BLIND CARRIER SYNCHRONIZATION AND CHANNEL IDENTIFICATION FOR OFDM COMMUNICATIONS

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ABSTRACT

In OFDM communications, the loss of orthogonality due to carrier offset must be compensated before DFT-based demodulation can be performed. In this paper, we present a high accuracy blind carrier offset estimation algorithm and a blind channel equalizer which exploit the intrinsic structure information of OFDM signals. The latter method allows the receiver to perform coherent demodulation in changing environments without the overhead required for additional pilots.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has received increasing attention in wireless broadcasting systems for its ability to mitigate frequency-dependent distortion across the channel bandwidth. Despite its promises, studies have shown that OFDM is very sensitive to inaccurate frequency references [3, 4]. A carrier offset at the receiver can cause loss of subcarrier orthogonality, and thus introduce interchannel interference (ICI) and severely degrade the system performance [3, 4]. High accuracy carrier offset estimation and compensation is of paramount importance in OFDM communications.

The use of differential phase-shift keying (DPSK) in OFDM systems permits the use of noncoherent demodulation. However, this results in a 3 dB signal-to-noise (SNR) penalty [5] which may be significant on some occasions. The receiver could perform coherent demodulation and/or use multilevel signaling schemes if the channel is equalized. This is particularly important when the input signals are coded, i.e., COFDM [6].

Most existing carrier estimation techniques in OFDM rely on the periodic transmission of reference symbols, which inevitably reduces the bandwidth efficiency. Schmidl and Cox [7] introduced a blind approach which is capable of estimating the carrier offset towards its closest subcarrier. However the algorithm requires the constellation on each subcarrier to have points equally spaced in phase. Moreover, the length of the guard interval must be chosen from a subset of allowed values.

This paper first proposes a new solution to the carrier offset estimation problem without using reference symbols. The technique developed here provides a high accuracy carrier estimate by taking advantage of the inherent orthogonality among OFDM subchannels. We show in the following that the virtual carriers of the OFDM signal enable the formulation of a cost function which provides a closed-form estimate of the carrier offset.

Also presented in this paper is a method for blind equalization of the received OFDM signals. Unlike [1], we make use of the cyclic prefix, commonly used in OFDM systems, to construct a blind equalizer. The proposed method is not a filter bank solution, as in [2], and exploits the algebraic structure of the signal which is a function of the channel. This method makes channel estimation in dynamic mobile communication systems computationally affordable.

2. PROBLEM FORMULATION

We begin with the data model of a baseband discrete-time OFDM signal. Denote $\mathbf{s}(k) \stackrel{\text{def}}{=} [s_0(k) \cdots s_{N-1}(k)]^T$ as the kth block of data. OFDM modulation is implemented by applying an IDFT operator to the data stream $\mathbf{s}(k)$. Using matrix representation, the resulting N-point time domain signal is given by

$$\mathbf{x}(k) \stackrel{\text{def}}{=} [x_0(k)\ x_1(k) \cdots x_{N-1}(k)]^T = \mathbf{W}\mathbf{s}(k), \qquad (1)$$

where $\mathbf{W}$ is the IDFT matrix $\mathbf{W}$. For DFT-based OFDM, a cyclic prefix is added to the multiplexed output of the IDFT before it is transmitted through a fading channel [6] as shown in Figure 1. The cyclic extension is chosen to

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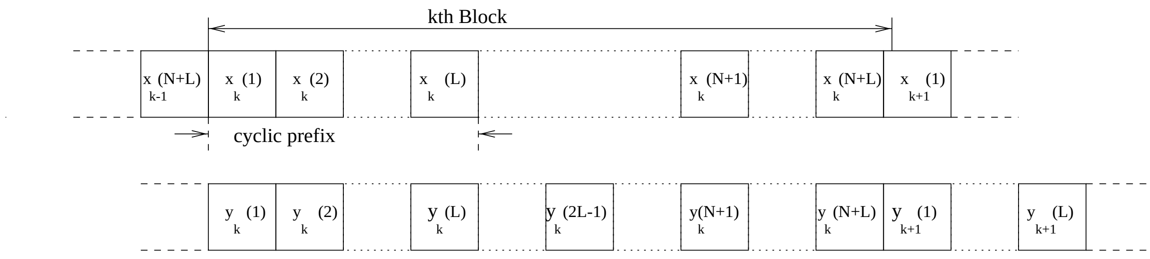

Figure 1: The Transmitted Sequence (above) and the Received Sequence (below)

be longer than the impulse response to avoid interblock interference and preserve orthogonality of the subchannels. Therefore, we assume the channel to have finite impulse response (FIR).

