USING PHASE INFORMATION TO DECORRELATE THE FILTERED-X ALGORITHM

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ABSTRACT

A well known algorithm in the field of active noise control is the filtered-x algorithm. As known in literature, the convergence properties of an adaptive algorithm can be improved by decorrelating its input signal. In this paper, the decorrelation needed for the filtered-x algorithm is discussed with the help of block frequency domain adaptive filters. It is shown that decorrelation of not only the input signal but also the amplitude response of the secondary acoustic path is necessary. While the former can be done by dividing the input signal in frequency domain by an estimate of the input signal power, the latter leads to a new method for improving convergence properties without any extra computation; by using only the phase information of the secondary path to calculate the filtered-x signal.

1. INTRODUCTION

Almost all systems based on the active noise cancellation concept utilize the filtered-x algorithm. Besides its direct use in active sound control systems, it is also applied in sound reproduction systems such as 'personal sound' and 'phantom sound sources'. These latter applications require large adaptive filters since they need to cover the whole audio frequency range and therefore the Block Frequency Domain Adaptive Filter (BFDAF) [1] is preffered. In this paper, the BFDAF will be used to explain the process of decorrelating that is needed in the filtered-x algorithm.

The concept of active noise cancellation is readily explained by the single point noise canceller shown in Fig. 1. The sound emitted from the primary loudspeaker is cancelled by another sound from the secondary loudspeaker. The secondary loudspeaker is driven by an adaptive filter $\underline{\mathbf{w}}$ which coefficients are updated to minimize the sound pressure at the microphone. The difference between this system and a normal adaptive filter is that the output of the adaptive filter $\hat{e}$ is first filtered by the secondary acoustic path $\underline{\mathbf{h}}_s$ before it is added in the microphone; therefore the so-called filtered-x algorithm [2] is needed. The update part of this algorithm uses a filtered version x_f instead of the input signal x itself. Usually this filtered-x signal x_f is obtained by filtering the input signal x by an estimate $\hat{\underline{\mathbf{h}}}_s$ of the secondary path.

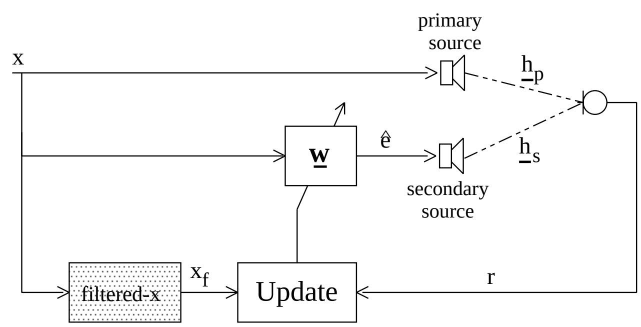

Figure 1: Single point noise canceller

As shown in [2] the convergence properties of an adaptive filter can be improved by decorrelating its input signal. This decorrelation removes the potential colouring in the input signal and results in convergence properties that is independent of the input signal statistics. However using the same decorrelation concept for the filtered-x algorithm will, in general, not result in a great convergence improvement since the colouring in the 'filtered-x' part is still present. Previous work in this area (see e.g. [3]) only deal with the normalization in time domain, which is a kind of scaling rather than decorrelation. To the best knowledge of the authors, no work has been done in the field of decorrelation of the filtered-x algorithm which is the subject of this paper.

The paper is further organized as follows: Section 2 describes the BFDAF implementation of the filtered-x algorithm. Decorrelation can be performed readily in frequency domain by normalizing each separate frequency bin of the update vector by its own input signal power. This concept is explained in section 3 for regular BFDAF and filtered-x BFDAF algorithms. In this section it is shown that the convergence properties of filtered-x algorithm can be significantly improved by using only the phase information in the secondary acoustic path to calculate the filtered-x signal. This work has been verified by simulations of a single point noise canceller using measured acoustic transfer functions. The results of these simulations are shown in section 4. Finally, conclusions and future research are to be found in section 5 .

In this paper, upper-case letters will be used for frequency domain and lower-case letters for time domain signals. Boldface letters will be used for matrices while underlined boldface letters for vectors. Furthermore we will assume, for simplicity, that all signals and systems are stationary.

