FINITE-STATE DIFFERENTIAL CODING FOR WIRELESS COMMUNICATIONS WITH MULTIPATH CHANNELS

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ABSTRACT

Differential coding allows signal demodulation without carrier phase estimation, and thus is commonly used to cope with phase ambiguity and residual carriers. In a wireless scenario where the system transfer function is FIR due to multipath reflections, channel estimation and equalization is usually required. Inspired by the recent work by Tong [1] on blind sequence estimation, we propose a vector differential coding scheme that allows instantaneous signal detection at the receiver without knowledge of the channel. The new technique can be regarded as a generalization of the standard differential coding method for removing convolutional ambiguities.

1. INTRODUCTION

Differential coding is widely used in communication systems with phase ambiguity or residual carrier offsets [2]. By encoding the information in phase differences between successive signal transmissions, a differentially encoded phase-modulated signal allows demodulation without the estimation of the carrier phase. Although relative to coherent demodulation, differential coding has slightly higher SNR requirement for the same BER performance, this small increase translates into a significant decrease in system complexity and sensitivity.

In wireless communications with frequency-selective fading channels, the system transfer function is generally an FIR filter rather than a complex scalar. The resulting convolutional ambiguity involves multiple adjacent signals which renders standard differential coding techniques inapplicable. Existing signal recovery techniques usually rely on channel identification (either using training sequences or through blind estimation) [2, 3], and channel equalization [4].

Tong in [1] proposed a blind sequence estimation algorithm to accomplish signal recovery without channel identification. By estimating the deterministic source correlation from the oversampled observations, the input symbols are reconstructed through correlation optimization using the Viterbi algorithm. While this direct approach avoids the identifiability and channel inversion problems commonly encountered in channel equalization, it does possess the well-known limitations of trellis search, *i.e.*, complexity and delays in signal decision. Also noticed is that the decoding scheme may suffers from catastrophic error propagation similar to catastrophic convolutional code, especially under low SNR.

Inspired by the ideas of differential coding and blind sequence estimation in [1], we introduce in this paper a generalized differential encoding technique to recover input symbols from the convolutional ambiguity (FIR channel) without channel equalization or trellis searching. The new scheme, which we term *finite-state differential coding*, injects certain ambiguity resistance at the transmitter by modulating the information bits through finite-state encoding. Instantaneous symbol recovery can be accomplished at the receiver with a low cost generalized differential decoding procedure. Other advantages of the proposed technique include robustness against carrier drifts and time varying channels.

2. FORMULATION

Consider the following baseband signals from M $(M > 1)$ receivers,

$$
\begin{aligned}
\mathbf{y}(n) &= [y_1(n) \cdots y_M(n)]^T \\
&= \sum_{l=0}^{L} \mathbf{h}(l)s(n-l) + [v_1(n) \cdots v_M(n)]^T .
\end{aligned}
$$

The composite vector channel, $\mathbf{h}(l) = [h_1(l) \cdots h_M(l)]^T$, characterizes the transfer function between the transmitter

and a plurality of physical and/or virtual receivers (due to antenna array, fractional oversampling, or transmission redundancy) [3, 4, 5]. $s(n)$ is the transmitted signal, and $v_i(n)$ is the noise element from the ith receiver. In wireless communications, it is generally plausible to model the channel as a vector FIR filter with maximum order L known to the system [2].

For presentational simplicity, we will consider noise-free data in the remainder of this paper. Stacking K receiver vectors into one column:

$$\mathbf{x}(n) \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{y}(n) \\ \vdots \\ \mathbf{y}(n+K-1) \end{bmatrix} = \mathbf{H} \begin{bmatrix} s(n-L) \\ \vdots \\ s(n+K-1) \end{bmatrix},$$

we obtain a smoothed data vector as follows,

$$\mathbf{x}(n) = \mathbf{H}\mathbf{s}(n). \tag{1}$$

Let $P = L + K$, $\mathbf{H} \in \mathcal{C}^{MK \times P}$ is a block Toeplitz matrix of $\{\mathbf{h}(l)\}_{l=0}^{L}$. We invoke the following common assumptions:

1. $K > L/(M-1)$;

2. The channel matrix $\mathbf{H}$ is of full column rank;

3. The transmitted sequence $\{s(n)\}$ has no less than $K + L$ modes.

3. FINITE-STATE DIFFERENTIAL CODING

If the channels are memoryless, the received signal from the ith receiver reduces to

$$y_i(n) = h_i s(n)$$

where h_i represents the phase ambiguity and attenuation. By differentiating successive received signals, it is seen that the deterministic correlation of the *normalized* transmitted signals, $s^*(n)s(n-1)$, is readily available from the outputs. Conventional differential coding exploits this very idea by modulating the information bit, denoted as $b(n)$, into the phase differences: $s(n) = s(n-1)b(n)$.

