BROADBAND BEAMFORMING USING ELEMENTARY SHAPE INVARIANT BEAMPATTERNS

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ABSTRACT

This paper presents a new method of designing a beamformer having a desired broadband beampattern with focusing capability to operate at any radial distance from the array origin. An important consequence of our result is that the beamformer processing can be factored in to three levels of filtering: (i) beampattern independent elementary beamformers; (ii) beampattern shape dependent filters; and (iii) radial zooming filters where a single parameter can be adjusted to zoom-focus the array to a desired radial distance from the array origin. As an illustration the method is applied to the problem of producing a practical array design that achieves a frequency invariant beampattern over the frequency range of 1:10.

1. INTRODUCTION

Consider the problem of designing a microphone array for speech acquisition. Not only does the array requires a narrow main beam, but it should operate uniformly over a large bandwidth and be able to cope with nearfield sources. Whilst there has been a deal of progress in designing broadband arrays, having them operate well in the nearfield is a challenge. In this paper, we will present a systematic way of designing nearfield broadband sensor arrays. In particular, we will explicitly show how to parameterize the filter coefficients inorder to be able to focus the array to practically any operating radius from the array origin whilst maintaining a predetermined broadband angular specification.

Most of the array processing literature assumes a farfield source having only plane waves impinging on the sensor array. However in many practical situations such as microphone arrays in car environments [1], the source is well within the nearfield. The use of farfield assumptions to design the beamformer in these situations can severely degrade the beampattern.

There is little work in the literature on nearfield beamforming. In [2], time delays were applied to compensate for differing propagation delays due to spherical propagation. However this is only an approximation and ignores the variation of the magnitude with distance and angle and assumes a point source. In [3] there was consideration initially for nearfield theoretical development but this was ignored in the actual array design. A novel technique useful in array design has been recently developed [4]. It is based on writing the solution to the wave equation in terms of spherical harmonics and allows a nearfield beampattern specification to be transformed to the farfield. Well understood farfield designs can then be used to design the nearfield beamformer. Farfield broadband beamforming has been reviewed in [5].

2. THEORY OF ELEMENTARY SHAPE INVARIANT BEAMPATTERNS

In [4], the nearfield-farfield transformation is obtained by solving the physical problem governed by the classical wave equation in the spherical co-ordinate system. Let r denote radial distance, ϕ and θ be the azimuth and elevation angles in our spherical coordinate system. Then a general solution of the wave equation for engineering applications can be constructed by combination of all possible modes of the form

$$E_n^m(r,\theta,\phi;k) \triangleq r^{\frac{1}{2}}\, H^{(1)}_{n+\frac{1}{2}}(kr)\, P_n^{|m|}(\cos\theta)\, e^{jm\phi}, \qquad (1)$$

where integers $n \geq 0$ and m $(|m| \leq n)$ index the modes, $k = \frac{2\pi f}{c}$ is the wave number, f is the frequency of the wave and c is the speed of wave propagation. The functions $P_n^m(\cdot)$ are associated Legendre functions and $H^{(1)}_{n+\frac{1}{2}}(\cdot)$ is the half odd integer order Hankel function of the first kind. We assume that the propagation speed c is independent of frequency, implying k is a constant multiple of frequency f and throughout this paper we will often refer to k as "frequency". By combining modes for all possible n and m the general solution to the wave equation in the *beampattern* form is given by

$$b_r(\theta,\phi;k) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \alpha_n^m(k), E_n^m(r,\theta,\phi;k) \qquad (2)$$

where $\{\alpha_n^m(k)\}$ is a set of frequency dependent coefficients. From this point onwards, we refer to (2) as the modal representation of beampatterns. It has been shown [4] that the $\alpha_n^m(k)$ coefficients can be obtain from the *analysis* equation:

$$\alpha_n^m(k) = \frac{(n+\frac{1}{2})}{2\pi r^{\frac{1}{2}}\, H^{(1)}_{n+\frac{1}{2}}(kr)} \frac{(n-m)!}{(n+m)!} \int_0^{2\pi}\int_0^{\pi} b_r(\theta,\phi;k) P_n^{|m|}(\cos\theta) e^{-jm\phi} \sin\theta d\theta d\phi. \qquad (3)$$

Since we can invert the representation (2) via (3) we conclude that the $\alpha_n^m(k)$ uniquely represent an arbitrary beampattern.