2. BFDAF IMPLEMENTATION OF FILTERED-X

The BFDAF implementation of the single point noise canceller is shown in Fig. 2. This is almost equivalent to the

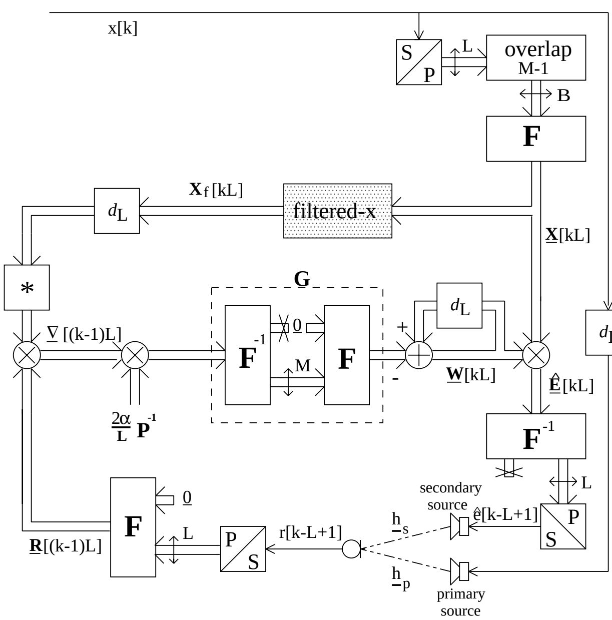

Figure 2: BFDAF implementation of filtered-x

original BFDAF except that the input signal is filtered, in the ‘filtered-x’ box before it enters the update part of the algorithm. A direct consequence of the block processing approach is a processing delay of L samples (with $L \geq 1$), which has to be compensated, for synchronization reasons, in the primary acoustic path ‘d_L’. Performing the convolution in frequency domain produces a cyclic convolution result. By using M (time domain) adaptive weights this results in an overlap of $M-1$ samples and thus the block length equals $B = M + L - 1$ samples. As depicted in Fig. 2, the blocks with B input signal samples are transformed to the frequency domain vector $\underline{\mathbf{X}}[kL]$. The transformation is done with a Fast Fourier Transform (FFT) of length B. The $(k,l)^{th}$ element of this $B \times B$ Fourier matix $\mathbf{F}$ is defined as $(\mathbf{F})_{k,l} = \mathrm{e}^{-\mathrm{j}kl\frac{2\pi}{B}}$. The convolution of the input signal and the adaptive coefficients is performed in the figure by element-wise multiplication of the vectors $\underline{\mathbf{X}}[kL]$ and $\underline{\mathbf{W}}[kL]$. By using $\underline{\mathbf{X}}[kL]$ as the diagonal elements of the diagonal matrix $\mathbf{X}[kL] = \mathrm{diag}\{\underline{\mathbf{X}}[kL]\}$ we can describe this convolution mathematically as follows:

$$\hat{\underline{\mathbf{E}}}[kL] = \mathbf{X}[kL] \cdot \underline{\mathbf{W}}[kL] \quad (1)$$

By defining the diagonal matrix $\mathbf{H}_s = \mathrm{diag}\{\underline{\mathbf{H}}_s\}$, in which $\underline{\mathbf{H}}_s$ is the transformed secondary impulse response, the filtered-x operation is mathematicaly given by:

$$\mathbf{X}_f[kL] = \mathbf{X}[kL] \cdot \mathbf{H}_s \quad (2)$$

in which all matrices are diagonal.

