$, the spectrum estimate (2) is analogous to filtering $\mathbf{x}$ into the subbands $\{[f_0 - W, f_0 + W], [f_1 - W, f_1 + W], \cdots, [f_M - W, f_M + W]\}$, as shown in figure 2, and then calculating the average energy in each band.

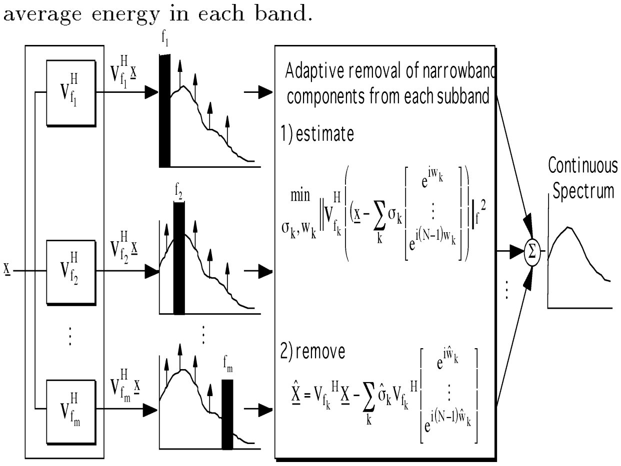

Figure 2. Depiction of multiple taper method and tonal removal.

1.2. Removal of Tonals

The multiple taper method provides a simple and effective way of locating and removing tonals from the underlying continuous BB spectrum component. To avoid mistaking local BB spectral peaks as NB components and for avoiding interference from adjacent tonals, Thomson [2, 3] proposed that the estimation and removal of tonals be done separately within each subband, as depicted in figure 2. The main idea is that the projection of $\mathbf{x}$ onto V_k effectively isolates the frequency band $[f_k - W, f_k + W]$ from out of band tonals, and if W is properly chosen, the background noise spectrum is approximately locally *flat* or *white*. Thus, effects from out of band tonals and problems of locking onto local spectral peaks of the broadband component are minimized.

2. SEPARATION OF NB AND BB TIME SERIES

How can we use the methodology of MTSE to recover the NB $n(t) = \sum_{k=1}^{P} n_k(t)$ and BB $b(t)$ time series components? Observing that if we concatenate the matrix filter banks V_k into matrix $[V_1|V_2|\cdots|V_h]$, the total filter bank output can be written as $\mathbf{z} = V^H\mathbf{x}$. If $rank[V] = N$ (where N is the number of time series samples in $\mathbf{x}$), then we can exactly reconstruct $\mathbf{x}$ from $\mathbf{z}$ using $\mathbf{x} = (V^H)^{\#}\mathbf{z}$, where operator # means psuedo-inverse. This suggests that an estimate of the broadband time series might be $\mathbf{x}_{BB} = (V^H)^{\#}\mathbf{z}_{clean}$ where

$$\mathbf{z}_{clean}^T = [clean(V_1^H\mathbf{x})^T | clean(V_2^H\mathbf{x})^T \cdots | clean(V_m^H\mathbf{x})^T] \qquad (4)$$

and the operator $clean(\cdot)$ depicts the tonal removal procedure depicted in figure 2, applied to the subbands where tonals are present.

Two problems with the above procedure are that (1) the NB components may not necessarily be pure tones (they could be from processes with bandwidth), and (2) leakage of NB components into adjacent subbands is not dealt with, since it is not possible to construct ideal passband filters. To alleviate these two concerns, we estimate and remove the NB components using the PCI method [5, 7] and then develop a simple method to estimate and remove leakage components. We now outline the new procedure for removing the BB time series from the NB components.

2.1. Procedure for Estimating the BB Time Series Component

1) Construct data matrix

In order to exploit the low rank structure of the NB components in the data [5, 7], we arrange the time series samples into the forward-backward linear predictor matrix

$$X = \begin{bmatrix} x_1 & \cdots & x_{N-L+1} & \bar{x}_L & \cdots & \bar{x}_N \\ x_2 & \cdots & x_{N-L+2} & \bar{x}_{L-1} & \cdots & \bar{x}_{N-1} \\ \vdots & & \vdots & \vdots & & \vdots \\ x_L & \cdots & x_N & \bar{x}_1 & \cdots & \bar{x}_{N-L+1} \end{bmatrix} \qquad (5)$$

2) Subband decomposition

Assuming that there are NB components present in the kth subband, decompose the columns of X into an *in-band*

$$Z_k^j = V_k^H X \qquad (6)$$

and *out-of-band* component

$$Z_k^{\dagger j} = V_k^{\dagger H} X = [V_1|V_2|\ldots|V_{k-1}|V_{k+1}|\ldots|V_h]^H X \qquad (7)$$

as illustrated in figure 3. The kth matrix filter V_k is a passband filter designed for the band $[f_k - W, f_k + W]$ as described earlier using the principal DPSS's. If desired, one can use more sophisticated filter design methods. We would

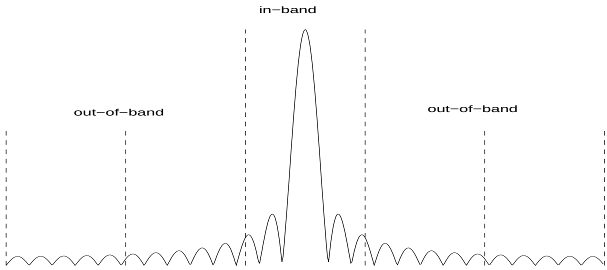

Figure 3. Depiction of division into local subbands and leakage.

also like to point out that the filters V_k do not have to be orthogonal and can be overlapping.