Using the analysis equation (3), any arbitrary beampattern can be decomposed in to modes which are characterized by the coefficients $\alpha_n^m(k)$. These modes $E_n^m(r,\theta,\phi;k)$ can be factored into two parts:

$$E_n^m(r,\theta,\phi;k) = R_n(r,k)\, \epsilon_n^m(\theta,\phi), \qquad (4)$$

where, $R_n(r,k) = r^{\frac{1}{2}} H^{(1)}_{n+\frac{1}{2}}(kr)$ and $\epsilon_n^m(\theta,\phi) = P_n^{|m|}(\cos\theta)e^{jm\phi}$. The quantity $\epsilon_n^m(\theta,\phi)$ can be considered as an elementary beam shape and $R_n(r,k)$ is a complex function parameterized by r and k. Therefore, the shape of the beampattern is invariant with frequency as well as with distance. Hence we call these beampatterns Elementary Shape Invariant Beampatterns (ESIB).

3. BROADBAND CONTINUOUS SENSOR DESIGN

3.1. Elementary Continuous Sensors

In previous sections we have developed a new method for decomposing a given beampattern into elementary shape invariant beampatterns (ESIBs). As an engineering problem, now we will develop relevant theory to physically realize these ESIBs using an array of sensors. Initially we will begin with the concept of a *continuous sensor*, so that an exact relationship between ESIB and aperture illumination can be developed. Then the illumination function of the continuous sensor will be approximated by a discrete sensor array to permit implementation.

In order to be able to completely describe results for different array configurations we introduce the notation $\rho(\vec{x},k)$ for the broadband aperture illumination or the response of the aperture at a point $\vec{x}$ and for a frequency k. Then the response of a continuous sensor to planar waves (i.e., generating from a farfield point source) impinging from an angle (θ,ϕ) is,

$$b_\infty(\vec{u},k) = \int_{-\infty}^{\infty} \rho(\vec{x},k)e^{jk\vec{u}\cdot\vec{x}}d\vec{x}, \tag{5}$$

where the vector $\vec{u}$ depends on the dimension of the sensor and its orientation. For example, in the case of a one dimensional array aligned to z axis, $\vec{u} = \cos\theta$ and $\vec{x} = z$. Equation (5) is a Fourier transform relating the farfield beampatterns and the aperture illumination for a fixed frequency k. The inverse Fourier transform corresponding to (5) is given by

$$\rho(\vec{x},k) = (\frac{k}{2\pi})^D \int_{-\infty}^{\infty} b_\infty(\vec{u},k)e^{-jk\vec{u}\cdot\vec{x}}d\vec{u} \tag{6}$$

where D is the dimension of the array. However, the beampattern $b_\infty(\vec{u},k)$ is defined only for $\theta \in [0,\pi]$ and $\phi \in [0,2\pi]$, and so the integration in (6) must have finite limits. Let $\vec{u} \in (\vec{u_1},\vec{u_2})$ be the range of $\vec{u}$ corresponding to the physical range of the angles. In order to establish an exact relationship between the ESIBs and the aperture illumination function, we write an arbitrary farfield beampattern in the modal representation (2) as

$$b_\infty(\vec{u},k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \alpha_n^m(k)\, E_n^m(\infty,\theta,\phi;k), \tag{7}$$

where $\{\alpha_n^m(k)\}$ are the decomposition weights. Substituting (7) in to (6) and rearranging, we obtain the desired relationship as

$$\rho(\vec{x},k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \alpha_n^m(k)\, R_n(\infty,k)\varrho_n^m(\vec{x},k) \tag{8}$$

where

$$\varrho_n^m(\vec{x},k) \triangleq (\frac{k}{2\pi})^D \int_{\vec{u}_1}^{\vec{u}_2} \epsilon_n^m(\theta,\phi)\, e^{-jk\vec{u}\cdot\vec{x}}\, d\vec{u}. \tag{9}$$

We can consider $\varrho_n^m(\vec{x},k)$ as the elementary aperture illumination functions corresponding to each $\epsilon_n^m(\theta,\phi)$. Note that these elementary aperture functions are independent of the specific beampattern and they can be calculated beforehand in a practical situation.

The weights $\alpha_n^m(k)$ have two interpretations: (i) they decompose the beampatterns (7) to a weighted sum of ESIBs, and (ii) they construct the aperture illumination (8) as a weighted sum of elementary aperture illumination functions.

3.2. Nearfield Equivalence

In this section, we generalize the above result for broadband beampatterns at any radial distance from the array origin using the nearfield-farfield transformation technique [4].

