A NOVAL WAVELET-BASED GENERALIZED SIDELOBE CANCELLER

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ABSTRACT

This paper presents a novel narrowband adaptive beamformer with the generalized sidelobe canceller (GSC) as the underlying structure. The new beamformer employs the regular M-band wavelet filters in the design of the blocking matrix of the GSC, which, as justified analytically, can indeed block the desired signals as required, provided the wavelet filters have sufficiently high regularity. Additionally, the eigenvalue spreads of the covariance matrices of the blocking matrix outputs, as demonstrated in various scenarios, decrease, thus accelerating the convergence speed of the succeeding least mean squares (LMS) algorithm. Also, the new beamformer belongs to a specific type of partially adaptive beamformers, wherein only a portion of weights is utilized in the adaptive processing. Consequently, the computational complexity is substantially reduced as compared with previous approaches. The issues of choosing the parameters involved for superior performance are addressed as well. Simulation results are furnished to justify this new approach.

1. INTRODUCTION

The design of adaptive beamformers is of importance in various disciplines of signal processing applications such as radar, sonar and geophysical explorations [1]. In many applications, it is not uncommon to use lots of sensors to achieve better interference rejection as well as resolution. To alleviate the computational overhead, two approaches have been advocated in the literature.

The first approach is based on the technique of partial adaptivity [1], in which only a fraction of the adjusting weights is employed, thus leading to lower computational complexity per iteration in adaptive processing. The second approach is to accelerate the convergence speed. The popular LMS algorithm has been notorious for its slow convergence rate, especially for signals whose covariance matrices have widely diverse eigenvalues. To overcome this difficulty, several cascade preprocessors such as the Gram-Schmidt orthogonalization[1], the discrete Fourier (cosine) transform [2], and the wavelet transform have been suggested. The recently introduced wavelet transform, which forms a frequency adaptive window on the time-scale plane, has in particular received a great amount of attention [3].

The wavelet transform has also been incorporated in the adaptive beamformer termed WASPAB [4]. The WASPB, which employs the GSC [5] as the underlying structure, puts a wavelet transform processor preceding the LMS algorithm and indeed exhibits a faster null-steering process. In this paper, we consider a more succinct approach by ingeniously combining the blocking matrix and the wavelet transform process into a single unit.

This new unit is constituted by a set of regular M-band wavelet filters [3] and, as justified analytically, can also block the desired signals as the traditional blocking matrix, provided that the wavelet filters employed have sufficiently high regularity. This new unit, also being referred to as a blocking matrix, encompasses the widely used one with ones and minus ones along the diagonals as a special case. In addition, it possesses two advantageous features. First, the eigenvalue spreads of the covariance matrices of the blocking matrix outputs, as observed in various scenarios, are decreased as compared with previous approaches, thus leading to a faster convergence speed of the succeeding LMS algorithm. Second, the new beamformer belongs to a specific type of partially adaptive beamformers, wherein higher-dimensional adaptive weights are mapped into lower-dimensional ones, thus further reducing the computational overhead. As a consequence, the computational complexity called for is substantially reduced.

The eigenvalue spread and the dimension reduced are determined by the parameters of the wavelet filters as well as the matrix structure of the blocking matrix. To facilitate the choices of these parameters, some suggestive guidelines are also provided, aiming at superior performance.

2. THE WAVELET-BASED GENERALIZED SIDELOBE CANCELLER

2.1. Background Review

Consider an equispaced linear array composed of N omnidirectional sensor elements. The narrowband beamformer output at time instant k, $y(k)$, can be expressed as $y(k) = \mathbf{w}^H\mathbf{x}(k)$, where $\mathbf{w}$ and $\mathbf{x}(k)$ denote the weight vector and the response vector, respectively, and the superscript ^{H} denotes the Hermitian transpose. The response vector $\mathbf{x}(k)$, which is assumed to consist of a single signal under the interference of J jammers, can then be represented by

$$\mathbf{x}(k) = s_s(k)\mathbf{a}_S(\theta_s) + \sum_{i=1}^{J} s_i(k)\mathbf{a}_J(\theta_i) + \mathbf{n} \qquad (1)$$

where $s_s(k)$ and $s_i(k), i = 1, \cdots, J$, are the waveforms of the desired signal and jammers, respectively, $\mathbf{a}_S(\theta_s) = [e^{j(1-n_0)\mu_s}, e^{j(2-n_0)\mu_s}, \cdots, e^{j(N-n_0)\mu_s}]^T$ (^{T} denotes matrix transpose) is the signal vector with an arrival angle of θ_s, n_0

*† This work was supported by National Science Council of R.O.C. under contract NSC 87-2213-E-011-014 and 87-2213-E-019-006.

is the reference point of the linear array, $\mu_s = \frac{2\pi}{\lambda_w} d_w \sin\theta_s$ with λ_w and d_w being, respectively, the wavelength and sensor distance, and $\mathbf{n}$ is the additive receiver white noise.

