KURTOSIS-BASED CRITERIA FOR ADAPTIVE BLIND SOURCE SEPARATION

*Constantinos B. Papadias**

Lucent Technologies/ Bell Labs Innovations
791 Holmdel Keyport Rd., P. O. Box 400, Holmdel, NJ 07733-0400, USA

ABSTRACT

We consider the problem of separating adaptively p synchronous user signals that are received by an m-element antenna array without the use of training sequences. We establish a set of necessary and sufficient conditions for perfect recovery of all the transmitted signals. Based on these conditions we propose optimization criteria that lead to adaptive algorithms for efficient blind source separation of non-Gaussian signals. Convergence analysis shows important global convergence properties of the proposed techniques. Combined with their low complexity, these features make the proposed algorithms good candidates for adaptive source separation.

1. INTRODUCTION

The problem of separating a number of independent source signals that are transmitted in the same frequency band and propagation environment is one of increasing practical importance. Interesting applications arise in the field of wireless communications in systems that employ CDMA (code division multiple access) or channel RWC (reuse within cell) - also called SDMA (space division multiple access) systems – see [4]. Given the desire to design bandwidth-efficient systems, as well as receivers that are able of tracking adaptively the channel changes, it is of interest to perform the source separation *blindly*, i.e. without the use of training sequences.

Whereas popular source separation methods such as MUSIC [5] and ESPRIT [3] stem from the so-called *subspace approach*, here we consider adaptive alternatives. An important issue when such techniques are used is their ability to find simultaneously all the transmitted signals. In order to derive suitable optimization criteria for algorithm development, we first establish a set of necessary and sufficient conditions for perfect recovery of all the signals. These conditions involve both fourth- and second-order moments of the receiver outputs. The study leading to the derivation of these conditions parallels the one performed by Shalvi and Weinstein in [6] for the dual problem of single-user channel equalization.

Based on the derived conditions, we propose optimization criteria for blind source separation. The analysis of these criteria shows that (in the case of $p = 2$ users) they do not contain undesirable stable stationary points. This makes the corresponding algorithms good alternatives to other techniques (such as [7], [1]) that may converge to solutions that recover only some of the transmitted signals.

*This work was performed while the author was with Information Systems Laboratory, Stanford University, Stanford, CA 94305, USA.

The rest of the paper is organized as follows. In Section 2 we present our assumptions and we derive the necessary and sufficient conditions for perfect recovery of all the transmitted signals. In Section 3 we propose and analyze some optimization criteria based on the conditions of Section 2, whereas Section 4 contains some computer simulation results that verify the algorithms' expected behavior. Finally, Section 5 contains our conclusions.

2. IDENTIFIABILITY CONDITIONS

We assume that p i.i.d. and mutually independent zero-mean discrete-time sequences $a_i(k)$ $i = 1, \ldots, p$ that share the same statistical properties are transmitted through a $p \times m$ memoryless MIMO (multiple input - multiple output) linear channel. The m channel outputs are subsequently filtered by an $m \times p$ spatial filter whose outputs $z_j(k)$, $j = 1, \ldots, p$ should ideally match the transmitted signals $a_i(k)$. The receiver outputs can then be written (in baseband) as:

$$z_j(k) = G_j^T A(k) \tag{1}$$

where $A(k) = [a_1(k) \ \cdots \ a_p(k)]^T$, $G_j = [g_{j1} \ \cdots \ g_{jp}]^T$ (T denotes transpose of a vector or matrix). G_j represents the channel/beamformer cascade that links the p input signals to the j-th output. Notice that due to the above assumptions A is stationary – however in order to obtain $A(k-1)$, $A(k)$ must be shifted by p positions downwards. Notice that we have side-stepped the received signal model as we will work in the channel/beamformer cascade domain (G).

In analogy to [6] we can write for the receiver outputs:

$$E|z_j(k)|^2 \ = \ \sigma_a^2 \sum_{l=1}^{p} |g_{jl}|^2 \ \ , \ \ j = 1, \ldots, p \tag{2}$$

$$K\left(z_j(k)\right) \ = \ K_a \sum_{l=1}^{p} |g_{jl}|^4 \ \ , \ \ j = 1, \ldots, p \tag{3}$$

where K_a is the *kurtosis* and σ_a^2 the variance of any $a_i(k)$ (since they are i.i.d.), defined as

$$\begin{array}{l} K_a = K(a_i) = E\left(|a_i|^4\right) - 2E^2\left(|a_i|^2\right) - |E\left(a_i^2\right)|^2 \\ \sigma_a^2 \ = \ E(|a_i|^2) \end{array} \tag{4}$$

We are interested in retrieving $a_i(k)$, $i = 1, \ldots, p$, based only on the statistics of the equalizer outputs $z_j(k)$, $j = 1, \ldots, p$. In the following we will denote the imaginary j by $\sqrt{-1}$ in order to avoid confusion with other indices.