Theorem 1 *Let $c(\theta,\phi;k)$ be an arbitrary broadband beampattern specification. Then the aperture illumination, $\rho^{(r)}(\vec{x},k)$ of a continuous sensor which realizes this beampattern at a radius r from the sensor origin is given by*

$$\rho^{(r)}(\vec{x},k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \alpha_n^m(k)\, \frac{[R_n(\infty,k)]^2}{R_n(r,k)}\, \varrho_n^m(\vec{x},k) \tag{10}$$

where $\alpha_n^m(k)$ are the coefficients of the modal equation representation of the given beampattern $c(\theta,\phi;k)$ specified in the farfield.

The proof is given in [6].

3.3. One dimensional Sensor

The broadband array theory developed in the previous section is sufficiently general to capture quite arbitrary three dimensional sensor geometries. In an attempt to bring the result into focus and provide a more concrete presentation of the ideas we examine a linear sensor aligned with z axis. In this case, the beampattern is rotationally symmetric with respect to ϕ, and a farfield beampattern can be expressed as $b_\infty(\theta,\phi;k) = b_\infty(\theta;k)$.

The only non-zero components are those for which $m = 0$, which leads to the following simplified set of equations:

$$b_\infty(\theta;k) = \sum_{n=0}^{\infty} \alpha_n(k)\, R_n(\infty,k)\, P_n^0(\cos\theta). \tag{11}$$

and

$$\rho^{(r)}(z,k) = \sum_{n=0}^{\infty} \alpha_n(k)\, \frac{[R_n(\infty,k)]^2}{R_n(r,k)}\, \varrho_n^0(z,k) \tag{12}$$

where $\alpha_n(k) \triangleq \alpha_n^0(k)$ and $\varrho_n^0(z,k)$ are the elementary aperture functions and $\rho^{(r)}(z,k)$ is the aperture illumination which will realize the desired response at a radius r from the array origin. By evaluating the integral in (9) for this case, we obtain a closed form expression for the elementary aperture functions for a linear sensor aligned with the z axis as:

$$\varrho_n^0(z,k) = \frac{k}{\sqrt{2\pi}}\,(-j)^n\, \frac{J_{n+\frac{1}{2}}(kz)}{\sqrt{kz}}. \tag{13}$$

4. BROADBAND DISCRETE ARRAY DESIGN

4.1. Approximation

Having developed the theory of a general broadband continuous sensor in terms of elementary aperture functions, we will now describe the implementation of a broadband array. From this point onward, we only consider one dimensional sensor arrays, however, the results can be generalized to higher dimensions. We consider a double sided array aligned to the z axis.

The continuous aperture distribution described by (12) is not practical for beamforming with finite number of point sensors. The problem of obtaining a desired broadband response using a discrete set of sensor locations reduces to a numerical approximation of the following integral representation, which gives the output frequency response of the ideal continuous sensor for an arbitrary signal $S(z,k)$ at frequency k impinging on the array at position z

$$Y(k) = \int_{-\infty}^{\infty} \rho^{(r)}(z,k)\, S(z,k)\, dz. \tag{14}$$

We follow the approach introduced in [5] to approximate (14) by

$$\tilde{Y}(k) = \sum_{i=-L}^{L} g_i\, \rho^{(r)}(z_i,k)\, S(z_i,k), \tag{15}$$

where $\{z_i\}_{i=-L}^{L}$ is a set of $2L+1$ discrete sensor locations and g_i is a spatial weighting term which is used to account for the (possibly) nonuniformly spaced sensor locations.

4.2. Beamformer Structure

We can consider $\rho^{(r)}(z_i,k)$ in (15) as the frequency response of a filter attached to the sensor at point z_i. By combining (12) and (15) we write,

$$\tilde{Y}(k) = \sum_{i=-L}^{L} g_i\, S(z_i,k) \sum_{n=0}^{\infty} \alpha_n(k)\, G_n(r,k)\, F_n^0(z_i,k), \tag{16}$$

where

$$F_n^m(z_i,k) \triangleq \frac{\sqrt{2\pi}}{k}\, \varrho_n^m(z_i,k), \tag{17}$$

$$G_n(r,k) \triangleq \frac{k}{\sqrt{2\pi}}\, \frac{[R_n(\infty,k)]^2}{R_n(r,k)}. \tag{18}$$