The effective linearly constrained minimum variance (LCMV) beamformer determines the weight $\mathbf{w}$ by minimizing the output power under some appropriate linear weight constraints and can be expressed as

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_x \mathbf{w} \quad \text{subject to} \quad \mathbf{C}^T \mathbf{w} = \mathbf{f} \tag{2}$$

where $\mathbf{R}_x \stackrel{\Delta}{=} \mathcal{E}\{\mathbf{x}(k)\mathbf{x}(k)^H\}$ is the data covariance matrix with $\mathcal{E}\{\cdot\}$ denoting the expectation operator, $\mathbf{C}$ and $\mathbf{f}$ are an $N \times S$ (full column rank) constraint matrix and an $S \times 1$ filter response vector, respectively. In particular, if we consider the mainbeam derivative constraints, which have been utilized to achieve a flatter mainbeam response so that the array is less sensitive to the steering errors [6, 7], the $(S-1)$ order derivative constraints then require that

$$\mathbf{C}_S^T \mathbf{w} = [1, 0, \cdots, 0]^T \tag{3}$$

where the $N \times S$ matrix $\mathbf{C}_S$ corresponds to the $(S-1)$ order derivative constraint matrix as

$$\mathbf{C}_S = [\mathbf{c}_0, \mathbf{c}_1, \cdots, \mathbf{c}_{S-1}] \tag{4}$$

with $\mathbf{c}_i = [(1-n_0)^i, \cdots, (N-n_0)^i]^T, i = 0, 1, \cdots, S-1$. The GSC reformulates the LCMV to facilitate more efficient implementations and performance analysis [5]. The basic principle of the GSC is to decompose the weight vector $\mathbf{w}$ into two orthogonal components as $\mathbf{w} = \mathbf{w}_f - \mathbf{B}\mathbf{w}_a$. The first component $\mathbf{w}_f$ stands for the fixed target signal filter of the GSC, whereas the second component $-\mathbf{B}\mathbf{w}_a$ denotes the adaptive part. $\mathbf{B}$ satisfies the constraint of $\mathbf{C}^T\mathbf{B} = \mathbf{0}$ and can prevent the desired signal from entering this path, thus being referred to as a blocking matrix.

2.2. The Proposed Wavelet-Based GSC

Consider an $N \times (\lfloor \frac{N-MP}{d} \rfloor + 1)(M-1)$ ($\lfloor \alpha \rfloor$ denotes the largest integer smaller than or equal to α) matrix $\mathbf{B}$ which is constituted by a set of P-regular M-band wavelet filters (of minimal length MP) [3] with coefficients $[h_m(0), h_m(1), \cdots, h_m(MP-1)]$, $m = 1, 2, \cdots, M-1$, as

$$\mathbf{B}^T = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_{M-1} \end{bmatrix} \tag{5}$$

where $\mathbf{H}_m$, $m = 1, \cdots, M-1$, is a $(\lfloor \frac{N-MP}{d} \rfloor + 1) \times N$ matrix

$$\mathbf{H}_m = \begin{bmatrix} h_m(0) & \cdots & h_m(MP-1) & \cdots & \\ \mathbf{o}_d^T & h_m(0) & \cdots & 0 & \\ \vdots & \ddots & \ddots & \vdots & \\ \mathbf{o}_d^T & \cdots & \cdots & h_m(MP-1) & \end{bmatrix} \tag{6}$$

in which $\mathbf{o}_d$ is a $d \times 1$ zero vector and d is a prespecified integer. It can be readily shown that all of the $(\lfloor \frac{N-MP}{d} \rfloor + 1)(M-1)$ columns of $\mathbf{B}$ form a linearly independent set by using the orthogonality of the unitary wavelet filters. Such a choice of $\mathbf{B}$ possesses a distinctive feature of "nulling" out the first few terms of the Taylor's series expansion of the desired signal. More specifically, invoking the extended "sum rule" for P-regular wavelet filters,[8] viz.

