ROBUST EQUALIZATION FOR SPREAD-RESPONSE PRECODING SYSTEMS

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ABSTRACT

The problem of equalization for spread-response precoding systems based on minimum mean-square error (MMSE) estimates of the fading channel coefficients is considered. These systems are attractive, low complexity alternatives to the combination of interleaving and error-control coding for achieving time diversity in fading environments. To make the performance of these systems robust to channel estimation errors, we derive the linear equalizer at the receiver that maximizes the effective signal-to-noise-and-interference ratio (SNIR) subject to uncertainty in the channel measurements. We examine the bit-error rate performance and develop fixed and dynamic solutions to the associated problem of optimal power allocation between the data transmissions and channel measurements. The effectiveness of these algorithms is demonstrated through measurements obtained from an indoor wireless setting.

1. INTRODUCTION

Due to channel distortions such as intersymbol interference (ISI) and signal fading, transmissions over wireless channels often exhibit bit-error performance dramatically inferior to transmissions over traditional additive Gaussian white-noise channels. The combination of interleaving with error-control coding is a common approach for combating temporal variations of these channels [1]; however, spread-response precoding systems [2], [3] represent a particularly appealing alternative to interleaving in terms of their performance-complexity considerations. These systems temporally distribute the energy of each symbol by means of dispersive linear filtering before transmission, allowing the receiver to effectively average the variations of the channel through appropriate equalization and matched-filtering. Conveniently, spread-response precoding systems also do not require additional power or bandwidth.

In developing the theory of spread-response precoding, [2] determines the linear equalizer that maximizes the effective signal-to-noise-and-interference ratio (SNIR) based on complete knowledge of the fading channel response. In practice, many equalization algorithms rely on an estimate of the channel, and their performance can be severely degraded by even small channel estimation errors. To prevent such sensitivity in spread-response precoding systems, the problem of determining the optimal linear equalizer at the receiver based on minimum mean-square error (MMSE) estimates of the fading channel is addressed in Section 3. In addition, the system performance strongly depends on the transmitter's power allocation among the data transmissions and channel measurements. Based on a representative channel evolution model described in Section 4, the optimal allocation strategy is developed in Section 5 and compared to fixed allocation strategies. We also include results from an implementation of these algorithms in an indoor wireless communications system testbed.

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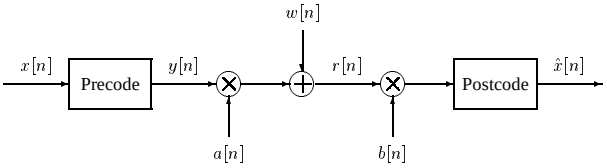

Figure 1: Narrowband spread-response precoding system model.

2. SYSTEM MODEL

Fig. 1 depicts a discrete-time, baseband equivalent model for the class of systems we consider for narrowband communication over time-selective wireless channels. The symbol sequence $x[n]$ is a zero-mean, white sequence with energy $\mathcal{E}_s$. Channel coding is performed by the spread-response precoder, a linear filter that generates the transmit sequence $y[n]$ from $x[n]$. The channel corrupts $y[n]$ with complex-valued fading $a[n]$ and additive noise $w[n]$, to produce the received sequence $r[n]$. At the receiver, symbol estimates $\hat{x}[n]$ are obtained by means of equalization (multiplication by $b[n]$) and finally postcoding with the inverse of the precoding filter.

2.1. Rayleigh Fading Channel

The wireless channel model represented in Fig. 1 captures the effects of time-selective fading and additive noise. Such a model is appropriate for wireless communication indoors, where the coherence bandwidth of the channel is generally larger than the signal bandwidth, and the coherence time of the channel is typically on the order of several symbol periods.

Statistically, we model the fading $a[n]$ as a zero-mean, stationary, complex-valued, circularly-symmetric Gaussian sequence with variance σ_a^2. (For simplicity of exposition, we refer to all such sequences simply as "complex Gaussian" sequences, and we point out explicit deviations from these properties whenever necessary.)

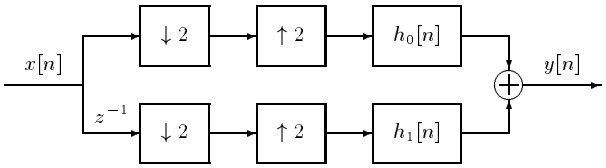

Figure 2: Filter bank structure of the linear, periodically time-varying spread-response precoder.