As shown in Fig. 2 the BFDAF update algorithm can now be described as follows:

$$\underline{\mathbf{W}}[kL] = \underline{\mathbf{W}}[(k-1)L] - \frac{2\alpha}{L}\mathbf{G}\mathbf{P}^{-1}\underline{\nabla}[(k-1)L] \quad (3)$$

in which the decorrelation, described in more detail in the following section, takes place by the inverse of the diagonal matrix $\mathbf{P}$ while the adaptation constant $\frac{2\alpha}{L}$ is used to control the convergence of the adaptive filter. The $B \times B$ constrained window matrix $\mathbf{G}$ is needed to take care of the cyclic operations to become linear ones [1]. Furthermore the gradient is estimated as:

$$\underline{\nabla}[kL] = \mathbf{X}_f^*[kL] \cdot \underline{\mathbf{R}}[kL] \quad (4)$$

By defining $\underline{\mathbf{H}}_p$ as the Fourier transform of the primary acoustic path and $\mathbf{V}$ as an appropriate window matrix we will use later on the following alternative description of the residual signal vector:

$$\underline{\mathbf{R}}[kL] = \mathbf{V} \cdot \mathbf{X}[kL] \cdot \underline{\mathbf{D}}[kL] \quad (5)$$

$$\underline{\mathbf{D}}[kL] = \mathbf{H}_s \cdot \underline{\mathbf{W}}[kL] + \underline{\mathbf{H}}_p \quad (6)$$

Finally note that with identity matrix $\mathbf{I}_\bullet$ (dimension $\bullet \times \bullet$) and the zero matrix $\mathbf{0}$, of appropriate dimension, the $B \times B$ window matrices $\mathbf{G}$ and $\mathbf{V}$ are defined as:

$$\mathbf{G} = \mathbf{F}\begin{pmatrix} \mathbf{I}_M & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\mathbf{F}^{-1} \qquad \mathbf{V} = \mathbf{F}\begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_L \end{pmatrix}\mathbf{F}^{-1}$$

3. DECORRELATION

Decorrelation of the input signal can be performed very efficiently by power normalization of each separate frequency bin in the frequency domain [1]. In subsection 3.1 we will shortly review this decorrelation concept for the BFDAF and in subsection 3.2 this concept is used for the filtered-x algorithm, which results in a new simplified algorithm.

3.1. Decorrelation of BFDAF

In order to describe the normalization process that is needed for the BFDAF algorithm we will use Fig. 2. For this case the box 'filtered-x' is a short circuit since no filtering takes place, thus $\mathbf{X}_f[kL] = \mathbf{X}[kL]$. With this and using the alternative expression (5,6), the gradient (4) becomes:

$$\underline{\nabla}[kL] = \mathbf{X}^*[kL] \cdot \mathbf{V} \cdot \mathbf{X}[kl] \cdot \underline{\mathbf{D}}[kL] \qquad (7)$$

In [1] it is shown that the following approximation holds:

$$E\{\mathbf{X}^*[kL] \cdot \mathbf{V} \cdot \mathbf{X}[kL]\} \approx L\mathbf{P}_X \qquad (8)$$

with $\mathbf{P}_X$ the power matrix of the (stationary) input signal

$$\mathbf{P}_X = E\{\mathbf{X}[kL] \cdot \mathbf{X}^*[kL]\} \qquad (9)$$

and $E\{\cdot\}$ is the mathematical expectation operation. Using these results and calculating the mathematical expectation of (3) we obtain, without taking into account the constrained window $\mathbf{G}$ (for deteails we refer to [1]), the convergence properties of the BFDAF algorithm (3) can be made, on average, independent of the input signal statistics by defining the power normalization matrix as:

$$\mathbf{P} = \mathbf{P}_X \qquad (10)$$

Thus for BFDAF the decorrelation requires for each separate frequency bin a division (normalization) of the update constant by, an estimate of, the input signal power.

3.2. Decorrelation of filtered-x

In this subsection we will concentrate on the operations that are needed in order to decorrelate the filtered-x algorithm. Now we have $\mathbf{X}_f[kL] = \mathbf{X}[kL] \cdot \mathbf{H}_s$ and with (5,6) the gradient vector (4) can be described as:

$$\underline{\nabla}[kL] = \mathbf{H}_s^* \cdot \mathbf{X}^*[kL] \cdot \mathbf{V} \cdot \mathbf{X}[kl] \cdot \underline{\mathbf{D}}[kL] \qquad (11)$$