3) Estimate NB components

The PCI method of reduced-rank interference nulling [5, 7] is used to estimate the NB component in the kth subband by solving

$$\min_{S_k subj.\ to\ rank[S_k]=r} \|Z_k - S_k\|_F^2 \tag{8}$$

A solution can be obtained from the SVD [5, 7] of Z_k and is given by

$$\hat{S}_k = \sum_{k=1}^{r} \hat{\sigma}_k \hat{\mathbf{u}}_k \hat{\mathbf{v}}_k^H \tag{9}$$

where the $\hat{\mathbf{u}}_k$ and $\hat{\mathbf{v}}_k$ are the r principal left and right singular vectors respectively of Z_k, and the $\hat{\sigma}_k$ are the r respective principal singular values.

The idea here is that if r tonals are present in the kth subband, then the rank of $V_k^H X$ will also be r, and the matrix corresponding to the tonals can be estimated by finding the best rank r approximation filtered data matrix. Non-tonal NB components can also be effectively modeled as low rank (see Tufts and Scharf [8] for details).

4) Estimation of out-of-band leakage and removal of NB component

Since it is not possible to design perfect passband filters V_k, NB components in the kth subband will *leak* into the adjacent subbands (see figure 3). This is not desirable. However, we now show that one can estimate or extrapolate the NB out-of-band leakage from the in-band NB estimate $\hat{S}_k$ and then remove it.

Denoting the BB component in X as N, and the NB component as S, the in-band and out-of-band components can be written as

$$Z_k^H = X^H V_k = \underbrace{S_k^H V_k}_{NB} + \underbrace{N^H V_k}_{BB} \tag{10}$$

and

$$Z_k^{\dagger H} = X^H V_k^{\dagger} = \underbrace{S_k^H V_k^{\dagger}}_{NB} + \underbrace{N^H V_k^{\dagger}}_{BB} \tag{11}$$

respectively. We start by rewritting (11) as

$$Z_k^{\dagger H} = B^H Q + N^H V_k^{\dagger} \tag{12}$$

where B is a $r \times 2(N-L+1)$ matrix whose rows span the row space of S and Q is some $r \times K$ (K is the number of columns of concatenated matrix filter bank V) matrix such that $S^H V_k^{\dagger} = B^H Q$. We want to estimate the leakage $V_k^{\dagger H} S$. At first glance, this seems impossible since it requires knowledge of S, which we do not know. However, step 3 has provided us an estimate of the row space of S_k from the SVD of Z_k (see (9)), $\hat{B} = [\hat{v}_1|\cdots|\hat{v}_r]$. Therefore to estimate $S^H V_k^{\dagger}$, we simply substitute the estimate of $\hat{B}$ in place of B in (12), and solve for Q by least squares fitting to $Z_k^{\dagger H}$ using (12). The final estimate of the out-of-band leakage is

$$\hat{Z}_k^{\dagger} = Z_k^{\dagger j} \hat{B}^H (\hat{B}\hat{B}^H)^{-1} \hat{B} \tag{13}$$

where j is the iteration number.

We now remove the in-band NB component

$$Z_k^{j+1} = Z_k^j - \hat{S}_k \tag{14}$$

and out-of-band NB component

$$Z_k^{\dagger(j+1)} = Z_k^{\dagger j} - \hat{Z}_k^{\dagger} \ , \tag{15}$$

and increment the counter $j = j+1$.

5) Repeat steps 2,3,4 until NB components from all subbands are removed.

6) Reconstruct BB time series

The BB time series is obtained from the cleaned Z, denoted as Z_{clean}, by first reconstructing the BB time series

$$X_{BB} = (V^H)^{\#} Z_{clean} \tag{16}$$

present in the columns of X using the psuedo-inverse of V^H and then arithmetically averaging all elements in X_{BB} which correspond to the same time series sample using the method developed in [5] for reduced-rank signal enhancement, which is denoted by the operation

$$\mathbf{x}_{BB} = \text{Average}[X_{BB}] \tag{17}$$

The NB time series estimate is then

$$\mathbf{x}_{NB} = \mathbf{x} - \mathbf{x}_{BB} \tag{18}$$

Weighted averaging proposed in [6] could be used to obtain even better performance.