The filters $F_n^0(z_i,k)$ depend on the elementary beam shapes and the position of the sensors. They are given by

$$F_n^0(z_i,k) = (-j)^n\, \frac{J_{n+\frac{1}{2}}(kz_i)}{\sqrt{kz_i}}, \tag{19}$$

where $J_{n+\frac{1}{2}}(\cdot)$ is the half odd integer order Bessel function. We will call $F_n^m(z_i,k)$ the elementary filters. As in the case of ESIBs, these elementary filters are same for all beamformers, thus they may be useful in developing effective parameterization for adaptation of beampatterns. We now demonstrate an important result regarding the elementary filters as a consequence of (19). Note that in (19), $F_n^0(z_i,k)$ is a symmetric function of spatial variable z_i and of the frequency variable k. Thus, these elementary filters have a *dilation property*. We state this property as a theorem.

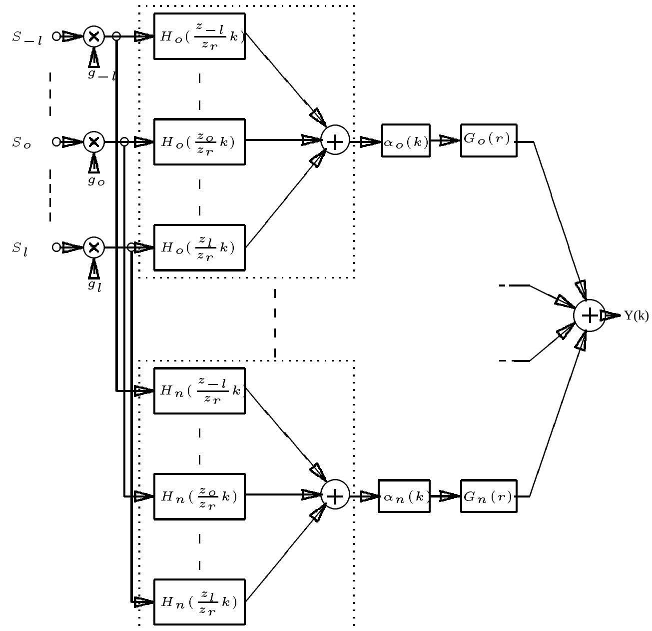


Figure 1: Block diagram of a general one-dimensional broadband beamformer

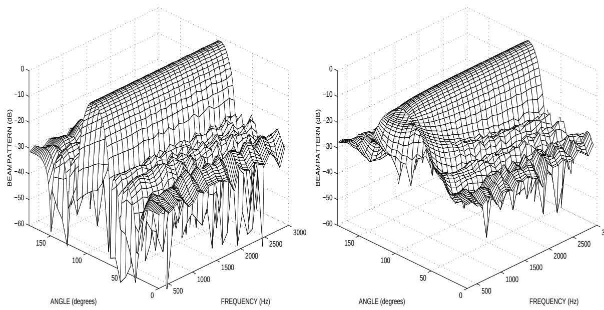
Theorem 2 *All elementary filter responses $F_n^0(z_i,\cdot)$ of the same mode n at different sensor locations z_i are identical up to a frequency dilation.*

With the double-sided one dimensional broadband array as defined in (16), we are led to a block diagram as shown in Fig. 1.

The proposed general beamformer has three levels of filtering associated with it. The first level consists of elementary beamformers, which are shown inside the dashed-line boxes in the Fig. 1. Each of the elementary beamformers consists of elementary filters of the same mode which are connected to different sensors but are related by the dilation property. As a consequence, we have a set of unique beamformers for each and every mode n. The characteristic coefficients $\alpha_n(k)$ form the second level of filtering. Since the $\alpha_n(k)$ determine the shape of the beampattern, we call them *Beam Shape Filters*. The final set of filters $G_n(r,k)$ are independent of sensor locations but dependent on the operating radius r and the mode, and from (18) can be simplified to

$$G_n(r,k) = \frac{(-1)^{n+1}}{\pi} \sqrt{\frac{2}{\pi r}}\, \frac{e^{j2(k-k_0)}}{H^{(1)}_{n+\frac{1}{2}}(kr)}, \tag{20}$$

where k_0 is an arbitrary chosen nominal frequency. By adjusting the parameter r in $G_n(r,k)$, the beamformer can be zoomed to a particular operating radius r. To highlight this important property we call the filters $G_n(r,k)$ *Radial Zooming Filters*. Our general beamformer structure has three interesting properties: (i) the elementary beamformers are same for all beamformers, (ii) beam shape filters control the shape of the beampattern and (iii) radial zooming filters zoom-focus the beamformer to the desired operating radius. Because of these properties, our design is readily convertible to adaptive implementations, where only the beam shape filters and radial zooming filters need to be adapted.