The effectiveness of channel estimation, and therefore, system performance, will generally depend on the dynamics of $a[n]$, and we consider a convenient model for these dynamics in Section 4.

The additive noise $w[n]$ often represents disturbances caused by receiver thermal noise, and more importantly, other sources of interference. We model $w[n]$ as a white, complex Gaussian sequence with variance $\mathcal{N}$.

2.2. Spread-Response Precoding

Spread-response precoding mitigates the fading distortion by applying a linear, periodically time-varying (LPTV) system of the form shown in Fig. 2 to the symbol sequence at the transmitter. Using the filter design algorithm in [3], the impulse responses $h_0[n]$ and $h_1[n]$ are of finite-length and take values

$$h_m[n] = \pm 1/\sqrt{N}, \quad \text{for } n = 0, 1, \ldots, N-1,\ m = 0, 1.$$

since unit-energy sequences of this form most effectively distribute each symbol's energy over N samples. Furthermore, the particular sequences are chosen so that the system in Fig. 2 constitutes an orthogonal transformation of the input sequence. For example, sequences of length $N = 8$ are given in Table 1. Prototypical impulse responses can be upsampled to match the coherence characteristics of the channel with no additional cost in computation, as pointed out [3].

At the receiver, a linear equalizer multiplies the received sequence $r[n]$ by the sequence $b[n]$ to partially compensate for the effects of fading. Following equalization, the precoding process is inverted by the associated LPTV postcoder [3].

With spread-response precoding systems, the composite channel formed by the precoder, fading channel, equalizer, and postcoder is effectively transformed into an additive marginally Gaussian white-noise channel [2], [3]. Specifically, as the length of a precoding filter becomes large compared to the coherence time of the channel, the symbol estimates may be approximated as

$$\hat{x}[n] \approx \mathrm{E}[ab]\ x[n] + z[n], \quad \text{for each } n, \tag{1}$$

where we have dropped the dependence of the channel and equalizer statistics on n due to stationarity. Moreover, in the limit as $N \to \infty$, the approximation in (1) becomes exact in the mean-square sense. Here $z[n]$ is a zero-mean, complex marginally Gaussian white-noise sequence that is uncorrelated with the input $x[n]$. The variance of $z[n]$ is given by

$$\sigma_z^2 = \mathrm{Var}[ab] + \mathcal{N}\left(\mathrm{Var}[b] + |\mathrm{E}[b]|^2\right) ,$$

where the first term is due to ISI and the second term is due to the additive noise $w[n]$. Accordingly, we may compute the effective SNIR in the symbol estimates as

n	0	1	2	3	4	5	6	7
$\sqrt{8}\,h_0[n]$	+1	+1	+1	-1	+1	+1	-1	+1
$\sqrt{8}\,h_1[n]$	+1	+1	+1	-1	-1	-1	+1	-1

Table 1: Example impulse responses for $N = 8$.

$$\gamma(b) = \frac{|\mathrm{E}[ab]|^2}{\mathrm{Var}[ab] + \xi\left(\mathrm{Var}[b] + |\mathrm{E}[b]|^2\right)} , \tag{2}$$

where $\xi = \mathcal{N}/\mathcal{E}_s$ and where we have used notation to explicitly reflect the dependence on the choice of equalizer.

3. OPTIMAL EQUALIZERS BASED ON MMSE CHANNEL ESTIMATES

In practice, the bit-error probability for spread-response precoding systems decreases monotonically with effective SNIR; thus, a natural approach is to select the equalizer that maximizes (2). If we constrain the equalizer to be a function of the MMSE estimate $\hat{a}[n]$ of the channel coefficient, we will show that the optimal equalizer satisfies

$$b[n] \propto \frac{\hat{a}^*[n]}{|\hat{a}[n]|^2 + \xi'} , \tag{3}$$

where $\xi' = \xi + \sigma_e^2$, and σ_e^2 represents the mean-square error in the channel estimates. Before developing this result, we make several preliminary remarks. First, the optimal equalizer depends on the mean-square error of the channel estimate, implying that the two problems of channel estimation and equalization are coupled. Second, (3) is consistent with the known channel solution: as $\sigma_e^2 \to 0$, we have $\hat{a}[n] \to a[n]$ and $\xi' \to \xi$, which gives the form originally derived in [2]. Third, (3) corresponds to the optimal equalizer for a "known channel" $\hat{a}[n]$ with additive noise of intensity $\mathcal{E}_s\,\sigma_e^2 + \mathcal{N}$. Here the first term is due to channel estimation error and is uncorrelated with the input sequence, while the second term is due to the original additive noise [4]. These observations suggest that the approximation (1) still holds if the equalizer employs MMSE estimates of the channel.