As is typically assumed in blind deconvolution, we will allow each transmitted signal to be recovered up to a unitary scalar rotation. Therefore blind recovery will be achieved if (after suitable reordering of the equalizer outputs) the following holds:

$$z_j(k) = e^{\sqrt{-1}\phi_j} a_j(k) \tag{5}$$

for some $\phi_j \in [0, 2\pi)$ and all $j \in \{1, \ldots, p\}$. Based on (2) and (3) we can formulate the following theorem:

Theorem I: If each $a_i(k)$, $i = 1, \ldots, p$ is an i.i.d. zero-mean sequence, $\{a_i(k)\}$, $\{a_j(k)\}$ are statistically independent for $i \neq j$ and share the same statistical properties, then the following set of conditions are necessary and sufficient for the recovery of all the transmitted signals at the equalizer outputs:

$$\begin{array}{ll} \text{(C1)} & |K(z_j(k))| = |K_a| \quad , \; j = 1, \ldots, p \\ \text{(C2)} & E|z_j(k)|^2 = \sigma_a^2 \; , \; j = 1, \ldots, p \\ \text{(C3)} & E(z_i(k) z_j^*(k)) = 0, i \neq j \end{array}$$

Proof:

Necessity: To achieve perfect recovery (5) must hold, from which (C1) and (C2) follow immediately and (C3) follows since $E\left(a_i(k) a_j^*(k)\right) = 0$ for $i \neq j$ ($*$ denotes complex conjugate).

Sufficiency: From (3) and (C1) we get $\sum_{l=1}^{p} |g_{jl}|^4 = 1$. From (2) and (C2) we get $\sum_{l=1}^{p} |g_{jl}|^2 = 1$. Therefore G_j must be of the form

$$G_j = [0 \cdots e^{\sqrt{-1}\phi_j} \; 0 \cdots 0]^T \tag{6}$$

where the single non-zero element can be at *any* position. Combining (C3) with (1) we get:

$$G_i^H G_j = 0 \; , \quad i \neq j \tag{7}$$

where H denotes Hermitian transpose. According to (6) and (7), G_i and G_j ($i \neq j$) cannot have a non-zero element at the same position. This results to a set of G_j's $j \in \{1, \ldots, p\}$ whose non-zero elements are in positions that correspond to (possibly rotated) versions of the p different inputs $a_j(k)$. Therefore, after re-ordering, we obtain (5) and perfect signal separation has been achieved. □

3. BLIND SEPARATION CRITERIA

In the following we denote $r_{ij}^2 = |g_{ij}|^2$. Theorem I suggests the following optimization criterion:

$$\left\{ \begin{array}{ll} \min\limits_{\mathbf{G}} & F_1(\mathbf{G}) = -\sum\limits_{j=1}^{p} |K(z_j)| \\ \text{subject to:} & \mathbf{G}^H \mathbf{G} = \mathbf{I} \end{array} \right. \tag{8}$$

where $\mathbf{G} = [G_1 \cdots G_p]$ and $\mathbf{I}$ is the $p \times p$ identity matrix. $\mathbf{G}$ can be written in terms of the $p \times m$ and $m \times p$ channel and beamformer matrices $\mathbf{C}$, $\mathbf{W}$, respectively, as

$$\mathbf{G} = \mathbf{C}\,\mathbf{W} \tag{9}$$

Notice the full symmetricity in (8) which implies that if $\mathbf{G}$ is a stationary point of the cost function (8), then $\mathbf{G}^H$ will be a stationary point too. We now investigate the stationary points of (8) in the case of $p = 2$ users.

