MODELING OF ACTIVE REVERBERATION BY TIME DELAY ESTIMATION

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ABSTRACT

This paper explores the statistical properties of underwater reverberation present in active sonar systems. The interference to signal processing which results from reverberation can be extensive, and is particularly acute when the boundaries (surface, bottom) of the water column are nearby. Of particular interest are situations where there may be weak targets masked by reverberation dominating the returning signal. The reverberation will be represented as the output of a linear system with the transmitted signal as an input. The random nature of the reverberation will be accounted for by using random parameters in the linear model, the most important of which are those parameters impacting the spatial distribution of the reverberation. Time delay estimation will be used to analyze reverberant signals obtained from a sonar system operating in a shallow water environment. The statistics of the linear models obtained from these analyses will be computed and discussed.

1. DISCUSSION

In this work, an analysis of active sonar data collected in a shallow water environment will be presented. The examples taken will be from data collected monostaticly by a high frequency array. Due to the proximity of the surface and bottom in this type of environment, these data are heavily corrupted by unwanted backscattering from the boundaries. The ultimate goal of this work is to recover weak signals due to targets from returns dominated by these reverberant fields. To this end, it is necessary to model the 'channel' realized by the environment, viewing the reverberation as having been generated by the linear system,

$$x_r[n] = c_r[n] * s[n] \; , \tag{1}$$

where $c_r[n]$ is the reverberant channel, $s[n]$ is the transmitted signal, and $x_r[n]$ is the return due to reverberation. This approach is in the spirit of the models proposed by Middleton [1, 2], which describe reverberation as the linear sum of multiple simple scattering processes. It is convenient, therefore, to model the target return $x_t[n]$ similarly, giving the system

$$x[n] = (c_t[n] + c_r[n]) * s[n] + m_n[n] \; , \tag{2}$$

as a data model. In Equation 2, c_t is the linear model for the scattering due to the target, if present. The measurement noise, m_n, will be assumed to be complex Gaussian, i.e.

$$m_n \sim \mathcal{N}\left(0, (\sigma^2/2)I\right) + j\mathcal{N}\left(0, (\sigma^2/2)I\right) \; . \tag{3}$$

1.1. Time Delay Estimation

To process these signals, the approach has been taken to model the linear systems rather than process the data by matched filters. Many sorts of matched filters could be attempted. For example, filters matched either to $s[n]$ (traditional matched filter) or to $c_t[n] * s[n]$ (matched target filter) may be tried. If $c_t[n]$ is extended to include the effects of multipath and propagation, the matched target filter becomes a matched field processor. The linear processes are modeled by performing time delay estimation on the data record $x[n]$. For a basic explanation of time delay estimation, see reference [3]. This work has also been extended to account for the effects of biasing in the presence of very closely spaced signal components [4].

This approach is a fundamental departure from much of the current work, and is summarized (in the frequency domain) in Table 1. Note that the traditional matched filter, T_{mf}, will generate a test statistic which is the autoambiguity function of the transmit convolved with the composite target and reverberation channel. The matched target filter, T_{mt}, will be similar to the standard matched filter, including an additional convolution due to the target model. The third statistic, T_{tde}, which is due to time delay estimation, will generate an estimate of the underlying linear models $c_t[n]$ and $c_r[n]$. This is because

$$t_{tde}[n] \approx \mathcal{FT}^{-1}\left\{C_t + C_r + M_n/S\right\} \; , \tag{4}$$

where the approximation indicates that the form of the channel model may be restricted for either the target or the reverberation. In addition, the spectral division implied by

M_n/S requires that processing only be attempted in frequency bands occupied by the transmission S.

The restriction imposed on the channel model by time delay estimation is that the channel be impulsive. This means that the signals generated by the reverberant process are linear sums of time delayed and scaled transmits. An impulsive channel is modeled as follows,

$$c[n] = \sum_{i=1}^{L} a_i \delta[n - \tau_i] \,, \tag{5}$$

where a mixed notation has been used to represent the sampled signal $c[n]$. By including a continuous time variable, τ_i, in the discrete time index of $\delta[n]$, it is implied that $\delta[n]$ is the sampled signal resulting from the delay of the underlying continuous time signal. This may be accomplished by interpolation and decimation, or by application of sampling theory, which is presented in the next section.