3.1. Encoding

When the channels are FIR, $s^*(n)s(n-1)$ in no longer available due to the convolutional effects of $\{\mathbf{h}(l)\}_{l=0}^{L}$. However, as will be shown (also see [1]), the deterministic correlation of $\mathbf{s}(n)$ or vector differentiation of $\{s(n)\}$

$$\begin{aligned} r(n) &\stackrel{\text{def}}{=} \mathbf{s}^H(n)\mathbf{s}(n-1) \\ &= s^*(n)s(n-1) + \cdots + s^*(n+P)s(n+P-1), \end{aligned}$$

can be calculated from $\{\mathbf{y}(n)\}$ through orthogonalization without the channel information. This observation suggests
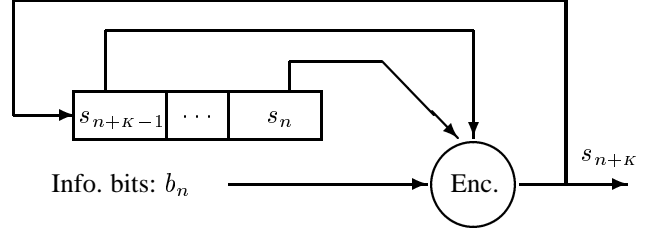

Figure 1: Finite-state differential encoding

one to modulate $b(n)$ into $r(n)$ at the transmitter so that it can be recovered blindly at the receiver using generalized differential decoding.

To elaborate, consider Figure 1 which depicts the encoding diagram. A shift register stores P previously transmitted signals, $s(n), \cdots, s(n+P-1)$. For every new bit, $b(n)$, the encoder selects the next output signal $s(n+P)$ so that $r(n)$ carries the information of $b(n)$. At the receiver end, the decoder calculates $r(n)$ using the algorithm described in the next section and provides instantaneous detection on $b(n)$.
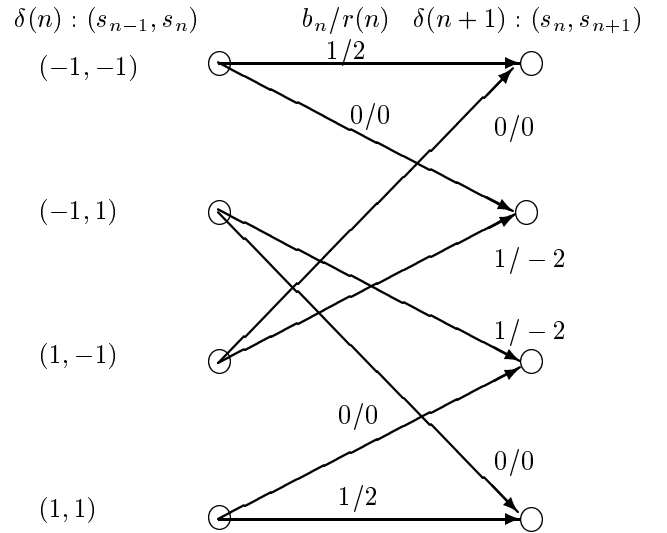

Figure 2: The encoding state transition diagram

The above idea can be illustrated using a simple example with $P = 2$. For simplicity, we consider BPSK output, *i.e.*, $s(n) \in \pm 1$. The encoding state transition diagram is shown in Figure 2. $\delta(n) = (s(n-1), s(n))$ denotes the current state. Each transition path from $\delta(n)$ to $\delta(n+1)$ is labeled by $b(n)/r(n)$. Based on the current setup, the detection rules at the receiver are given by

$$\begin{cases} b(n) = 1 & \text{if } r(n) = \pm 2 \\ b(n) = 0 & \text{if } r(n) = 0 \end{cases}$$

3.2. Decoding

The above encoding scheme modulates the information bits into the correlation function of the transmitted signals. In this section we will show that $r(n) = \mathbf{s}^H(n)\mathbf{s}(n-1)$ can be directly calculate from the received signals [1].