For any candidate stationary point $\mathbf{G}$ we consider the following perturbations:

$$\mathbf{G}' = \mathbf{G}U \tag{10}$$

where U can be any of the following four 2×2 matrices

$$\begin{aligned} U_{I,II} &= \begin{bmatrix} \sqrt{1-\epsilon} & \mp\sqrt{\epsilon} \\ \pm\sqrt{\epsilon} & \sqrt{1-\epsilon} \end{bmatrix} \\ U_{III,IV} &= \begin{bmatrix} \sqrt{1-\epsilon} & \mp\sqrt{-1}\sqrt{\epsilon} \\ \mp\sqrt{-1}\sqrt{\epsilon} & \sqrt{1-\epsilon} \end{bmatrix} \end{aligned} \tag{11}$$

Notice that all four matrices in (11) are unitary, as is required to guarantee $\mathbf{G}'^H \mathbf{G}' = \mathbf{I}$. Expressing $|K(z_j)| = \sum_{l=1}^{p} |g_{jl}|^4$ perturbations I and II give

$$\begin{aligned} F_1(\mathbf{G}') &\simeq F_1(\mathbf{G}) \mp 2\sqrt{\epsilon}\,\Delta_I \\ \Delta_I &= \left\{(r_{11}^2 - r_{21}^2)\mathrm{Re}(g_{11}^* g_{21}) + (r_{12}^2 - r_{22}^2)\mathrm{Re}(g_{12}^* g_{22})\right\} \end{aligned} \tag{12}$$

whereas perturbations III and IV give

$$\begin{aligned} F_1(\mathbf{G}') &\simeq F_1(\mathbf{G}) \mp 2\sqrt{\epsilon}\,\Delta_{II} \\ \Delta_{II} &= \left\{(r_{11}^2 - r_{21}^2)\mathrm{Im}(g_{11}^* g_{21}) + (r_{12}^2 - r_{22}^2)\mathrm{Im}(g_{12}^* g_{22})\right\} \end{aligned} \tag{13}$$

Combining (12) and (13) it turns out that a necessary condition for a solution to (8) to be a stable stationary point is:

$$(r_{11}^2 - r_{21}^2) g_{11}^* g_{21} + (r_{12}^2 - r_{22}^2) g_{12}^* g_{22} = 0 \tag{14}$$

Invoking the orthogonality constraint $G_1^H G_2 = 0$ Eq. (14) gives

$$[r_{11}^2 - r_{21}^2 - r_{12}^2 + r_{22}^2] g_{11}^* g_{21} = 0 \tag{15}$$

From (15) it turns out that the only possible non-desirable solutions on $\mathbf{G}^H \mathbf{G} = \mathbf{I}$ satisfy:

$$\begin{bmatrix} r_{11}^2 & r_{21}^2 \\ r_{12}^2 & r_{22}^2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \tag{16}$$

We will now show that solutions of the type (16) cannot be local minima of the cost function (8). For any perturbation $\mathbf{G}'$ of a stationary point $\mathbf{G}$ that satisfies (16) we can write

$$\begin{bmatrix} r_{11}'^2 & r_{21}'^2 \\ r_{12}'^2 & r_{22}'^2 \end{bmatrix} = \begin{bmatrix} 1/2+\epsilon_{11} & 1/2+\epsilon_{21} \\ 1/2+\epsilon_{12} & 1/2+\epsilon_{22} \end{bmatrix} \tag{17}$$

As $\mathbf{G}'$ must lie on $\mathbf{G}'^H \mathbf{G}' = \mathbf{I}$, (17) gives

$$\begin{aligned} \epsilon_{11} + \epsilon_{12} = 0 \\ \epsilon_{21} + \epsilon_{22} = 0 \end{aligned} \tag{18}$$

Using (18) and (16) we get (up to $|K_a|$):

$$\begin{aligned} F_1(\mathbf{G}') &= -\textstyle\sum_{j,l=1}^{2} r_{jl}'^4 = -\sum_{j=1,2}^{2} r_{jl}^4 - \sum_{j=1,2}^{2} \epsilon_{jl}^2 \\ &= F_1(\mathbf{G}) - \textstyle\sum_{j=1,2}^{2} \epsilon_{jl}^2 < F_1(\mathbf{G}) \end{aligned} \tag{19}$$

Hence, no solution satisfying (16) can be a local minimum of the cost function (8).