Let $x_n(k) = x((k-1)(N+L)+n)$ and $x(m)$ be the sample sequence to be transmitted. The input to the receiver at the kth block is given by $y(m) = x(m) * h(m)$. The use of a cyclic prefix that is longer than the channel, will transform the linear convolution of the channel to a cyclic convolution after removing the cyclic prefix. We can show for the kth block:

$$\begin{aligned} \mathbf{y}(k) &= [y_0(k) \cdots y_{N+L}(k)]^T = \mathbf{W}\mathbf{H}\,\mathbf{s}(k) \\ &= \mathbf{W}\mathrm{diag}\,(H(0) \cdots H(N-1))\,\mathbf{s}(k), \end{aligned} \tag{2}$$

where $H(i),\ i = 1 \cdots N$ is the frequency response of the channel as obtained by the DFT of $\{h(m)\}$. Clearly, each subchannel can be recovered to a scalar ambiguity by applying a DFT to $\mathbf{y}(k)$:

$$\mathbf{W}^H\mathbf{y}(k) = \mathbf{H}\,[s_0(k) \cdots s_{N-1}(k)]^T. \tag{3}$$

If $\{s_i(k)\}$ are differentially encoded, the information bearing symbols can be perfectly recovered even when the channel information is not available.

The first problem addressed in this paper is the estimation of the carrier offset, ϕ, from the receiver outputs, $\{\mathbf{y}(k)\}$, without the use of a training sequence or known input symbols. The estimation of the carrier offset does not depend on the channel information and offset compensation is necessary before we perform channel identification. The second problem addressed is blind estimation of the equalizer for the channel.

3. BLIND CARRIER OFFSET ESTIMATION

In a practical OFDM system, the number of subchannels that carry information is generally smaller than the size of the DFT block, *i.e.*, $P < N$, for various reasons including oversampling [1] and/or virtual carriers [6], e.g. in the European DAB [8] standard local mode, there are 256 virtual carriers out of a total of 1024 carriers.Without loss of generality, we assume carriers no. 1 to P are used for data transmission and $s_{P+1}(k) = \cdots = s_N(k) = 0$. We further assume $\mathbf{H} = \mathbf{I}$ for notational simplicity. It will become clear that our approach can readily be applied to fading channels.

Let $\mathbf{W}_P = [\mathbf{w}_0 \cdots \mathbf{w}_{P-1}]$, where $\mathbf{w}_i$ is ith the columns of $\mathbf{W}$. Therefore,

$$\begin{aligned} \mathbf{y}(k) &= \mathbf{W}\ [s_0(k) \cdots s_{P-1}(k)\ \ 0 \cdots 0]^T \\ &= \mathbf{W}_P\ [s_0(k) \cdots s_{P-1}(k)]^T. \end{aligned} \tag{4}$$

In the presence of a carrier offset, $e^{j\phi}$, however, the output becomes $\mathbf{y}(k) = \mathbf{E}\mathbf{W}\mathbf{s}(k)e^{j\phi(N+L)}$, where $\mathbf{E} = \mathrm{diag}(1,\ e^{j\phi}, \cdots, e^{j(N-1)\phi})$ and L is the length of the prefix. Since $\mathbf{W}^H\mathbf{E}\mathbf{W} \neq \mathbf{I}$, the $\mathbf{E}$ matrix destroys the orthogonality among the subchannel carriers and thus introduces ICI. To recover $\{\mathbf{s}(k)\}$, the carrier offset, ϕ, needs to be estimated and compensated before performing the DFT. The demodulation can be described as the following concatenated matrix operations:

$$e^{-j\phi(N+L)}\mathbf{W}^H\mathbf{E}^H\mathbf{y}(k) = \mathbf{W}^H\mathbf{E}^H\mathbf{E}\mathbf{W}\mathbf{s}(k) = \mathbf{s}(k).$$

Collecting K sample vectors and arranging them in a matrix form,

$$\mathbf{Y} = [\mathbf{y}(0)\ \mathbf{y}(1) \cdots \mathbf{y}(K-1)], \tag{5}$$

our goal is to derive a high efficiency algorithm to estimate ϕ directly from $\mathbf{Y}$.

Since $\mathbf{W}_P$ consists of a subset of the columns of $\mathbf{W}$, the IDFT matrix, $\mathbf{W}^\perp = [\mathbf{w}_{P+1} \ldots \mathbf{w}_N]$, its orthogonal complement, is known *a prior*. Hence in the absence of the carrier offset, *i.e.*, $\phi = 0$ and for $1 \leq i \leq N-P$,

$$\mathbf{w}_{P+i}^H\mathbf{Y} = \mathbf{w}_{P+i}^H\mathbf{W}_P\,[\mathbf{s}(0) \cdots \mathbf{s}(K-1)]$$

must be zeros. Such is not true when $\phi \neq 0$. However, if we let $\mathbf{Z} = \mathrm{diag}(1,\ z,\ z^2, \cdots, z^{N-1})$, it can easily be shown that

