FILM GRAIN NOISE REMOVAL AND GENERATION FOR COLOR IMAGES

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ABSTRACT

In this paper, we propose a noise filtering scheme, which is based on a multichannel homomorphic transformation, for color photographic images corrupted by signal–dependent film grain noise. The proposed method performs the estimation of the noise parameter using the higher–order statistics (skewness or kurtosis) of the corrupted image and the filtered image statistics. This parameter estimation technique can be used to generate color film grain noise that has applications in motion picture productions. After a theoretical description of the method employed, experimental results are provided.

1. INTRODUCTION

The extraction of information from noisy signal is a common problem in image processing. Traditional techniques for processing corrupted images have assumed the noise to be additive and signal independent. However, film grain noise, that is present in photographic images, has signal dependent statistics, therefore a different model has to be used. Since the formation of an image on a photographic film is a highly complex optical and chemical process, modeling this process with a high degree of accuracy, if at all possible, often results in models that are too complex to be managed. On the other hand, the use of oversimplified models may lead to very suboptimal algorithms. Different models have been proposed in [5]. Here we'll represent the signal dependence of the noise through a model proposed in [8], adapted to represent color images:

$$\begin{bmatrix} r_R \\ r_G \\ r_B \end{bmatrix} = \begin{bmatrix} s_R \\ s_G \\ s_B \end{bmatrix} + \begin{bmatrix} k_R \\ k_G \\ k_B \end{bmatrix} \cdot \begin{bmatrix} s_R^p \\ s_G^p \\ s_B^p \end{bmatrix} \cdot \begin{bmatrix} n_R \\ n_G \\ n_B \end{bmatrix} \quad (1)$$

where, the symbol $\cdot$ indicates dot product and for notation simplicity, we have omitted the dependence from the pixel's position (m_1, m_2).The subscripts $i = R, G, B$ represent the generic channel (Red, Green, Blue) of the color image, s_i is the *i-th* component of the noiseless image, k_i is the *i-th* component of the scanning constant, p is an exponent that depends on film, usually taking the value 0.5, and n_i is the *i-th* component of Gaussian noise with zero mean and unit variance. The eq.(1) assumes negligible measurement noise because in many practical cases the main source of degradation for photographic images is film grain noise. The suppression of signal dependent noise, for color image, can be performed by generalizing to the multichannel case the standard techniques designed for additive noise, properly modified to take into account the dependence of noise from the signal. Examples are the Wiener filter [8], and statistical estimators [8] for the signal model of eq.(1). However due to the nonlinearity of the model employed, the statistical estimators (MMSE and MAP) have a complicated form and involve numerical integration at every pixel. Moreover these techniques assume that the parameter $\mathbf{k} = [k_R\ k_G\ k_B]^T$ is known a priori.

In this paper, a generalization of the approach outlined in [11], to the multichannel case, is proposed. A multichannel homomorphic transformation is performed on the image to decouple the signal from the noise so that the noise becomes additive signal–independent, and then conventional additive noise filtering techniques are applied. A new multichannel adaptive linear filter, that is based on the local–statistics of the image, is also proposed. It performs better than the Wiener filter, that employs the global–statistics of the image.

2. MULTICHANNEL GENERALIZED HOMOMORPHIC ADAPTIVE FILTERING

Homomorphic processing was initially applied to multiplicative noise. Arsenault et al. [2], derived a generalized transformation for signal dependent noise of arbitrary form.

Consider a noisy image r, with film grain noise, having a probability distribution $p(r)$. Since the noise is signal–dependent, the standard deviation of r is a known function of the mean value μ_r, and so we can write $\sigma_r = H(\mu_r)$ where $H(\cdot)$ represents the signal–dependence functionality of σ_r. The objective is to find a transformation (generalized homomophic transformation), $w = g(r)$, such that w will contain statistically signal–independent noise (*i.e.* constant variance noise) over a wide range of image values. It can be easily shown [1], [10], that the wanted transformation is

$$w = g(r) = K \int [H(r)]^{-1} dr \quad (2)$$

For any signal-dependent noise, once the relationship between the variance and the signal has been determined, the above formula can be used to find a transformation that will make the noise additive and signal independent. With the noise model in eq.(1), it can be demonstrated that

$$w = g(r) = \frac{K r^{(1-p)}}{k(1-p)} \quad (3)$$

where K is a constant. For the special case $p = 0.5$, the above expression becomes

$$w = g(r) = \frac{2K\sqrt{r}}{k} \quad (4)$$

and transforms the signal–dependent noise into additive independent noise with variance K^2. The transformed image model (for $K = 1$) then becomes, [3], $w = g(r) \simeq g(s) + n = u + n$. Notice that the noise in the transformed domain is approximately additive Gaussian and signal–independent.