Processor	Test Statistic
matched filter (transmit)	$T_{mf} = \{(C_t + C_r)S + M_n\}S^*$
matched filter (target)	$T_{mt} = \{(C_t + C_r)S + M_n\}C_t^* S^*$
time delay estimation	$T_{tde} \approx \{(C_t + C_r)S + M_n\}/S$

Table 1: Summary of Detection Processors

1.2. Sampled Signals Delayed by Arbitrary Continuous Time

Convolution of the transmitted signal $s[n]$ with $\delta[n-\tau_i]$ will yield the signal $x[n] = s[n - \tau_i]$. It can be shown that this signal can be represented by

$$s[n - \tau_i] = \frac{1}{N} \sum_{k=0}^{N-1} S_k e^{+j2\pi(n\frac{k}{N} - f_k \tau_i)} \,, \tag{6}$$

where S_k is the Fourier series expansion coefficient from the k^{th} frequency bin of $s[n]$. The frequency f_k is evaluated at the analysis frequency indicated by the continuous time signal,

$$\begin{aligned} f_k &= \tfrac{f_s}{D} * \tfrac{k}{N} + f_c \,, & 0 \leq k \leq \tfrac{N}{2} - 1 \\ f_k &= \tfrac{f_s}{D} * (\tfrac{k}{N} - 1) + f_c \,, & \tfrac{N}{2} \leq k \leq N - 1 \,, \end{aligned} \tag{7}$$

where f_s is the sampling frequency, f_c is the center frequency of the system, and D is the decimation rate. These equations provide the fundamental result required to perform time delay estimation. Most importantly, time delays are not restricted to integer multiples of the sampling interval. This implies that resolution of very closely spaced signal components may be possible.

2. DATA ANALYSIS

A series of active transmissions made in a shallow water channel of approximately 120 feet of water have been analyzed. This is high frequency data, generated from a cw transmit of approximately 10 milliseconds. The array data has been beamformed in a direction containing a target, and a portion of the beam selected which is dominated by reverberation. These data records were analyzed by time delay estimation to recover the impulsive channel model. Impulses in the model near the target have been manually removed to restrict attention to the reverberant portion of the signals.

2.1. Reverberation Model

A model of the reverberation was sought to perform system design. Such a model would have the advantages that different sonar transmissions could be simulated without reverting to an in water test, and that independent trials could be generated. To develop this model, each component (impulse) in the estimated channel is interpreted as the backscattering from a simple point target. Next, the sonar equation is used to calculate the equivalent target strength TS of each scatterer, given the amplitude and delay (range) estimates from the time delay estimation.

$$RL = XL - TL + TS \tag{8}$$

The component amplitude is related to the receive level RL by $a_i = 10^{(RL)/20}$. The transmission loss for this system is given by $TL = 2 * (\alpha \; 20\log_{10} r + .004 \; r)$, where r is the range to the component in yards. XL is the transmit level. The variable α is included in the transmission loss to modify the spreading law for components from time delay estimation. All quantities in equation 8 are in dB.

Figure 1 gives a composite picture of the locations and strengths of the reverberation components for this data set. Each point represents the estimate of one impulse in the channel model $c_r[n]$. The delay is expressed as a range along the x axis, and the amplitude is expressed in dB along the y axis. The upper figure shows the result for a spherical spreading law ($\alpha = 1$). This had to be modified to a value of $\alpha = .1244$ for the components generated by time delay estimation, and is shown in the lower figure. This modification is necessary because, as range increases, the components from the time delay estimation span larger numbers of actual scatterers and have artificially larger amplitudes.

Figure 2 shows the distribution of the amplitudes of the reverberation components. The amplitudes are approximately Gaussian in the dB domain, implying the log-normal distribution

$$p_a(a) = \frac{1}{2\pi\sigma_a a} e^{-\frac{1}{2\sigma_a^2}(\ln a - \mu_a)^2} \,. \tag{9}$$

