SUBSPACE-BASED DETECTION FOR CDMA COMMUNICATIONS

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ABSTRACT

The oblique projection supports the framework to resolve a signal space into desired signal and interference subspaces. This paper presents subspace-based detection methods using the oblique projection for the CDMA channel. For the synchronous case, it is shown that this detector represents the geometrical form of the decorrelating detector, and performs a complete rejection of interfering signals. This paper also suggests the approach of combining the subspace-based detection with the MUSIC algorithm for an asynchronous CDMA channel. It is shown that the BER performance of this detection approach, which depends on the accuracy of the code timing acquisition, is better than that of the blind adaptive demodulation technique.

1. INTRODUCTION

Direct-Sequence Code-Division Multiple Access (DS-CDMA) has received much attention as an alternative to TDMA and FDMA for wireless communication. Different from TDMA or FDMA, a specific signature waveform is assigned to each user, and all users can occupy the same bandwidth simultaneously in CDMA. The main drawback of CDMA communication is the near-far problem. The received power of the desired user may be much smaller than those of other interfering users when the receiver is closer to the interfering users. In this situation, the strong interfering powers dominate the output of the correlator so that the detection and parameter estimation for the desired user may not be reliable any more. The performance of the conventional detectors which rely on the single user demodulation techniques may be drastically degraded by even small near-far effect. This problem has initiated the development of multiuser detectors which are robust to multiple access interferences(MAI). The decorrelating detector[3] is a relatively simple method which can reject MAI completely.

In recent work for parameter estimation in a CDMA channel, it has been shown that subspace-based algorithm, like MUSIC, is very robust to the near-far effect[7], since it exploits the relations of signal and noise space. This is the motivation for the development of a subspace-based detection in a CDMA channel. This paper describes the subspace-based detection using oblique projection [1] which can resolve the signal space into two independent subspaces, i.e, the desired signal and interference subspaces. This paper also describes the relation with the decorrelating detector. The approach of combining the subspace-based detection with the MUSIC algorithm is suggested for an asynchronous CDMA channel. Different from most multiuser detectors which require exact timing information, in this approach, the code timing is acquired by the MUSIC algorithm with negligible error, and then, the bit information is detected after resolving the signal space in the MUSIC algorithm into two independent but not orthogonal subspaces.

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2. SIGNAL MODEL FOR CDMA CHANNEL

In a CDMA channel, several users transmit simultaneously over a shared channel. The baseband received signal can be modeled as the superposition of K active users with additive channel noise.

$$r(t) = \sum_{k=1}^{K} A_k d_k(i) s_k(t-\tau_k) + n(t), \tag{1}$$

where τ_k is the delay, which is uniformly distributed over a bit interval. $n(t)$ is assumed to be white Gaussian noise with zero mean and power spectral density of σ^2, A_k is the k_{th} user's signal amplitude. $s_k(t)$ denotes the signature waveform of the k_{th} user which is given by

$$s_k(t) = \sum_{n=0}^{N-1} \Pi_{T_c}(t-nT_c) c_k^{(n)}, \tag{2}$$

where Π is a rectangular pulse, T_c is the chip duration($T_c = T/N$, N:number of chips per symbol) and $\mathbf{c}_k \in \mathbf{R}^N$ is a spreading code. The spreading codes are not necessarily orthogonal but independent.

After chip matched filtering which is given in the form of an integrator during one chip interval, an equivalent discrete-time model with the observation vector length N is obtained.

$$\mathbf{y}_i = \sum_{k=1}^{K} \{a_k(i-1)\mathbf{u}_k^r + a_k(i)\mathbf{u}_k^l\} + \mathbf{n}_i, \tag{3}$$

where $\mathbf{n}_i$ is white Gaussian noise with variance σ^2/T_c, and $a_k(i)$ includes the power and the transmitted data symbols at time index i. Each vector, $\mathbf{u}_k^r$ and $\mathbf{u}_k^l$, denotes the delayed spreading code associated with the previous or current data symbol respectively, since each observation vector contains the consecutive symbols for all users. Note that a delayed spreading code with chip delay, $p \in \{0, \cdots, N-1\}$, and inter-chip delay, $\delta \in [0,1)$, such that (τ_k/T_c) mod $N = p + \delta$, will be a convex combination of the two adjacent chip delay codes, since the chip matched filter is just an integrator.