Denoting

$$\mathbf{R}_x = E\left[\mathbf{x}(n)\mathbf{x}^H(n)\right],$$

we have,

$$\mathbf{R}_x = \mathbf{H}\mathbf{H}^H = \mathbf{U}_s \Sigma_s \mathbf{U}_s^H, \qquad (2)$$

where $\mathbf{U}_s \Sigma_s \mathbf{U}_s^H$ is the eigenvalue decomposition of $\mathbf{R}_x$. Here we use the fact that the encoder output $s(n)$ is white, since the modular operation incurred by the encoder generally does not change the statistics of information sequence $b(n)$. The SVD of $\mathbf{H}$ can be expressed as $\mathbf{H} = \mathbf{U}_s \Sigma_s^{\frac{1}{2}} \mathbf{V}_s^H$. Defining

$$\mathbf{T} = \Sigma_s^{-\frac{1}{2}} \mathbf{U}_s^H, \qquad (3)$$

it is readily shown that

$$\mathbf{z}(n) \stackrel{\text{def}}{=} \mathbf{T}\mathbf{x}(n) = \mathbf{V}_s^H \mathbf{s}(n). \qquad (4)$$

Note that $\mathbf{V}_s^H$ is a unitary matrix. The above procedure essentially normalizes the received signals. Consequently, in noise-free case

$$\mathbf{z}^H(n)\mathbf{z}(n-1) = \mathbf{s}^H(n)\mathbf{s}(n-1) = r(n). \qquad (5)$$

Equation (5) asserts that by normalizing and differentiating the received signal vectors, the correlation function $r(n)$, which contains the bit information, can be reconstructed from the received signal without the channel information.

The decoding process is outlined as follows,

1. Estimate $\mathbf{R}_x$ from the received data vectors;

2. Perform an eigenvalue decomposition on $\mathbf{R}_x$ to obtain $\mathbf{U}_s$ and Σ_s;

3. Normalize the received data sequence by $\mathbf{z}(n) = \Sigma_s^{-\frac{1}{2}} \mathbf{U}_s^H \mathbf{x}(n)$;

4. Calculate the correlation function of the normalized data vector to obtain $r(n) = \mathbf{z}^H(n)\mathbf{z}(n-1) = \mathbf{s}^H(n)\mathbf{s}(n-1)$;

5. Detect the information bit $b(n)$ from $r(n)$ based on the encoding rules.

Remark 1: The above encoding and decoding schemes can be regarded as generalized differential coding techniques for FIR channel ambiguities - they reduce to regular differential coding when the channels are memoryless. It is important to realize that unlike the scalar ambiguity, the convolutional ambiguity can only be removed when redundancy is available at the receiver, as well-known in the system identification literature.

Remark 2: In contrast with the blind sequence estimation algorithm in [1], which also involves normalization and differentiation, the scheme here avoids the computational demanding trellis searching (with 2^P states). Furthermore, by encoding the information into correlation at transmitter side, one can avoid the error propagation problem in [1]. The current approach offers instantaneous decision on the information bits. Such is extremely important to delay sensitive communications. On the other hand, it is reasonable to expect some performance loss of the new algorithm. Similar loss occurs in standard differential coding relative to coherent demodulation.

3.3. Implementation Issues

Several implementation issues are further addressed in this section.

First of all, the way encoding is performed, information bits are modulated into transmission data vectors of length P ($P = K + L$) to accommodate FIR channels with maximum order L. P is fixed by the transmitter. The order of the channel however, varies in practice at the receiver. One should change the smoothing factor accordingly so that the length of $\mathbf{s}(n)$ in (1) remains a constant. In other words, if the true channel order is $L' \leq L$, the K value must be $P - L'$. This can be easily accomplished by checking the rank condition of $\mathbf{R}_x$.

On the complexity side, the main computational cost at the receiver is the eigenvalue decomposition of $\mathbf{R}_x$. The development of fast subspace decomposition techniques provide computationally efficient, easily parallelizable methods to obtain $\mathbf{T}$ [6]. Recently studies show that subspace decomposition can be implemented adaptively, even when the rank of the matrix varies [7]. Subspace tracking algorithms can be applied to the proposed technique to handle time-varying channels. Due to its differential nature, the decoding scheme here has inherent resistance against environmental variations and error propagation. In addition, it can be shown that similar to the standard differential coding technique, the new method is also insensitive to residual carrier offset, which makes it particularly attractive to wireless communications.