(a) farfield response

(b) nearfield response at a radius $3\lambda_l$

Figure 2: Response of the farfield focussed 25dB Chebyshev beamformer over 300-3000Hz

4.2.1. Frequency Invariant Beamforming

In this section, we consider the design of frequency invariant beamformers as a special case of the general beamforming theory developed above. An arbitrary beampattern over an arbitrary bandwidth can be expressed (in the farfield) by (11). It can be easily seen from (11) that if there is a sequence $\{\beta_n\}$ of mode dependent constants such that

$$\alpha_n(k) = \frac{\beta_n}{R_n(\infty, k)},$$

for a range of frequencies $k \in [k_l, k_u] \subset (0, \infty)$, then the beampattern is frequency invariant over $k \in [k_l, k_u]$. This simplifies the general beamformer structure in Fig. 1, and in particular the product $G_n(r,k)\,\alpha_n(k)$ appearing in (16) becomes

$$G_n(r,k)\,\alpha_n(k) = \begin{cases} \beta_n \frac{k}{\sqrt{2\pi}} & \text{for farfield} \\ \beta_n \frac{(-j)^{n+1}}{\pi} \sqrt{\frac{k}{r}} \, \frac{e^{j(k-k_0)}}{H^{(1)}_{n+\frac{1}{2}}(kr)} & \text{for nearfield} \end{cases} \tag{21}$$

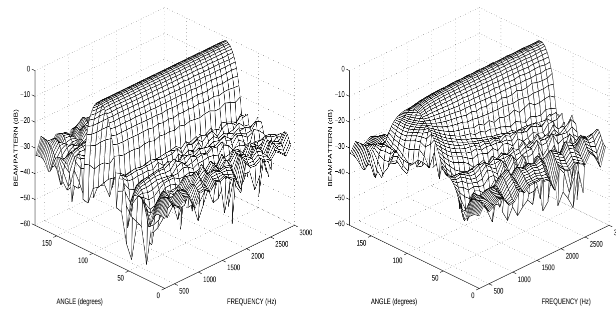
Here, to determine β_n, we only need to calculate $\alpha_n(k)$ for a nominal frequency $k_0 \in [k_l, k_u]$.

5. DESIGN EXAMPLE

Suppose we wish to design a broadband beamformer having the desired design frequency range 300-3000Hz, which is suitable for speech applications. Let us limit the number of modes N to be 15. Thus we assume all beampatterns of our interest can be approximately decomposed to 15 ESIBs. We have designed the first 15 elementary filters according to (19). A double sided array of 41 non-uniformly spaced sensors are located according to [6].

Now we consider an example beampattern which is for a beamformer having a constant Chebyshev 25dB beampattern over the desired frequency range. The example chosen is a frequency invariant beampattern, although we stress that our design method is not restricted to frequency invariant beamformers. The frequency response of the combined filters $G_n(r,k)\alpha_n(k)$ are given by (21).

The resulting beamformer is zoomed to the farfield by setting the parameter $r = 100\lambda_l$ in the zooming filter $G_n(r,k)$ (Here $\lambda_l = \frac{2\pi}{k_l}$). The response of the beamformer to a farfield source is given in Fig. 2(a), which is close to the desired response. The response of the same farfield focussed beamformer to a nearfield source at a radius $3\lambda_l$ is given Fig. 2(b). It is evident from this figure that the farfield assumption severely degrades the nearfield performance. Next we zoom-focus the same beamformer to the nearfield by adjusting the variable r in the zooming filter $G_n(r,k)$ to $3\lambda_l$. The resulting beamformer is simulated in the nearfield and we observe an improved response in the Fig. 3(a). For completeness, we find the response of nearfield zoomed beamformer to a farfield source and show this in Fig. 3(b). This demonstrates that generally a beamformer designed to operate in farfield does not perform accordingly in the nearfield, and a nearfield beamformer will not produce the desired farfield response. However, in our design, a single parameter can be adjusted to zoom focus a beamformer to a desired operating radial distance from the array origin.



(a) Nearfield Response at $3\lambda_l$

(b) farfield Response

Figure 3: Response of the 25dB Chebyshev beamformer which is focussed to the nearfield at $r = 3\lambda_l$

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