To verify (3), we first rewrite (2) in a manner analogous to that used in [2], namely,

$$\gamma(b) = \frac{1}{1/\phi(b) - 1} , \tag{4}$$

where

$$\phi(b) = \frac{|\mathrm{E}[ab]|^2}{\mathrm{E}\left[\left(|a|^2 + \xi\right)|b|^2\right]} , \tag{5}$$

and note that maximizing (5) is equivalent to maximizing (4).

We substitute the relationship

$$a[n] = \hat{a}[n] + e[n] ,$$

into (5), where $e[n]$ is the channel estimation error, to obtain

$$\phi(b) = \frac{|\mathrm{E}[\hat{a}b] + \mathrm{E}[eb]|^2}{\mathrm{E}\left[|\hat{a}b|^2\right] + 2\,\mathrm{Re}\left\{\mathrm{E}\left[\hat{a}e^*|b|^2\right]\right\} + \mathrm{E}\left[|eb|^2\right] + \xi\mathrm{E}\left[|b|^2\right]} . \tag{6}$$

Since $\hat{a}[n]$ is an MMSE estimate of the channel, it possesses useful properties that simplify the expression (6). Recall that $\hat{a}[n]$ is unbiased, so that

$$\mathrm{E}[e[n]] = 0, \quad \text{for each } n. \tag{7}$$

Furthermore, from the orthogonality principle, $e[n]$ is uncorrelated with any function of the observations, and therefore, any function of $\hat{a}[n]$, *i.e.*,

$$\mathrm{E}[f(\hat{a}[n])\, e^*[n]] = 0, \quad \text{for each } n \tag{8}$$

and any $f(\cdot)$. Applying (8) to (6), we obtain

$$\phi(b) = \frac{|\mathrm{E}[\hat{a}b]|^2}{\mathrm{E}\left[\left(|\hat{a}|^2 + \xi'\right)|b|^2\right]}\,, \tag{9}$$

with $\xi' = \xi + \sigma_e^2$. Finally, by the Schwartz inequality, we have

$$\begin{aligned} |\mathrm{E}[\hat{a}b]|^2 &= \left|\mathrm{E}\left[\frac{\hat{a}}{\sqrt{|\hat{a}|^2 + \xi'}} \cdot b\sqrt{|\hat{a}|^2 + \xi'}\right]\right|^2 \\ &\le \mathrm{E}\left[\frac{|\hat{a}|^2}{|\hat{a}|^2 + \xi'}\right] \cdot \mathrm{E}\left[\left(|\hat{a}|^2 + \xi'\right)|b|^2\right]\,, \end{aligned}$$

with equality if and only if $b[n]$ is of the form (3).

The effective SNIR performance for this equalizer can readily be computed [4]

$$\gamma_{\max} = \frac{1}{\zeta' e^{\zeta'} E_1(\zeta')} - 1\,, \tag{10}$$

where $\zeta' = \xi'/\sigma_{\hat{a}}^2$ and $E_1(\cdot)$ denotes the exponential integral

$$E_1(\nu) = \int_\nu^\infty \frac{e^{-t}}{t} dt, \quad \nu > 0\,.$$

As the approximation (1) suggests, a good estimate of the bit-error probability for symbol-by-symbol detection associated with the optimal equalizer (3) can be obtained from the corresponding bit-error probability for symbol-by-symbol detection over an additive Gaussian white-noise channel with signal-to-noise ratio (SNR) given by (10). For example, if $x[n]$ is a quadrature phase-shift keying (QPSK) sequence, we can approximate the bit-error probability of these systems as [1]

$$\Pr(e) \approx \mathcal{Q}(\sqrt{\gamma_{\max}}) = \frac{1}{\sqrt{2\,\pi}} \int_{\sqrt{\gamma_{\max}}}^\infty e^{-t^2/2} dt\,. \tag{11}$$

We should emphasize that similar results can in principle be obtained for the frequency-selective channel if the error kernel corresponds to a wide-sense stationary, uncorrelated scattering channel [4]. However, the validity of this uncorrelated scattering approximation for the error kernel warrants further investigation.