$$\sum_k (k_0 + k)^r h_m(k) = 0 \tag{7}$$

for any integer k_0, $m = 1, \cdots, M-1$, and $r = 0, \cdots, P-1$, leads to the following theorem [8]:

Theorem 1

The matrix $\mathbf{B}$ blocks the first $(P-1)$ order Taylor's series expansion of the desired signal components, i.e.

$$\mathbf{B}^T \mathbf{a}_{S(P-1)}(\theta_s) = \mathbf{0} \tag{8}$$

where $\mathbf{a}_{S(P-1)}(\theta_s)$ denotes the first $(P-1)$ order Taylor's series expansion of $\mathbf{a}_S$ with respect to the look direction θ_0

$$\mathbf{a}_{S(P-1)}(\theta_s) = \mathbf{a}_S(\theta_0) + \sum_{i=1}^{P-1} \frac{1}{i!} \frac{\partial^i \mathbf{a}_S(\theta_s)}{\partial \theta_s^i}\bigg|_{\theta_s = \theta_0} (\theta_s - \theta_0)^i \tag{9}$$

Theorem 1 implies that if the desired signal is well approximated by the first P terms of the Taylor's series expansion (or the wavelet filters in $\mathbf{B}$ have sufficiently high regularity), then the desired signal will be "blocked" by the $\mathbf{B}$ matrix as required by the blocking matrix of the GSC structure. To follow, we will then employ such $\mathbf{B}$ as the blocking matrix in our proposed GSC.

As a special case, when $M = 2$, $P = 1$, and $d = 1$, $\mathbf{B}$ becomes an $N \times (N-1)$ matrix $\mathbf{B}_0$ as

$$\mathbf{B}_0^T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \tag{10}$$

which simply corresponds to a normalized version of the widely used blocking matrix [5].

2.3. Relationship with the LCMV Beamformer

To follow, we consider the relationship between the wavelet-based GSC described above and the associated LCMV beamformer with the derivative constraints. First, we show that in this case, the columns of $\mathbf{B}$ are orthogonal to those of $\mathbf{C}_S$ [8].

Theorem 2

Assume that $\mathbf{C}_S$ is an $N \times S$ derivative constraint matrix of (4), then the wavelet-based matrix $\mathbf{B}$ as that of (5) (assume that $(\lfloor \frac{N-MP}{d} \rfloor + 1)(M-1) \leq N - S$) satisfies

$$\mathbf{B}^T \mathbf{C}_S = \mathbf{0} \quad \text{if} \quad S \leq P \tag{11}$$

As a consequence of Theorem 2, if $rank(\mathbf{C}_S) + rank(\mathbf{B}) = N$, then the proposed GSC is equivalent to the corresponding LCMV. If $rank(\mathbf{C}_S) + rank(\mathbf{B}) < N$, in which the columns of $\mathbf{C}_S$ in conjunction with those of $\mathbf{B}$ do not span $\mathcal{R}^N$, then $\mathbf{w}_{opt}$ of the LCMV can now be expressed as $\mathbf{w}_{opt} = \mathbf{w}_f - \tilde{\mathbf{B}}\mathbf{w}_a^o$, where $\tilde{\mathbf{B}} = [\mathbf{B}, \mathbf{A}]$ with $\mathbf{A}$ being an $N \times (N - S - (\lfloor \frac{N-MP}{d} \rfloor + 1)(M-1))$ matrix, which together with $\mathbf{B}$ and $\mathbf{C}_S$, form an orthogonal decomposition of $\mathcal{R}^N$. The adaptive term now becomes

$$\begin{aligned} -\tilde{\mathbf{B}}\mathbf{w}_a^o &= -\mathbf{B}(\mathbf{B}^T\mathbf{R}_x\mathbf{B})^{-1}\mathbf{B}^T\mathbf{R}_x\mathbf{w}_f \\ &\quad -\mathbf{Y}\mathbf{A}(\mathbf{A}^T\mathbf{R}_x\mathbf{Y}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Y}^H\mathbf{R}_x\mathbf{w}_f \end{aligned} \tag{12}$$