For color images, the following noise model is assumed after transformation:

$$\mathbf{w} = g(\mathbf{r}) = \mathbf{u} + \mathbf{n} \quad (5)$$

that can be explicitely expressed as

$$\begin{bmatrix} w_R \\ w_G \\ w_B \end{bmatrix} = \begin{bmatrix} g(r_R) \\ g(r_G) \\ g(r_B) \end{bmatrix} = \begin{bmatrix} u_R \\ u_G \\ u_B \end{bmatrix} + \begin{bmatrix} n_R \\ n_G \\ n_B \end{bmatrix} \quad (6)$$

where the function $g(\cdot)$ is given by eq.(4).

Given the additive signal–independent noise model of eq.(6), conventional linear filtering techniques can be applied. Example of a classical linear filter is the Wiener filter which takes a "global" view of the image statistics and applies the same filter to the entire image. In this way noise is smoothed but some details are also destroyed.

To overcome this problem it is desirable to use a filter whose coefficients are adjusted according to the local–statistics of the image in order to smooth noise and at the same time preserve the image details. The proposed filter is the generalization to the multichannel case of the filter employed in [11], and it assumes the following form:

$$\hat{\mathbf{u}} = h_{\mathbf{w}} + a[\mathbf{w} - h_{\mathbf{w}}] \quad (7)$$

where a is an adaptive variable constrained to have a value between 0 and 1, and $h_{\mathbf{w}}$ is a filter mask defined as a weighted window of **M** size $(2N+1)$x$(2N+1)$ centered on the current pixel (m_1, m_2):

$$h_{\mathbf{w}}(i,j) = \exp\left(-\frac{\|\mathbf{w}(i,j) - \mathbf{w}(m_1,m_2)\|^2}{b^2}\right) \quad (8)$$

and $\|\cdot\|$ is the Euclidean distance defined in the following fashion:

$$\|\mathbf{w}(i,j) - \mathbf{w}(m_1,m_2)\|^2 = [w_R(i,j) - w_R(m_1,m_2)]^2 + [w_G(i,j) - w_G(m_1,m_2)]^2 + [w_B(i,j) - w_B(m_1,m_2)]^2 \quad (9)$$

Using this approach, the filter coefficients are calculated using a discriminating function, which tends to 1 for pixels having "small" distance, in the sense specified by eq.(9), from the current pixel, and tends to 0 for the others. The filter mask $h_{\mathbf{w}}(i,j)$ is allowed to vary from pixel to pixel. The same weighted window is applied to the three channels (R, G, B).

The proposed filter can be interpreted as a combination of a lowpass component given by the term $h_{\mathbf{w}}$ and a highpass component, expressed as the difference between the corrupted image **w** and its lowpass component $h_{\mathbf{w}}$, weighted by a nonstationary variable a.

To determine the adaptive variable a, we minimize the following criterion:

$$min\ E[\|\hat{\mathbf{u}}(m_1,m_2) - \mathbf{u}(m_1,m_2)\|^2] \quad (10)$$

Minimization of eq.(10) yields:

$$a_{optimum} = \frac{E[\|\mathbf{u}\|^2] + E[\|h_{\mathbf{w}}\|^2] - 2E[\mathbf{u} \cdot h_{\mathbf{w}}]}{E[\|\mathbf{u}\|^2] + E[\|\mathbf{n}\|^2] + E[\|h_{\mathbf{w}}\|^2] - 2E[\mathbf{u} \cdot h_{\mathbf{w}}]}$$

with $\mathbf{u} \cdot h_{\mathbf{w}}$ denoting dot product.

The estimation of $a_{optimum}$ requires the knowledge of **u** that isn't available. In [10] it is shown that a suboptimal estimation, that provides a reasonable approximation of the optimum a, is

$$a_{sub\_optimum} = \frac{\sigma^2_{\mathbf{w}}(m_1,m_2) - \sigma^2_{\mathbf{n}}(m_1,m_2)}{\sigma^2_{\mathbf{w}}(m_1,m_2)} \quad (11)$$

The parameter b in the filter mask controls the width of the discriminating function, and is left as a design parameter. Through experimentation a good choice for b is the value $\sigma_{\mathbf{w}}$. If the variance is high, which corresponds to a large value of b and indicates a high level of noise, the slope of the function is small. This results in smoothing the image details. However, if the variance is low, then more emphasis is on edge preservation.