Using this for the update equation (3), again without taking into account the constrained matrix $\mathbf{G}$, the average convergence behaviour of this algorithm can be made independent of both the colouring of the input signal and the secondary path $\mathbf{H}_s$ by defining the power matrix $\mathbf{P}$ as follows:

$$\mathbf{P} = \mathbf{P}_X \cdot |\mathbf{H}_s| \qquad (12)$$

in which $|\mathbf{H}_s| = \text{diag}\{|H_{s,0}|, \cdots, |H_{s,B-1}|\}$ gives the amplitude spectrum of $\mathbf{H}_s$. Thus besides, an estimate of, the power matrix $\mathbf{P}_X$ we also need, an estimate of, the secondary acoustic path $\mathbf{H}_s$. We see that on one hand $\mathbf{H}_s$ is needed to calculate the filtered-x signal $\mathbf{X}_f$ while on the other hand the (inverse of the) amplitude spectrum $|\mathbf{H}_s|$ is needed to calculate the power normalization matrix $\mathbf{P}$, as defined above. This complexity can be reduced by combining these operations. and results in the following new method to decorrelate the BFDAF filtered-x algorithm:

$$\begin{aligned} \underline{\mathbf{W}}[kL] &= \underline{\mathbf{W}}[(k-1)L] - \frac{2\alpha}{L} \cdot \mathbf{G} \cdot \Lambda[(k-1)L] \\ \Lambda[kL] &= \mathbf{P}_X^{-1} \cdot \text{arg}\{\mathbf{H}_s^*\} \cdot \mathbf{X}^*[kL] \cdot \underline{\mathbf{R}}[kL] \qquad (13) \\ \text{arg}\{\mathbf{H}_s\} &= \text{diag}\{\mathrm{e}^{-\mathrm{j}\,\text{arg}(H_{s,0})}, \cdots, \mathrm{e}^{-\mathrm{j}\,\text{arg}(H_{s,B-1})}\} \end{aligned}$$

in which we used the fact that we can write the secondary acoustic path as:

$$\mathbf{H}_s = |\mathbf{H}_s| \cdot \text{arg}\{\mathbf{H}_s\} \qquad (14)$$

Thus the decorrelation of the BFDAF filtered-x takes place by normalizing the power of the input signal matrix by $\mathbf{P}_X$ and using only the phase information $\text{arg}\{\mathbf{H}_s^*\}$ of the secondary path. Or stated in another way: *decorrelation of the secondary path is embedded in the calculation of the filtered-x signal itself by using only the phase information.*

4. SIMULATION RESULTS

The above discussed method has been verified by simulating the one-point noise canceller system shown in Fig. 1 using Matlab. Measured acoustic transfer functions for $\underline{\mathbf{h}}_p$ and $\underline{\mathbf{h}}_s$, each of 512 coefficients at 32 kHz, have been used in this simulation. Fig. 3 shows the impulse, amplitude and phase

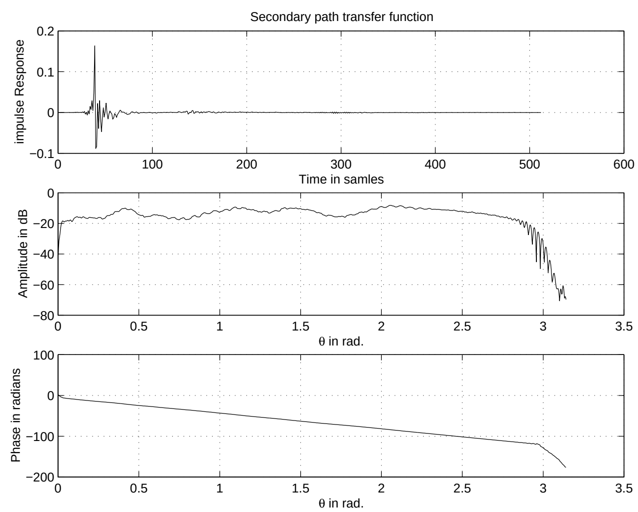

Figure 3: Secondary acoustic path transfer function.

responses of the secondary acoustic path $\underline{\mathbf{h}}_s$. The BFDAF filtered-x algorithm shown in Fig. 2, was used to implement the noise canceller with blocks of $B = 4096$ samples and an adaptive filter $\underline{\mathbf{w}}$ of $M = 2049$ weights [1]. Four simula-