where $\mathbf{Y} = \mathbf{I} - \mathbf{B}(\mathbf{B}^T\mathbf{R}_x\mathbf{B})^{-1}\mathbf{B}^T\mathbf{R}_x$. In this case, the proposed GSC belongs to a partially adaptive beamformer since only a portion of the $(N-S)$ adaptive dimension is utilized. The computational complexity called for is reduced as those of partially adaptive beamformers addressed previously. More specifically, it can be shown that the array output power of the proposed GSC, σ_y^2, can be expressed as

$$\sigma_y^2 = (\sigma_y^o)^2 + \|(\mathbf{A}^T\mathbf{R}_x\mathbf{Y}\mathbf{A})^{-\frac{1}{2}}\mathbf{A}^T\mathbf{Y}^H\mathbf{R}_x\mathbf{w}_f\|^2 \qquad (13)$$

where $(\sigma_y^o)^2 \triangleq \mathbf{w}_{opt}^H\mathbf{R}_x\mathbf{w}_{opt}$ denotes the output power of the LCMV and $\|\cdot\|$ is the Euclidean norm. Therefore, the output power of the proposed GSC, σ_y^2, is greater than that of the corresponding LCMV, $(\sigma_y^o)^2$.

2.4. Choices of Parameters

In this subsection, we treat the issues of determining the parameters M, P, and d involved in the proposed blocking matrix. Our consideration will be based on the misadjustment and the convergence rate of the LMS algorithm, and the output performance of the beamformer.

First, we show that under some appropriate conditions, the misadjustment and the eigenvalue spread of the blocking matrix output covariance matrix for the proposed GSC satisfy the following inequalities [8]:

Theorem 3

If the desired signal, the jammers, and the contaminated noise are uncorrelated, then the misadjustment $\mathcal{M}$ obeys the following inequality:

$$\mathcal{M} \le \frac{\eta}{2}\,(\left\lfloor\frac{N-MP}{d}\right\rfloor + 1)(M\sum_{i=1}^{J}\sigma_{Ji}^2 + (M-1)\sigma_n^2) \quad (14)$$

where η is the step-size used in the LMS algorithm, σ_{Ji}^2, $i = 1,\cdots,J$, and σ_n^2 stand for the power of the i^{th} jammer and contaminated white noise measured at each element of the array, respectively. Similarly, the eigenvalue spread of $\mathbf{R}_u$, $\mathcal{X}(\mathbf{R}_u)$, where $\mathbf{R}_u \triangleq \mathcal{E}\{\mathbf{u}\mathbf{u}^H\}$ and $\mathbf{u} = \mathbf{B}^T\mathbf{x}$, satisfies

$$\mathcal{X}(\mathbf{R}_u) \le \frac{M(\left\lfloor\frac{N-MP}{d}\right\rfloor + 1)\sum_{i=1}^{J}\sigma_{Ji}^2 + \sigma_n^2\lambda_{max}(\mathbf{B}^T\mathbf{B})}{\sigma_n^2\lambda_{min}(\mathbf{B}^T\mathbf{B})} \qquad (15)$$

where $\lambda_{max}(\mathbf{B}^T\mathbf{B})$ and $\lambda_{min}(\mathbf{B}^T\mathbf{B})$ denote the maximum and minimum eigenvalues of $\mathbf{B}^T\mathbf{B}$, respectively.

Then, following the same approach as that of [2], we will choose the parameters M, P, and d to minimize the upper bounds of the inequalities of (14) and (15). We can observe that both the upper bounds of (14) and (15) decrease as P increases (for fixed d and M). It follows therefore that the choice of wavelet filters with high regularity is preferred for smaller $\mathcal{M}$ and $\mathcal{X}(\mathbf{R}_u)$.