3. COLOR FILM GRAIN NOISE REMOVAL

The adaptive noise smoothing filter structure is represented in fig.(1)

Then, the noise removal procedure can be summarized by the following:

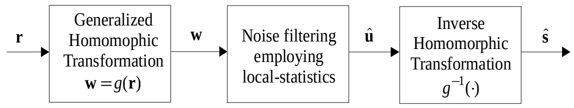

Figure 1: *Noise filtering scheme*

- perform the generalized homomorphic transformation on the corrupted image **r** given by eq.(1), in order to decouple noise form signal,
- filter the transformed image, **w**, using the proposed filter,
- the filtered image so obtained, **û**, is then homomorphically transformed back obtaining an estimation, **ŝ**, of the ideal image **s**.

4. COLOR FILM GRAIN NOISE GENERATION

As outlined in the Introduction, film grain noise generation has applications in television productions where computer generated images and video images are combined into one frame. In this process, film grain noise is added to the computer generated images to match the grain pattern of the film, thus obtaining images that appear to be obtained through the same photographic process. However to add the right amount of noise, the parameter **k** must be estimated. Assuming $p = 0.5$, the variance, skewness and kurtosis of each component r_i, with $i = R, G, B$, of the corrupted image **r**, are related to that of the corresponding components of the observed image **s** by the following equations:

$$\begin{aligned}\sigma^2_{r_i} &= \sigma^2_{s_i} + k_i^2 E[s_i] \\ c_3^{r_i} &= c_3^{s_i} + 3k_i^2\sigma^2_{s_i} \\ c_4^{r_i} &= c_4^{s_i} + 6k_i^2 c_3^{r_i} - 15k_i^4\sigma^2_{s_i}\end{aligned} \tag{12}$$

where $c_3^{r_i}$ and $c_4^{r_i}$, with $i = R, G, B$, represent the skewness and kurtosis of the *i-th* component of the observed image respectively and $\sigma^2_{s_i}$ is the variance of the *i-th* component of the signal. For $i = R, G, B$, the value of k_i, can then be obtained by substituting the statistics of the observed image $(\sigma^2_{r_i}, c_3^{r_i}$, and $c_4^{r_i})$ and the a prior image statistics $(\sigma^2_{\hat{s}_i}, c^3_{\hat{s}_i}$, and $c^4_{\hat{s}_i})$ into any of the above equations.

Then, given a noisy image, the noise generation procedure, that is depicted in fig.(2), can be summarized by the following:

- perform, on the given noisy image, the operations summarized in the previous section to obtain the ”ideal” image **ŝ**,
- estimate the value of **k**, using for each component k_i relations eq.(12),
- generate noise according to eq.(1) and add it to a noise free image to match the grain pattern of the noisy image.

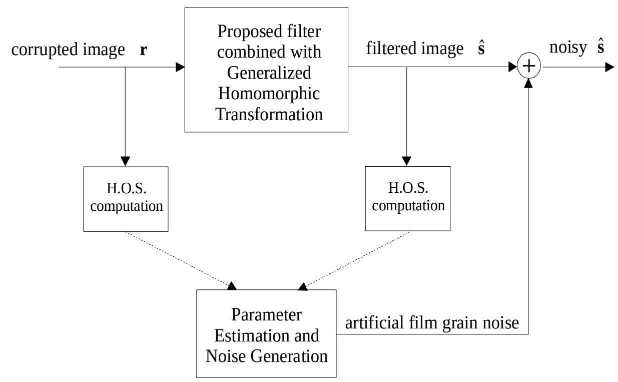

Figure 2: *Noise generation scheme*

5. SIMULATION RESULTS AND CONCLUSIONS

In order to verify the effectiveness of the proposed methods, for removal and generation of color film grain noise, two test images are used: ”Lenna” and ”Melon”.

The original images are corrupted, with film grain noise, accordingly to eq.(1); the resulting images are then filtered using the proposed method and another method,[6], for comparisons.