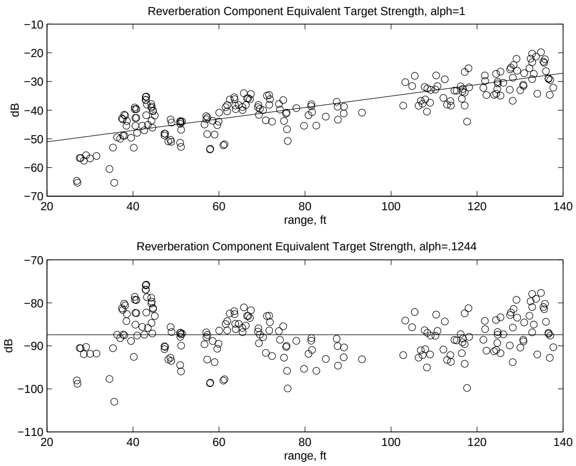

Figure 1: Aggregate of Delay Locations and Equivalent Target Strength for Eight Active Transmissions. A modified spreading law, $\alpha = .1244$, must be used to generate components with an equivalent target strength of -87.42 dB.

In Equation 9, $\sigma_a = \ln 10/20\sigma_{dB}$ and $\mu_a = \ln 10/20\mu_{dB}$, where μ_{dB} and σ_{dB} are derived from the distribution of the amplitudes in the dB domain.

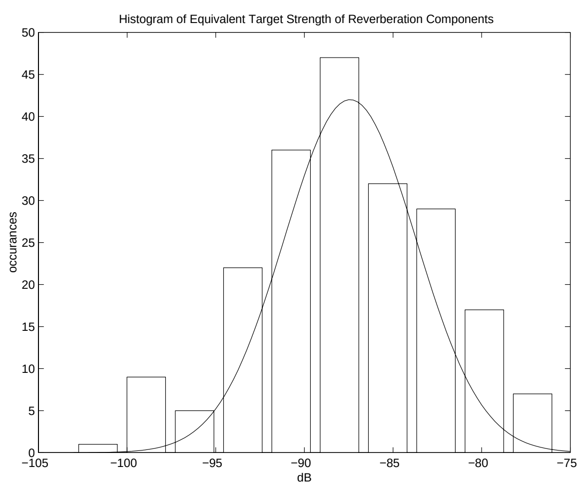

Figure 2: Histogram of Equivalent Target Strength for Reverberation Components. Amplitude is Normal when represented in dB, implying a Log-Normal distribution.

Figure 3 shows the statistics for the spatial distribution of the reverberation components. The upper figure shows a histogram of the reverberation components' delays, compiled for the eight pings in this experiment. This figure shows the delay locations to be approximately uniform over the processing interval. Delays corresponding to a target have been removed at approximately 100 feet, leaving that bin nearly empty. The concentration of components near 40 feet may be due to the specular returns from the bottom or surface, and would not be present in a data record further out in range. The bottom figure gives the spatial correlation of the reverberation, again using the delay locations from the time delay estimations from the eight pings. This figure is striking in that any structure to the spatial correlation is 20 dB down from the zero lag point, indicating that the reverberation is strongly (spatially) white.

These analyses characterize the statistics of the reverberation components obtained from time delay estimation. Since the ultimate interest is in using this information to enhance target detection, it is necessary to derive the properties of the random process implied by these statistics. Once that is understood, stratagem can be developed to process data corrupted by strong reverberation.

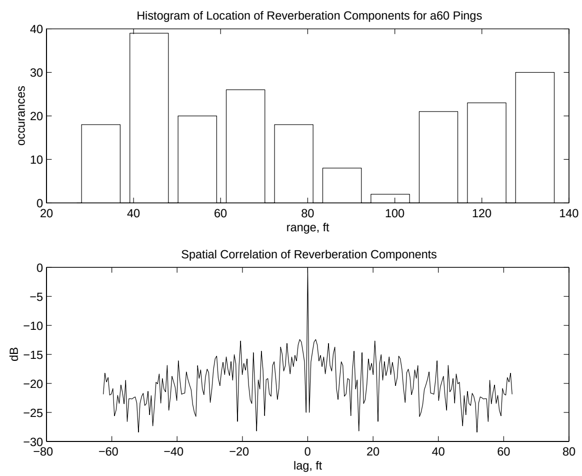

Figure 3: Correlation and Histogram of Spatial Data (delays) from Time Delay Estimation. Target has been removed at approximately 100 ft, data is spatially white and uncorrelated.