As the orthogonality constraint in (8) may not be easy to satisfy in practice, we may relax it by penalizing the non-orthogonality with an additive term in the cost function:

$$\left\{ \begin{array}{ll} \min\limits_{\mathbf{G}} & -\sum\limits_{j=1}^{p} |K(z_j)| \; + \; \beta \sum\limits_{i<j} |E(z_i z_j^*)|^2 \\ \text{subject to:} & G_j^H G_j = 1, \;\; \forall j \in \{1, \ldots, p\} \end{array} \right. \tag{20}$$

By choosing $\beta = 2|K_a|$ (20) gives

$$\begin{cases} \min_{\mathbf{G}} \quad F_2(\mathbf{G}) = -\sum_{j=1}^{p}\sum_{l=1}^{p}|g_{jl}|^4 \; + \; 2\sum_{i<j}^{p}|G_i^T G_j^*|^2 \\ \text{subject to:} \quad G_j^H G_j = 1, \;\; \forall j \in \{1,\ldots,p\} \end{cases} \tag{21}$$

We now investigate the stationary points of (21) in the case $p = 2$ by constructing the following Lagrange function:

$$\mathcal{L}(\mathbf{G},\lambda) = F_2(\mathbf{G}) - \lambda_1(\sum_{l=1}^{2}|g_{1l}|^2 - 1) - \lambda_2(\sum_{l=1}^{2}|g_{2l}|^2 - 1) \tag{22}$$

The stationary points of (21) satisfy the following system of equations resulting from $\frac{\partial \mathcal{L}(\mathbf{G},\lambda)}{\partial g_{ij}^*} = 0$:

$$\begin{aligned} g_{11}(|g_{11}|^2 - |g_{21}|^2 + \lambda_1/2) &= g_{12}g_{21}g_{22}^* && \text{(23a)} \\ g_{12}(|g_{12}|^2 - |g_{22}|^2 + \lambda_1/2) &= g_{11}g_{22}g_{21}^* && \text{(23b)} \\ g_{21}(|g_{21}|^2 - |g_{11}|^2 + \lambda_2/2) &= g_{11}g_{22}g_{12}^* && \text{(23c)} \\ g_{22}(|g_{22}|^2 - |g_{12}|^2 + \lambda_2/2) &= g_{12}g_{21}g_{11}^* && \text{(23d)} \\ |g_{11}|^2 + |g_{12}|^2 &= 1 && \text{(23e)} \\ |g_{21}|^2 + |g_{22}|^2 &= 1 && \text{(23f)} \end{aligned}$$

We now consider separately the two following cases:

- At least one coefficient g_{ij} is zero.

We assume e.g. that $g_{11} = 0$. Then (23e) gives $|g_{12}|^2 = 1$. Also because of (23a) at least one of g_{21} and g_{22} must be zero too. We first examine the case $g_{21} = 0$, which gives through (23f) $|g_{22}|^2 = 1$, corresponding to the following setting:

$$\mathbf{G} = \begin{bmatrix} 0 & 0 \\ e^{\sqrt{-1}\phi_{12}} & e^{\sqrt{-1}\phi_{22}} \end{bmatrix} \tag{24}$$

Notice that with the setting (24) both equalizer outputs retrieve the same input signal, it is therefore clearly not a desired solution. For $\mathbf{G}$ in (24) we have $F_2(\mathbf{G}) = 0$. Now we consider the following perturbation:

$$\mathbf{G}' = \begin{bmatrix} \sqrt{\epsilon} & 0 \\ g_{12}\sqrt{1-\epsilon} & g_{22} \end{bmatrix} \tag{25}$$

where ϵ is a small positive number. Clearly $\mathbf{G}'$ satisfies the constraints (23e),(23f). We also have:

$$F_2(\mathbf{G}') = -(\sqrt{\epsilon})^4 - (\sqrt{1-\epsilon})^4 - 1 + 2(1-\epsilon) = -2\epsilon^2 < 0 \tag{26}$$

Therefore $F_2(\mathbf{G}') < F_2(\mathbf{G})$ and hence the setting (24) cannot be a local minimum.

We now examine the case $g_{22} = 0$, for which $|g_{21}|^2 = 1$ yielding

$$\mathbf{G} = \begin{bmatrix} 0 & e^{\sqrt{-1}\phi_{21}} \\ e^{\sqrt{-1}\phi_{12}} & 0 \end{bmatrix} \tag{27}$$

which is clearly an optimum desired setting. For this setting $F_2(\mathbf{G}) = -2$ which is also the global minimum value of F_2. Therefore this point is a global minimum. The same results hold accordingly when any of the four coefficients is zero, yielding the absence of undesired local minima when at least one of the coefficients is nonzero.