To select the values of k_i, with $i = R, G, B$, used in eq.(1), several issues need to be considered. First, the choice of k_i, should reflect the relative noise levels in actual color–sensitive emulsion layers that lead to amount of noise in the different channels (R, G, B). Second, we should also take into account the human visual system so that, the channel with heavy noise would have the greatest effect on perceived graininess. With the above considerations, noise power in the Red channel should be the least, followed by the Blue and the Green channel [7]. In simulations, two sets of values for **k** are $\mathbf{k}_1 = [0.07\ 0.10\ 0.10]^T$ and $\mathbf{k}_2 = [0.10\ 0.15\ 0.15]^T$. The comparison method employed is the generalization to the multichannel case of the Lee's algorithm [6] that gives

$$\hat{\mathbf{u}} = m_{\mathbf{w}} + \frac{\sigma^2_{\mathbf{w}} - 1}{\sigma^2_{\mathbf{w}}}(\mathbf{w} - m_{\mathbf{w}}) \tag{13}$$

where, for notation simplicity, we have omitted the pixels position dependence, and **û** and **w** are the signals represented in fig.(1). In fact, the following different algorithms are selected for noise filtering:

- 3 independent runs of Lee's algorithm for single–channel filtering (Lee's S.)
- 1 run of Lee's algorithm for multichannel filtering (Lee's M.)
- 3 independent runs of the proposed Generalized Homomophic Adaptive Filtering algorithm for Single–channel (G.H.A.F. S.)
- 1 run of the proposed Generalized Homomophic Adaptive Filtering algorithm for Multi–channel (G.H.A.F. M.)

Two well known metrics used in evaluating the distance, and hence the performance, between the ideal image and the filtered image are the L_1 norm and the L_2 norm, defined as in [4]

$$L_1 = |s_R - \hat{s}_R| + |s_G - \hat{s}_G| + |s_B - \hat{s}_B|$$
$$L_2 = \sqrt{(s_R - \hat{s}_R)^2 + (s_G - \hat{s}_G)^2 + (s_B - \hat{s}_B)^2}$$

Results are summarized in Table[1] and Table[2].

	$\mathbf{k}_1$		$\mathbf{k}_2$	
Metric	L_1 norm	L_2 norm	L_1 norm	L_2 norm
No filtering	1.196e-1	9.443e-2	1.783e-1	1.415e-1
Lee's S.	7.287e-2	6.142e-2	9.857e-2	8.369e-2
Lee's M.	7.106e-2	6.001e-2	9.511e-2	8.053e-2
G.H.A.F. S	7.097e-2	5.997e-2	1.028e-1	8.643e-2
G.H.A.F. M	6.623e-2	5.604e-2	8.814e-2	7.337e-2

Table 1: Performance of different filtering schemes with color "Lenna"

	$\mathbf{k}_1$		$\mathbf{k}_2$	
Metric	L_1 norm	L_2 norm	L_1 norm	L_2 norm
No filtering	1.190e-1	9.449e-2	1.773e-1	1.409e-1
Lee's S.	7.029e-2	5.801e-2	9.351e-2	7.775e-2
Lee's M.	6.816e-2	5.605e-2	8.984e-2	7.402e-2
G.H.A.F. S	6.459e-2	5.276e-2	9.429e-1	7.793e-2
G.H.A.F. M	6.103e-2	4.989e-2	8.143e-2	6.599e-2

Table 2: Performance of different filtering schemes with color "Melon"

It can be observed that for both the moderate ($\mathbf{k}_1$) and the large ($\mathbf{k}_2$) noise power, the proposed multichannel scheme has excellent noise suppression properties over the single channel filtering scheme. This is because information between channels is utilized in computing the filter mask and the adaptive weight. Moreover, the proposed multichannel scheme outperforms the Lee's algorithm in both noise smoothing and edge preservation.

To test the color noise procedure generation, the image "Lenna" is used. Film grain noise, with $\mathbf{k}_1 = [0.07\ 0.1\ 0.1]^T$ and $p = 0.5$, is added to the ideal image and the proposed film grain noise generation procedure is applied. To compare the noise level of the original corrupted and the final image, multichannel mean square error (MSE) is used. It is defined as the sum of the three signal–channel MSE's. The obtained results using second, third, fourth order statistics according to eq.(12), are summarized in Table [3].

Multichannel MSE			
statistics	second order	third order	fourth order
	1.1550e-2	1.1570e-2	9.7017e-3

Table 3: MSE's of noise–added "Lenna" image. Multichannel MSE for the original corrupted color image is 8.92e-3.

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