[1] Much more than 512 coefficients are needed for the adaptive filter since $\underline{\mathbf{w}} \rightarrow (\underline{\mathbf{h}}_s)^{-1} \cdot \underline{\mathbf{h}}_p$

tion runs have been performed with an autoregressive input signal of order one having its pole at 0.75. For each simulation run, the adaptive filter was updated and the mean square error was plotted against time. The output of the four simulations are shown in Fig. 4 for comparison. The four cases

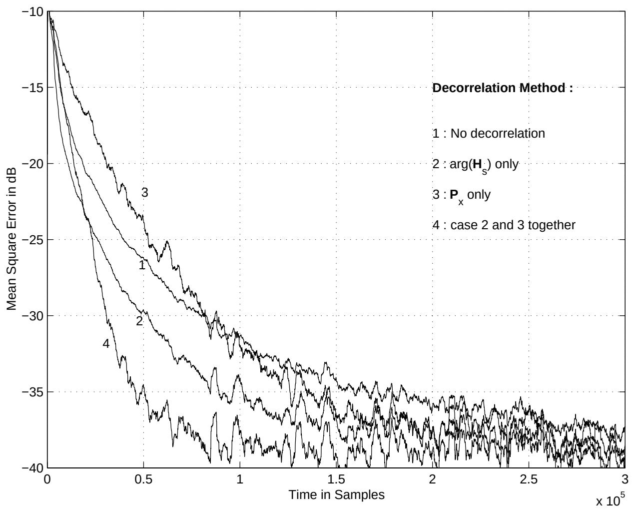

Figure 4: Influence of different decorrelation methods

in this figure are as follows:

1. The adaptive weights are updated using a filtered-x signal calculated by $\mathbf{X}_f[kL] = \mathbf{X}[kL] \cdot \mathbf{H}_s$ where $\mathbf{H}_s$ is an exact copy of the secondary acoustic path. No decorrelation of the input signal power is performed. This case corresponds to the original filtered-x algorithm and is included here as a reference.

2. The adaptive weights are updated using a filtered-x signal calculated by $\mathbf{X}_f[kL] = \mathbf{X}[kL] \cdot \arg\{\mathbf{H}_s\}$. No decorrelation of the input signal power is performed here either. This case shows the improvement in the convergence speed by decorrelating only the amplitude response of the secondary acoustic path. In general, this improvement increases as the non-regularities in $\mathbf{H}_s$ increase. Note the sharp decrease in $\mathbf{H}_s$ at high frequencies in Fig. 3; this is mainly responsible for the slow convergence of case 1 (in addition to the effect of the input signal colour, of course).

3. The adaptive weights in this simulation run are updated using the same filtered-x signal as in case 1; but the decorrelation of the input signal is performed as described in subsection 3.1. Note that not much improvement is possible by decorrelating only the input signal since the huge difference in $|\mathbf{H}_s|$ is dominant and much more effective than the effect of the AR(1) process.

4. The last simulation run shows the improvement when both decorrelation methods are used.

Note that the input signal decorrelation process requires estimation of the power vector and dividing the update constant by this vector; which makes it an expensive process compared to the secondary path decorrelation that is embedded in the filtered-x signal calculation; which has to be performed anyway. The decision whether to perform both decorrelation or only one of them depends on the input signal statistics (the signal to be cancelled) and the secondary acoustic path amplitude response.

5. CONCLUSIONS AND FUTURE WORK

In order to improve convergence properties of the filtered-x algorithm we derived in this paper the decorrelation (normalization) process that is needed for this purpose. This resulted in a new method in which the update algorithm needs only the phase information, in stead of the impulse response, of the secondary path. Simulation results show the strength of the method, even by taking only the phase of the secondary acoustic path and no power normalization (no divisions) resulted in a great convergence improvements.

At this moment we are applying this method to active noise cancelling for acoustics. As can be seen in Fig. 3 it follows that for this kind of applications the phase of the secondary acoustic path is *almost* linear. However, if the secondary path would have been *exact* linear phase, our method requires only one delay in the 'filtered-x' path. For this reason we are currently working on very simple (almost linear) 'phase-models' for the 'filtered-x' path in active noise cancellation problems.

6. REFERENCES

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