As for the choice of d, note that since $\mathrm{tr}(\mathbf{B}^T\mathbf{B}) = (\left\lfloor\frac{N-MP}{d}\right\rfloor+1)(M-1)$, where $\mathrm{tr}(\cdot)$ is the matrix trace operator, we can readily deduce that $\lambda_{max}(\mathbf{B}^T\mathbf{B}) \le \mathrm{tr}(\mathbf{B}^T\mathbf{B}) = (\left\lfloor\frac{N-MP}{d}\right\rfloor+1)(M-1)$ and $\lambda_{min}(\mathbf{B}^T\mathbf{B}) \le 1$. Also, since the upper bound of (15) is dictated mainly by the eigenvalue spread of $(\mathbf{B}^T\mathbf{B})$, we can then choose d to minimize this value. Recall that when d is a multiple of M, $\mathbf{B}^T\mathbf{B} = \mathbf{I}$ for which $\lambda_{max}(\mathbf{B}^T\mathbf{B}) = \lambda_{min}(\mathbf{B}^T\mathbf{B}) = 1$. As a result, we can choose d as a multiple of M which then yields a smaller upper bound of (15) (for fixed P and M). In particular, when $M = 2$, choosing $d = 2$ will yield a smaller upper bound of (15) as compared with that by using $d = 1$, which was employed by $\mathbf{B}_0$ of (10). This explains why the proposed approach in general converges more rapidly than that of previous work, as the latter is based on $\mathbf{B}_0$.

The chosen parameters should also maximize the output signal-to-interference-plus-noise-ratio (SINR). First, note that a wavelet filter with high regularity exhibits a fast decaying response. Since the wavelet filters stand for the highpass spatial filtering, high regularity of the wavelet filters will then form a sharper and wider null in the low spatial frequency part of the spatial response of the blocking matrix. As such, the blocking matrix will block not only the desired signal but also the interfering signals which are supposed to pass through. Therefore, the SINR will somehow degrade if the regularity P of the wavelet filters is chosen widely large. Additionally, from [3], we know that $(M-1)$ wavelet filters with a larger M will provide better energy compaction, leading to a narrow null in the low spatial frequency part of blocking matrix spatial response. Along the same line, a larger M is therefore preferred, as it will cause less degradation of the resulting SINR.

3. EXPERIMENTAL RESULTS

Some simulations are carried out in this section to access the proposed wavelet-based approach. To follow, we consider two examples, both of which are based on linear equispaced arrays consisting of N omnidirectional sensors spaced one half wavelength apart. The GSC, with various blocking matrices along with the derivative constraints, is utilized for the determination of beamformer weights. The resulting array output beampatterns are all based on an ensemble average of 50 independent trials.

Example 1 : The array considered is composed of 15 sensors. The interference environment consists of one jammer with an arrival direction of 55^o. The gain constraint, i.e. $\mathbf{C}_1$, and $\eta = 9\times10^{-6}$ are used for the GSC. The interference to noise ratio (INR) is 30 dB and the contaminated white Gaussian noise (WGN) is 10 dB. The resulting array output beampatterns by using $\mathbf{B}_0$ of (10) and by the proposed blocking matrix with $M = 3$, $d = 3$, and $P = 3$ ($\mathbf{B}$ is now a 15×6 matrix) are shown in Figs. 1, and 2, respectively.

Example 2 : The array considered is composed of 48 sensors. The interference environment now consists of two jammers arriving from directions of $(-45^o, 50^o)$. The 2^{nd} order derivative constraints, i.e. $\mathbf{C}_3$ (with $n_0 = 24$), and $\eta = 5\times10^{-6}$ are used for the GSC. The INR's are equal to 20 dB and 30 dB for jammers with directions of arrival -45^o and 50^o, respectively, whereas the WGN has SNR 10 dB. The resulting array output beampatterns by using a (normalized) $N\times(N-3)$ blocking matrix $\mathbf{B}_2$ [9], where

$$\mathbf{B}_2^T = \frac{1}{\sqrt{20}}\begin{bmatrix} 1 & -3 & 3 & -1 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & -3 & 3 & -1 \end{bmatrix}, \quad (16)$$

and by the proposed one with $M = 6$, $d = 6$, and $P = 7$ ($\mathbf{B}$ is now a 48×10 matrix) are shown in Figs. 3 and 4, respectively.

From both examples, we can find that the new wavelet-based GSC can form deep nulls more rapidly at the jammer directions. This is attributed to the fact that both the misadjustment and $\mathcal{X}(\mathbf{R}_u)$ are significantly reduced as compared with those of previous approaches [8]. Additionally, the dimensions of the weight vector $\mathbf{w}_a$ involved in the adaptive processing also decrease, thus calling for lower computational overhead in each iteration. As such, the overall computational complexity is substantially reduced.

4. CONCLUSION

In this paper, we describe a low complexity wavelet-based GSC, in which the blocking matrix is constituted by a set of regular M-band wavelet filters. This new blocking matrix can block the desired signal as required, provided that the employed wavelet filters are highly regular.