- All coefficients g_{ij} are nonzero

We now write the system of equations (15) as:

$$\begin{aligned} r_{11}^2(r_{11}^2 - r_{21}^2 + \lambda_1/2) &= \gamma && \text{(28a)} \\ r_{12}^2(r_{12}^2 - r_{22}^2 + \lambda_1/2) &= \gamma^* && \text{(28b)} \\ r_{21}^2(r_{21}^2 - r_{11}^2 + \lambda_2/2) &= \gamma^* && \text{(28c)} \\ r_{22}^2(r_{22}^2 - r_{12}^2 + \lambda_2/2) &= \gamma && \text{(28d)} \\ r_{11}^2 + r_{12}^2 &= 1 && \text{(28e)} \\ r_{21}^2 + r_{22}^2 &= 1 && \text{(28f)} \end{aligned}$$

where $\gamma = g_{11}^* g_{12} g_{21} g_{22}^*$. As

$$\frac{\partial^2 \mathcal{L}(\mathbf{G},\lambda)}{\partial g_{11}\partial g_{11}^*} = -r_{11}^2 + r_{21}^2 - \lambda_1 \tag{29}$$

must be real it turns out that λ_1 is real. Similarly, λ_2 is real too. Therefore γ is real too which gives $\gamma = \gamma^*$. We also find by combining (28a),(28b) and (28c),(28d):

$$\lambda_1 = \gamma\left(\frac{1}{r_{11}^2} + \frac{1}{r_{12}^2}\right) \tag{30a}$$

$$\lambda_2 = \gamma\left(\frac{1}{r_{21}^2} + \frac{1}{r_{22}^2}\right) \tag{30b}$$

By combining (28a),(28c) and (28b),(28d) we get

$$\frac{1}{r_{11}^2} + \frac{1}{r_{21}^2} = \frac{1}{r_{12}^2} + \frac{1}{r_{22}^2} \tag{31}$$

From (28e), (28f) and (31), r_{11}^2 and r_{22}^2 must satisfy

$$\nu r_{11}^4 - (2+\nu) r_{11}^2 + 1 = 0 \tag{32a}$$

$$\nu r_{22}^4 - (2+\nu) r_{22}^2 + 1 = 0 \tag{32b}$$

where ν is given by $\nu = \frac{1-2r_{11}^2}{r_{11}^2(1-r_{11}^2)}$. From (32a), (32b), r_{11}^2 and r_{22}^2 satisfy the same quadratic equation $\nu x^2 - (2+\nu)x + 1 = 0$. The corresponding binomial has always a positive discriminant, however only one of its two solutions is in the $(0,1)$ interval (where r_{11}^2 and r_{22}^2 must belong). Therefore the simultaneous satisfaction of (32a) and (32b) requires that

$$r_{11}^2 = r_{22}^2 \tag{33}$$

and hence

$$r_{12}^2 = r_{21}^2 \tag{34}$$

Now $F_2(\mathbf{G})$ can be written in terms of only one of the four coefficient magnitudes as

$$F_2(\mathbf{G}) = -2 + 4r_{11}^2\left(1 - r_{11}^2\right)(2 + \cos\phi) \tag{35}$$

where $\phi = \phi_{11} + \phi_{22} - \phi_{12} - \phi_{21}$ and $g_{ij} = r_{ij}e^{\sqrt{-1}\phi_{ij}}$. We now consider the following perturbation

$$\mathbf{G}' = \begin{bmatrix} \sqrt{r_{11}^2+\delta}\,e^{\sqrt{-1}\phi_{11}} & \sqrt{r_{21}^2-\delta}\,e^{\sqrt{-1}\phi_{21}} \\ \sqrt{r_{12}^2-\delta}\,e^{\sqrt{-1}\phi_{12}} & \sqrt{r_{22}^2+\delta}\,e^{\sqrt{-1}\phi_{22}} \end{bmatrix} \tag{36}$$

where δ is a small (positive or negative) number. For $\mathbf{G}'$ we have similarly to (35):

$$\begin{aligned} F_2(\mathbf{G}') &= -2 + 4(r_{11}^2+\delta)(1-r_{11}^2-\delta)(2+\cos\phi) \\ &= -2 + 4\left(r_{11}^2(1-r_{11}^2) + \delta(1-2r_{11}^2) - \delta^2\right)(2+\cos\phi) \end{aligned} \tag{37}$$

If $r_{11}^2 < 1/2$, then we get

$$F_2(\mathbf{G}') \left\{ \begin{array}{l} > F_2(\mathbf{G}) \ , \ \delta > 0 \\ < F_2(\mathbf{G}) \ , \ \delta < 0 \end{array} \right. \tag{38}$$

whereas if $r_{11}^2 > 1/2$, we get

$$F_2(\mathbf{G}') \left\{ \begin{array}{l} < F_2(\mathbf{G}) \ , \ \delta > 0 \\ > F_2(\mathbf{G}) \ , \ \delta < 0 \end{array} \right. \tag{39}$$

Hence all the settings where all four coefficients are non-zero and not equal between them are saddle points. If $r_{11}^2{=}1/2$, we obtain from (37) $F_2(\mathbf{G}') = -2{+}4\left(r_{11}^2(1{-}r_{11}^2){-}\delta^2\right)(2+\cos\phi) < F_2(\mathbf{G})$. Therefore this setting cannot be a local minimum either. Hence there are no local minima for settings all coefficients of which are non-zero. We summarize the above results in the following theorem.