4. CHANNEL MODELING AND ESTIMATION

The mean-square channel estimation error σ_e^2 depends on the particular channel dynamics in addition to the amount of transmitter power allocated to channel measurements. We examine the impact of these variations on the system performance in terms of the effective SNIR (10) in the context of a simple channel evolution model.

Specifically, we consider the following baseband equivalent, first-order state-space model for the fading process $a[n]$:

$$a[n+1] = \left(\frac{\tau - 1}{\tau}\right) a[n] + v[n] \tag{12a}$$

$$q[n] = \sqrt{\mathcal{E}_p}\, a[n] + \tilde{w}[n] \tag{12b}$$

In the state evolution equation (12a), τ is a measure of the channel coherence time (in symbols), and $v[n]$ is a white complex Gaussian sequence that is independent of $a[k]$ for $k \le n$. Note that $v[n]$ has mean zero and variance appropriately chosen so that $\mathrm{Var}[a[n]] = \sigma_a^2$. The observation equation (12b) corresponds to pilot-tone measurements of the channel, where $\mathcal{E}_p$ denotes the pilot-tone energy per symbol. The additive noise $\tilde{w}[n]$ is another white, complex Gaussian sequence that is independent of $a[n]$, $w[n]$, and $v[n]$ and has variance $\mathcal{N}$.

The Kalman filter associated with (12) provides the MMSE estimate $\hat{a}[n]$. Furthermore, techniques for joint estimation of the channel state $a[n]$ as well as the model parameters, *e.g.*, τ, based on the Expectation-Maximization (EM) algorithm are also readily developed, and can be extended to higher-order models. In addition, the performance of the estimator, in terms of the mean-square estimation error σ_e^2 as a function of pilot-tone energy $\mathcal{E}_p$, can be computed from the Riccati equation and used to characterize system performance [4].

5. POWER ALLOCATION AND SYSTEM PERFORMANCE

System performance ultimately depends upon allocation of transmitter power between data signaling and channel measurements. For a fixed amount of transmit energy $\mathcal{E} = \mathcal{E}_s + \mathcal{E}_p$ per symbol, we examine system performance as a function of the pilot-tone energy ratio $0 < \mathcal{E}_p/\mathcal{E} < 1$. Fig. 3 shows the result of numerically computing optimal power allocations to maximize the effective SNIR (10). As Fig. 3 indicates, the optimal allocation strategy varies with channel coherence time and, to a lesser extent, the receiver input SNR.

In Fig. 4, we compare the QPSK bit-error probability (11) for systems employing the optimal power allocations from Fig. 3 and the fixed power allocation $\mathcal{E}_p/\mathcal{E} = 0.15$. As Fig. 4 indicates, the fixed allocation strategy incurs a relatively modest penalty in system performance for a wide range of channel coherence times.

Fig. 5 illustrates results from a preliminary implementation of these spread-response precoding systems within the indoor wireless communications laboratory in the Digital Signal Processing Group at M.I.T. Fig. 5 shows scatter plots (before hard-limiting) from transmission of 2000 QPSK symbols over a time-varying indoor channel, both with and without spread-response precoding. We see from inspection of this figure that finite-length spread-response precoding significantly changes the interference characteristics at the output of the receiver. These results are consistent with those obtained via computer simulations.