Furthermore, the outputs of the blocking matrix not only have reduced dimensions, but their covariance matrices in general also have smaller misadjustments and eigenvalue spreads. Consequently, the array can form the desired null-steering beampatterns with substantially reduced computational complexity.

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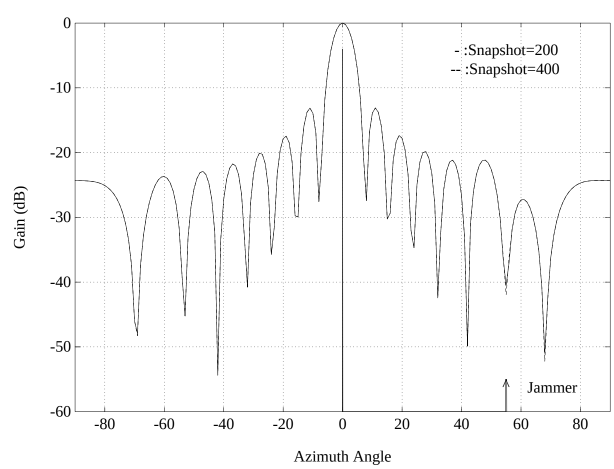

Figure 1. The outputbeam pattern of Example 1 by using the blocking matrix of (10).

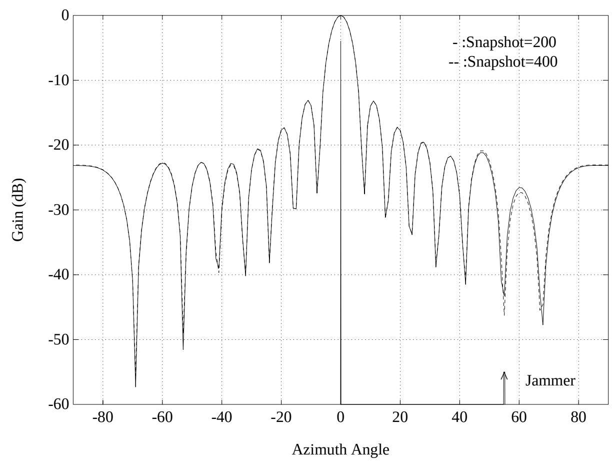

Figure 2. The output beampattern of Example 1 by using the proposed blocking matrix.

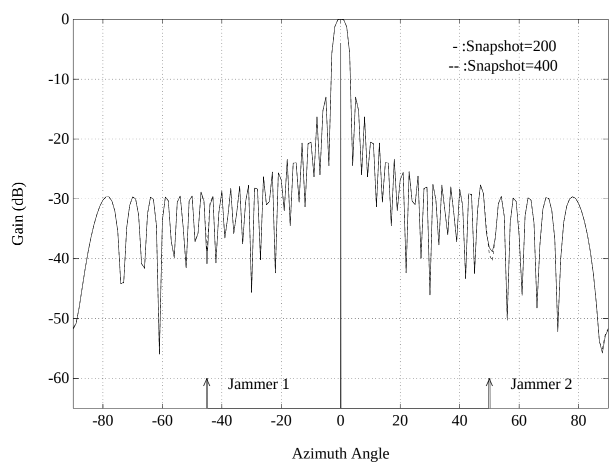

Figure 3. The output beampattern of Example 2 by using the blocking matrix of (16).

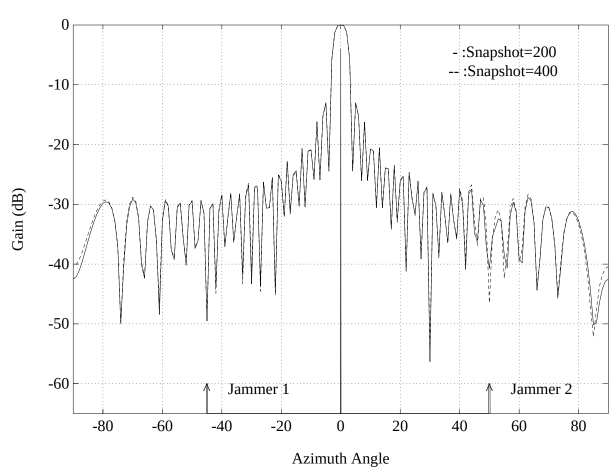

Figure 4. The output beampattern of Example 2 by using the proposed blocking matrix.