$$\mathbf{w}_{P+i}^H\mathbf{Z}^H\mathbf{Y} = \mathbf{w}_{P+i}^H\mathbf{Z}^H\mathbf{E}\mathbf{W}_P[\mathbf{s}(0) \cdots \mathbf{s}(K-1)],$$

are zeros when $\mathbf{Z} = \mathbf{E}$. This observation suggests that we can combine all possible equations as follows,

$$P(z) = \sum_{i=1}^{L} \mathbf{w}_{p+i}^H\mathbf{Z}^{-1}\mathbf{Y}\mathbf{Y}^H\mathbf{Z}\mathbf{w}_{p+i}, \tag{6}$$

where $L \leq N - P$. Clearly, $P(z)$ is zero when $z = e^{j\phi}$. Therefore one can find the carrier offset by evaluating $P(z)$ at all possible values of $\phi \in [0,\ 2\pi]$, as in the MUSIC algorithm in array signal processing. It is noted that $P(z)$ forms a polynomial of z with order $2(N-1)$. This allows a closed-form estimate of ϕ through root searching. In particular, $e^{j\phi}$ can be identified as the root of $P(z)$ on the unit circle. The proposed algorithm is summarized as follows:

1. Form the polynomial cost function as in (6) using the receiver inputs, $\{y_n(k)\}$.

2. Estimate the carrier offset as the null of $P(z)$ or the phase of the root of $P(z)$ on the unit circle.

3. In the presence of noise, the carrier offset is estimated as the minimum of $P(z)$ or the phase of the root of $P(z)$ closest to the unit circle.

4. BLIND CHANNEL ESTIMATION

Once the carrier offset is compensated, the remaining impairment is the channel effect. We use the structure of the cyclic prefix added OFDM signal to equalize the channel.

Let us consider the transmitted and received signals shown in Figure 1. We assume that the channel has length $K \leq L$ and that there is an ambiguity of up to L samples for the arrival time of the first sample of the OFDM block. We observe $N + 2L$ consecutive samples of $\mathbf{y}(k)$ as shown in Figure 1. Then, we can express the first $2L - 1$ received samples as follows:

$$\mathbf{Y}_h = \begin{bmatrix} y_k(L) & \cdots & y_k(1) \\ \vdots & \ddots & \vdots \\ y_k(2L-1) & \cdots & y_k(L) \end{bmatrix} = \underbrace{\begin{bmatrix} x_k(L) & \cdots & x_1(1) & \cdots & x_{k-1}(N+1) \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ x_k(2L-1) & \cdots & x_k(L) & \cdots & x_k(1) \end{bmatrix}}_{\mathbf{X}_h} \times \underbrace{\begin{bmatrix} h_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ h_L & \ddots & \vdots \\ 0 & \ddots & h_1 \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & h_L \end{bmatrix}}_{\mathbf{H}}, \quad (7)$$

and we can express the last $2L - 1$ samples as follows:

$$\mathbf{Y}_t = \begin{bmatrix} y_k(N+L) & \cdots & y_k(N) \\ \vdots & \ddots & \vdots \\ y_{k+1}(L-1) & \cdots & y_k(N+L) \end{bmatrix} = \underbrace{\begin{bmatrix} x_k(N+L) & \cdots & x_k(N-L+1) \\ \vdots & \ddots & \vdots \\ x_{k+1}(L-1) & \cdots & x_k(N) \end{bmatrix}}_{\mathbf{X}_t} \mathbf{H}. \quad (8)$$

We form the difference, $\mathbf{Y} = \mathbf{Y}_t - \mathbf{Y}_h$:

$$\mathbf{Y} = \mathbf{Y}_t - \mathbf{Y}_h = (\mathbf{X}_t - \mathbf{X}_h)\mathbf{H} \stackrel{\text{def}}{=} \mathbf{XH} \quad (9)$$

Due to the cyclic prefix, $\mathbf{X}_t$ and $\mathbf{X}_h$ share the Lth column. The Lth column of $\mathbf{X}$ is thus $\mathbf{0}$. This very structure can be exploited to form an equalizer below:

$$\mathbf{g} = [g_1 \cdots g_L]^T = \arg\min \|\mathbf{Yg}\|^2. \quad (10)$$

Since only one column of $\mathbf{X}$ is zero, deterministically, the equalizer found will be the solution of:

$$\mathbf{Yg} = \mathbf{XHg} = \mathbf{Xe}_L = \mathbf{0} \quad (11)$$

where $\mathbf{e}_L = [0, \cdots, 0, \alpha, 0, \cdots, 0]$ and α is a scalar. Since the ISI terms are suppressed, $\mathbf{g}$ is effectively an equalizer. Thus, the problem at hand reduces to finding the null space of $\mathbf{Y}$. We can find the null space of $\mathbf{Y}$ by performing SVD on $\mathbf{Y}$. The right eigen vector which corresponds to the minimum eigen value of $\mathbf{Y}$ is the solution to (10).