4. SIMULATION

In this section, we provide some computer simulations to compare the performance of our algorithm with Tong's method [1]. For illustration purpose, we choose a channel with length 2, so the state diagram depicted in Figure 2 is used for finite-state differential encoding. At receiver end, we use 4 antennas so no further data smoothing is needed. The

information sequence is binary, and BPSK signals are transmitted into the channels.

We measure the Bit Error Rate(BER) under different signal-to-noise ratio (SNR) defined by

$$SNR = 20 \log_{10} \frac{E\big(\|\mathbf{H}(k)\|\big)}{E\big(\|\mathbf{v}(k)\|\big)} \qquad (6)$$

where, the $\|\cdot\|$ stands for Frobenius norm.

The channels do not vary with time in our simulations, we use 100 symbols to calculate the transform matrix $\mathbf{T}$ in Equation (3) for normalization. Implementation of the proposed algorithm is straightforward. In Figure 3 we show $|r(n)|$ of received signals in Equation (5) under 15dB SNR.

To alleviate the effect of error propagation, we simulate Tong's algorithms in batch mode by collecting every 30 received signals and feeding them into decoding trellis. Note we assume the knowledge of initial state of trellis for each data block. Therefore, even there exists a catastrophic paths, it will not last more than 30 symbols. The performance of two algorithms is shown in Figure 4. It can be seen at low SNR, the proposed algorithm performs better. This is due to the error propagation effect in Tong's algorithm. Our algorithm is more robust under low SNR. When SNR exceeds certain threshold, the successive errors become less possible, the error correcting capability of Viterbi algorithm becomes dominant. Therefore Tong's algorithm outperforms the proposed approach at high SNR values.
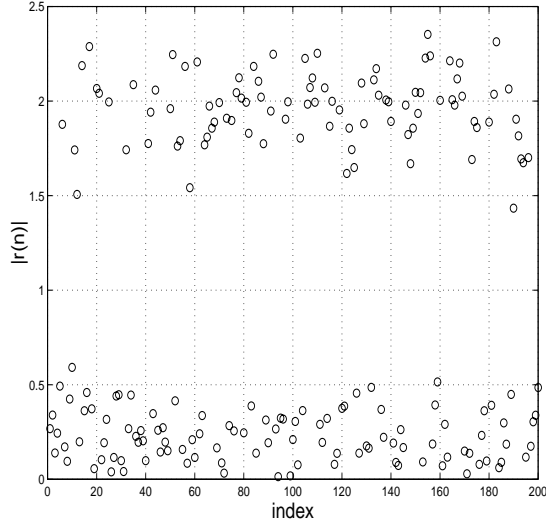

Figure 3: Values of $|r(n)|$

5. CONCLUSION

We presented in this paper a finite-state differential coding scheme that can remove the convolutional ambiguities due to FIR channels. By modulating information bits in differences between successive vector signal transmissions, instantaneous detection can be realized at the receiver through generalized differential decoding without knowledge of the channels. Numerical simulations show promises of the new approach in wireless communications. Further analytical studies are required to better understand the performance of this scheme.
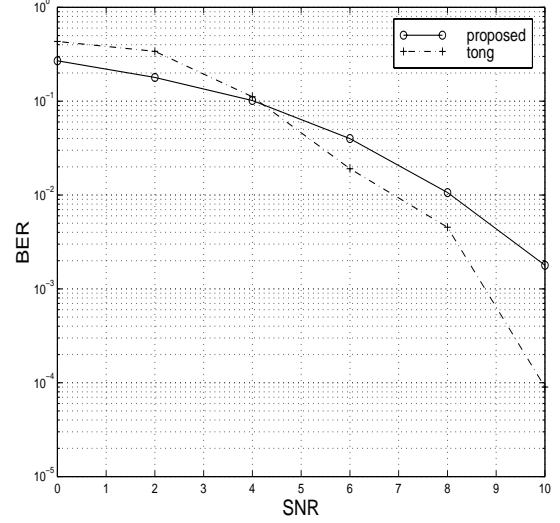

Figure 4: BER vs. SNR

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