2.2. Statistics of the Random Process due to Reverberation

To perform detection processing of a data record, the statistics of the vector of (complex) samples realized by the random process are required. The first two moments of this process will be a mean vector μ, which is the same length N as our data vector x, and a covariance matrix $R = [r_{p,q}]$ which is N by N. To evaluate the statistics of this random process, the moments of the random process containing just one component will be calculated. Since the reverberation is simply the sum of L independent and identically distributed components, the statistics of the random process follow by simple induction. Recall that the reverberation is modeled as L replicas of the transmit delayed by *random* delays, each scaled by *random* amplitudes.

The mean of a delayed signal, when the delay is a uniformly distributed random variable ($p(\tau) \sim \mathsf{U}[\tau_0 - \Delta\tau/2, \tau_0 + \Delta\tau/2]$), is:

$$\mathsf{E}\{s[n-\tau]\} = \frac{1}{N} \sum_{k=0}^{N-1} S_k e^{j2\pi(n\frac{k}{N} - f_k\tau_0)} \frac{\sin 2\pi f_k \Delta\tau}{2\pi f_k \Delta\tau} \; . \tag{10}$$

For single sideband systems with a high carrier frequency, $2\pi f_k \Delta\tau \gg 1$, and the mean value for the process is $\mathsf{E}\{s[n-\tau]\} = 0$. Under these conditions, the covariance matrix, $[r_{p,q}] = \mathsf{E}\{s[n-\tau]s^h[n-\tau]\}$, is given by

$$r_{p,q} = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} \{S_k S_l^* e^{j2\pi(p\frac{k}{N} - q\frac{l}{N})} * \int_{-\infty}^{+\infty} p(\tau) e^{-j2\pi(f_k - f_l)\tau} d\tau\} \; . \tag{11}$$

Furthermore, if the location of a reverberation component is equally likely anywhere in the processing interval, $p(\tau) \sim \mathsf{U}[0, N]/F_s$, and

$$r_{p,q} = \frac{1}{N^2} \sum_{k=0}^{N-1} |S_k|^2 e^{j2\pi(p-q)\frac{k}{N}} \; . \tag{12}$$

There are two basic cases which help interpret this result. Recall the data model given in equations 2 and 5 and the form of the traditional (whitened) matched filter,

$$t_{mf} = \frac{s^h R^{-1} x}{|s^h R^{-1} s|^{1/2}} \; . \tag{13}$$

The first is the wideband case. Assuming that the transmit signal has approximately uniform energy in the processing band, the covariance matrix for L components becomes $[r_{p,q}] = (L * \mathsf{E}\{|a|^2\}) \frac{1}{N} I$. The inverse of the covariance matrix, including the measurement noise is

$$R^{-1} = (\frac{1}{N}(L * \mathsf{E}\{|a|^2\}) + \sigma^2)^{-1} \; I \; . \tag{14}$$

Since this is simply a scaled identity matrix, the implication is that the traditional whitened matched filter is no better than a simple matched filter because $t_{mf} = s^h x / \sqrt{s^h s}$.

The second case is the narrowband case. For simplicity assume that the transmit has one non-zero frequency component S_k. This is equivalent to the subspace detection problem [5], with an interfering subspace of rank 1. The covariance matrix for L components in this case is $[r_{p,q}] = (L * \mathsf{E}\{|a|^2\}) u_k u_k^h$, where u_k is the Fourier vector at the frequency f_k. In this case u_k is simply a scaled version of the transmit signal s. The inverse of the covariance matrix, including the measurement noise is

$$R^{-1} = \frac{1}{\sigma^2} \left(I - \frac{L * \mathsf{E}\{|a|^2\}}{L * \mathsf{E}\{|a|^2\} + \sigma^2} \, u_k u_k^h \right) \; . \tag{15}$$

It can be shown that this result also gives a whitened matched filter statistic which is proportional to the simple matched filter.

In conclusion, observe that whitening, or subspace detection, is ineffectual against reverberant interference as it has been modeled in this paper. The problem is that detection of a signal is attempted in an environment composed of other copies of that signal. If the reverberation is such that components can be present at any location, there is no recourse in matched filter or subspace detection theory to improve detection performance over a simple matched filter.

3. SUMMARY

A model has been presented for the reverberation due to underwater active transmissions. This model is motivated by the classical view of reverberation as a scattering process and is based on the analysis of test data by time delay estimation. The analysis and model presented herein are attractive to the system designer in the simplicity of the result and in the immediate applicability of the statistics of the model. This analysis will be extended in future work to include processing for detection of structured targets in these types of interference.

4. REFERENCES

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