Theorem II: *The minimization problems (8) and (21) have no undesired local minima for $p = 2$, provided that the inputs $\{a_i\}$ are non-Gaussian ($K_a \neq 0$).*

The results of Theorem II will equally hold in the beamformer $\mathbf{W}$ parameter space if the channel matrix $\mathbf{C}$ in (9) is full column rank.

4. COMPUTER SIMULATION

In order to test the performance of the constrained multi-user criteria we have implemented the stochastic gradient algorithm corresponding to the criterion (21) [1] for the following 2×2 channel matrix:

$$\mathbf{C} = \left[\begin{array}{cc} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{array} \right]$$

(notice that $\mathbf{C}$ is full column rank). We initialize the 2×2 beamformer with $\mathbf{W}_o = \mathbf{I}$, so that at the beginning $r_{11}^2{+}r_{12}^2 = 1$, $r_{21}^2{+}r_{22}^2 = 1$. We then test the algorithm for the case that each of the two sources belongs equi-probably to a 4-QAM alphabet. Notice that this is a sub-Gaussian distribution ($K_a < 0$). We add to the channel output random Gaussian noise of SNR=30 dB and then run the stochastic-gradient algorithm corresponding to (21) with a step-size of $\mu = 0.01$.

Figure 1 shows the performance of the algorithm. The algorithm converges quickly to a setting that recovers both transmitted signals at its outputs. The two left-most plots show the eye patterns obtained at each output after convergence, whereas the two right-most plots show the evolution of each output's squared error with respect to the corresponding transmitted input (in order to obtain these plots we first had to remove the arbitrary phase shift introduced by the blind algorithm). We have run several other independent runs of this experiment (for both sub-Gaussian and super-Gaussian inputs), which all yielded similar results, thus verifying our theoretical expectations according to Theorem II.

5. CONCLUSIONS

We have studied the problem of multi-user blind signal separation in the case of independent inputs that have a common statistical distribution and are transmitted through a linear memoryless MIMO channel. We have first established sufficient and necessary conditions for perfect recovery of all the signals in this case. Based on these conditions we have developed optimization criteria for this problem that lead to stochastic gradient-type adaptive algorithms. The analysis of the two proposed constrained criteria in the simple case of two sources has yielded the important property of global convergence to perfect recovery settings, thus avoiding the problem of locking multiple times to the same input signal while missing other signals. These results have been obtained in the overall channel/beamformer cascade parameter space but hold also in the beamformer space alone if the channel matrix is full column rank. We also presented a computer simulation example that shows accordance with our theoretically expected behavior of the algorithms.

[1] Due to the lack of space we do not present here the stochastic gradient algorithms corresponding to our proposed criteria. The algorithms (which are easily derived from (8) and (21)) can be found in [2].

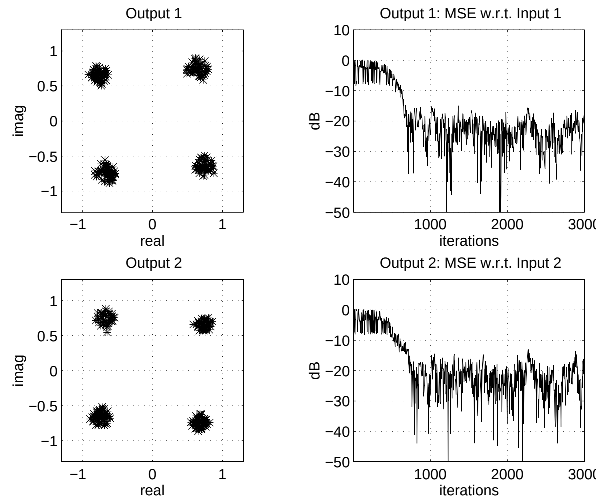

Figure 1. Blind separation of two sub-Gaussian inputs

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