The proposed algorithm is summarized as follows:

1. Form $\mathbf{Y}_h$ and $\mathbf{Y}_t$ and find $\mathbf{Y}$.

2. Apply SVD on $\mathbf{Y}$ and find the eigenvalues and right eigen vectors $\{\mathbf{v}_i\}$.

3. Estimate $\mathbf{g}$ as the $\mathbf{v}_i$ that corresponds to the smallest eigenvalue.

Note that the solution we get may not give the best performance. In particular, although $\mathbf{Hg} = \mathbf{e}_L$ qualifies $\mathbf{g}$ as an equalizer, the signal strength of the equalizer may be small. If the channel is known *a prior*, we can find the zero-forcing (ZF) equalizer by solving $\mathbf{g}_{ZF} = \mathbf{H}^{\dagger}\mathbf{e}_L$, with α fixed at 1.

For the proposed blind equalizer:

$$\begin{aligned} \mathbf{g}_{BL} &= \text{null}(\mathbf{Y}) = \text{null}(\mathbf{Y}^H\mathbf{Y}) \\ &= \text{null}(\mathbf{H}^H \mathcal{E}\{\mathbf{X}^H\mathbf{X}\}\mathbf{H}) \\ &= \text{null}(\mathbf{H}^H \text{diag}(L-1, \cdots, 0, \cdots, L)\mathbf{H}) \end{aligned} \quad (12)$$

5. PERFORMANCE ANALYSIS AND COMPUTER SIMULATIONS

To demonstrate the performance of the blind equalizer vs other linear equalizers, some computer simulations have been conducted. We simulated a multipath channel $h(z) = 0.1 - j0.005 + z^{-1} + (-0.51 - j0.32)z^{-2} + (0.17 - j0.09)$. The MSEs of the carrier offset, ϕ, estimates were obtained from the null spectrum by running 500 independent realizations and varying SNR from 0 to 30 dB. In Figure 2, we present the MSEs of carrier offset estimates for a system with $N = 128, P = 68, L = 5$, which is typical of OFDM with a large number of carriers and virtual channels.

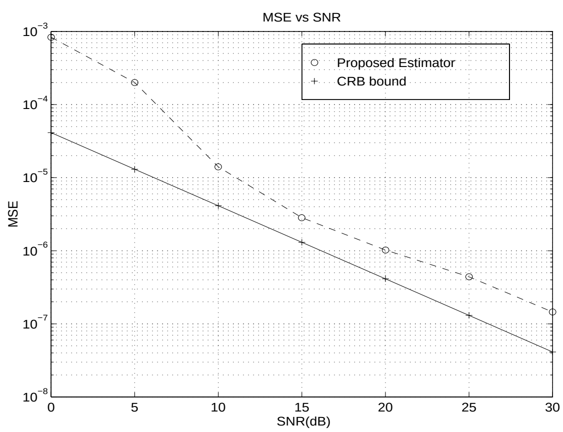

Figure 2: Carrier Offset Estimates' MSE vs SNR

The blind equalizer was used to obtain estimates of the transmitted symbols. For performance comparison, we also equalized the same data set using ZF and MMSE linear equalizers. The mean-square-error at the output of the equalizers is presented in Figure 3.

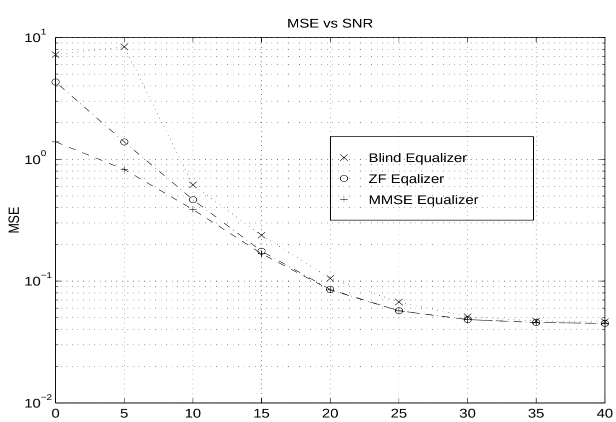

Figure 3: Input Sequence Estimates' MSE vs SNR

6. CONCLUSIONS

Carrier compensation is mandatory for the demodulation of OFDM signals. We have presented a low complexity/high performance carrier estimator to achieve high performance without requiring training sequences. For coherent demodulation and/or decoding coded OFDM signals, we have proposed a blind channel equalizer that performs very well even for a small number of blocks and can track a fast changing channel with affordable complexity.

7. REFERENCES

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