2.2. Discussion

An important advantage of the new BB and NB time series separation procedure is that it nearly eliminates the problem of mistaking local spectral peaks of the broadband component as narrowband components and application is greatly simplified.

Finally, the use of matrix passband filters rather than FIR filters allows the processing of very short record lengths since filter transient effects are avoided.

3. EXPERIMENTAL RESULTS

We start by computer generating a BB transient-like signal 130 samples long that has a triangular shaped spectrum and then adding 3 interfering complex tones. The signal plus interference spectrum is plotted in figure 4. We now apply our new procedure to remove the 3 interfering tones from the BB transient. Briefly, the Nyquist band is divided up into 10 non-overlapping subbands using matrix filters based on DPSS's sequences as described earlier. The spectrum of the estimated BB component for our new method and the PCI method is plotted in figure 4. Note that our new procedure has removed the tonals with little distortion to the BB transient, while for comparsison, the PCI method applied directly to the time series has caused considerable distortion to the transient by locking onto the transient spectral peak and missed one of the tonals.

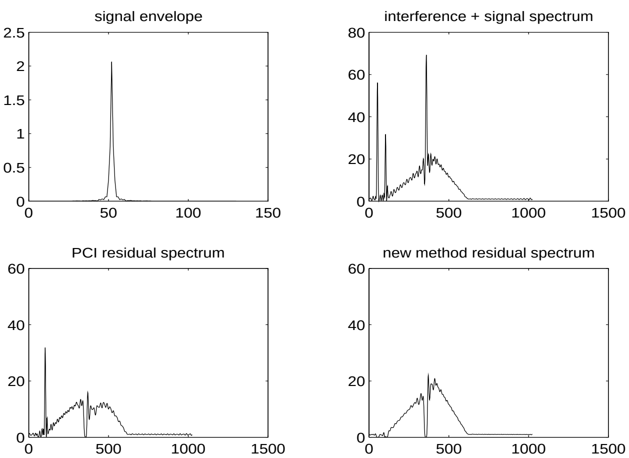

Figure 4. Synthetic transient example with 3 interfering tones injected. Original spectrum is triangular plus constant component.

Next, we compare our new method against the Wiener filter (assuming full knowledge of the NB and BB covariance) and the PCI method for estimating 3 complex tones scaled by complex Gaussian random variables embedded in colored Gaussian BB noise. The power spectrum of the 3 tonal signals and broadband noise for $SNR = 0db$ (SNR is measured relative to the power of the 3 tones and BB component) and division into subbands are plotted in figure 5. 100 independent simulation trials were performed using computer generated 130 sample segments of signal plus noise to measure the mean-square estimation error (MSE) of the signal estimate for the new method and PCI method assuming rank 3. The Wiener MSE was calculated theoretically. The MSE normalized by the total signal power is plotted for all 3 methods in figure 6 as a function of SNR (note that the BB noise component power is varied while the signal power was kept fixed).

At high SNR, the new method and the PCI method both perform well, being close to the Wiener MSE (see figure 6). However, at moderate to low SNR's, starting around $15db$ and lower, the new method significantly outperforms the PCI method, while the MSE is still close to the Wiener filter. This is because the PCI method begins to mistake the local spectral peaks of the BB component for the NB components, while the new method using local subband processing avoids this problem.

4. CONCLUSION

We have derived a new, robust procedure for adaptively separating time series into their constituent NB and BB time series components. The procedure is robust in that it can have performance near the Wiener filter and avoids mistaking local spectral peaks of the BB component as NB components.

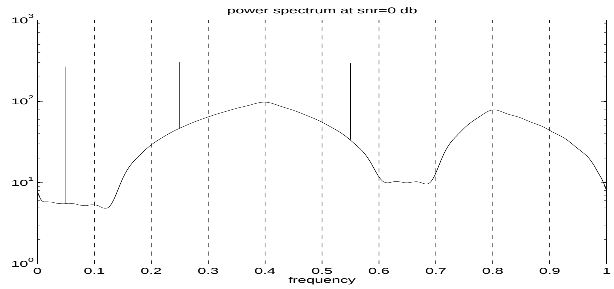

Figure 5. Power spectrum of simulated interference and signal at $SNR = 0db$ and division into subbands.

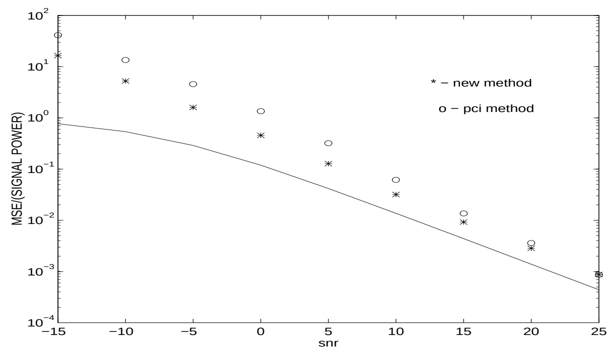

Figure 6. Normalized mean-square estimation error vs. SNR. Solid line is Wiener filter.

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