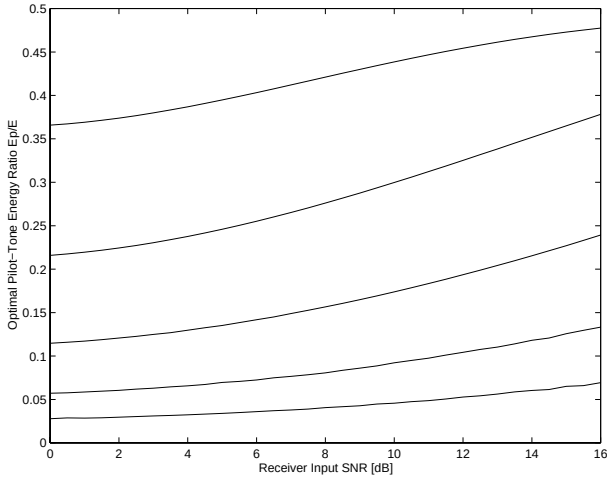

Figure 3: Optimal pilot-tone energy ratio $\mathcal{E}_p/\mathcal{E}$ versus receiver input SNR. The successively lower curves correspond to fading channels with coherence times $\tau = 10$, 100, 1000, 10000, and 100000 symbols, respectively.

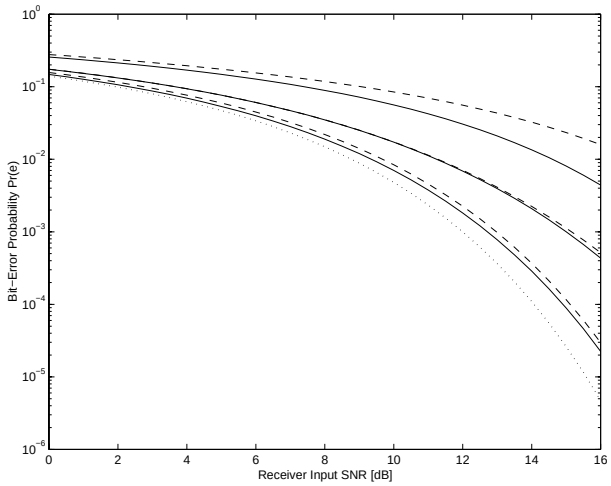

Figure 4: System performance with optimal and suboptimal transmitter power allocation. The successively lower solid curves correspond to the optimal bit-error performance of QPSK signaling for fading channels with coherence times $\tau = 10$, 1000, and 100000 symbols, respectively. The successively lower dashed curves denote the associated bit-error rates with the fixed power allocation $\mathcal{E}_p/\mathcal{E} = 0.15$. Finally, the dotted curve depicts the bit-error probability for the case in which the channel is perfectly known.

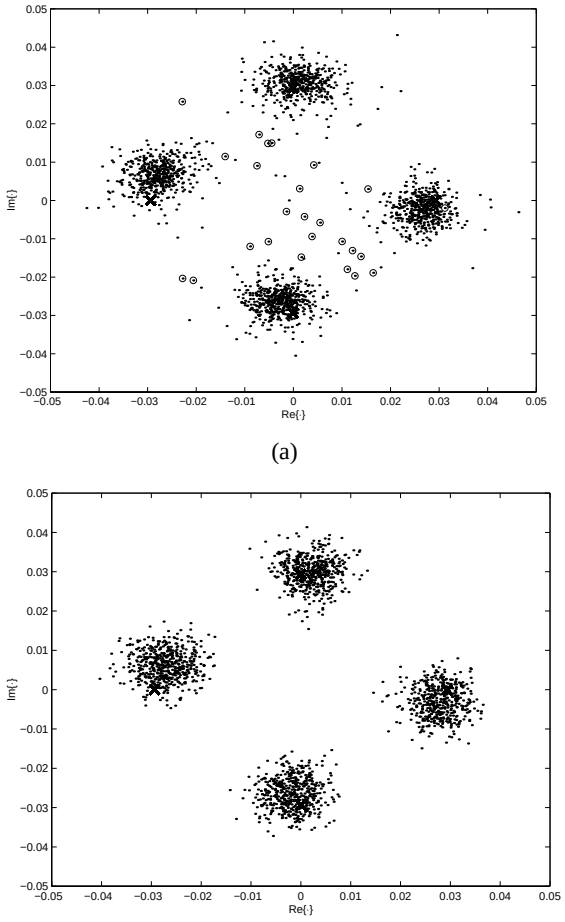

Figure 5: Laboratory receiver output scatter plots (a) without spread-response precoding, and (b) with length $N = 128$ spread-response precoding. The symbol estimates $\hat{x}[n]$ are indicated by "$\cdot$". Symbol estimates resulting in erroneous decisions after hard-limiting are indicated by "$\odot$".

6. REFERENCES

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