3. SUBSPACE-BASED DETECTION

3.1. Detection by Oblique Projection

The direct sum of the partial oblique projections results in the projected vector on the space, and the direct sum decomposition uniquely determines the partial oblique projections. The main distinguishing property of oblique projection is that the null space of a space is not necessarily orthogonal to the space. Resolving a projection onto the space $< \mathbf{\Gamma}\ \mathbf{\Sigma} >$ into two oblique projections,

$$\mathbf{P}_{\Gamma\Sigma} = \mathbf{L}_{\Gamma} + \mathbf{L}_{\Sigma}, \tag{4}$$

where the subscripts denote the ranges, then[1]

$$\mathbf{L}_{\Gamma} = \mathbf{\Gamma}(\mathbf{\Gamma}^H \mathbf{P}_{\Sigma}^{\perp} \mathbf{\Gamma})^{-1} \mathbf{\Gamma}^H \mathbf{P}_{\Sigma}^{\perp}. \tag{5}$$

$\mathbf{P}_{\Sigma}^{\perp}$ performs as a null-steering operator that nulls everything in the space $< \mathbf{\Sigma} >$.

Now, consider the signal model in a synchronous CDMA channel, where $\tau_k = 0$, that is $\mathbf{u}_k^l = \mathbf{c}_k$ in (3),

$$\mathbf{y}_i = a_1(i)\mathbf{c}_1 + \sum_{k=2}^{K} a_k(i)\mathbf{c}_k + \mathbf{n}_i. \tag{6}$$

Without loss of generality, omitting time index from the model, we can establish the corresponding spaces as follows,

$$\begin{aligned} \mathbf{y} &= \left[\begin{array}{c|ccc} \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_K \end{array}\right] \left[\begin{array}{c} a_1 \\ ---- \\ a_2 \\ \cdots \\ a_K \end{array}\right] + \mathbf{n} \\ &= \left[\begin{array}{c|c} \kappa & \mathbf{\Sigma} \end{array}\right] \left[\begin{array}{c|c} \mu & \nu \end{array}\right]^T + \mathbf{n}, \end{aligned} \tag{7}$$

where μ and ν are the parameters to be estimated in order to determine the received symbols. The spreading code $\mathbf{c}_1$ defines the desired signal space $< \kappa >$, while other users' codes constitute the interference space $< \mathbf{\Sigma} >$. Recall that the two spaces are not necessarily orthogonal. The ML estimation of $[\mu\ \nu]$ is given by

$$\left[\begin{array}{c} \hat{\mu} \\ \hat{\nu} \end{array}\right] = \left([\kappa\ \mathbf{\Sigma}]^T [\kappa\ \mathbf{\Sigma}] \right)^{-1} \left[\begin{array}{cc} \kappa & \mathbf{\Sigma} \end{array}\right]^T \mathbf{y}. \tag{8}$$

For the detection of desired user 1, we need only the estimation of μ. By the inversion formula for 2×2 block matrices[1],

$$\hat{\mu} = (\kappa^T \mathbf{P}_{\Sigma}^{\perp} \kappa)^{-1} \kappa^T \mathbf{P}_{\Sigma}^{\perp} \mathbf{y}. \tag{9}$$

Note that $\hat{\mu}\kappa = \mathbf{L}_{\kappa}\mathbf{y}$, that is, the oblique projection of the received signal vector $\mathbf{y}$ onto the desired signal space is the solution of the least square problem. The detector based on $\hat{\mu}$ is then simply given by

$$\hat{b}_1(i) = sgn\{\mu_i\}.$$

We can easily confirm from (9) that $\hat{\mu}$ has the following normal distribution,

$$\hat{\mu} \sim N[\mu, \frac{\sigma^2}{(\kappa^T \mathbf{P}_{\Sigma}^{\perp} \kappa)}], \tag{10}$$

where μ corresponds to the amplitude of the desired signal, A_1. Hence, the bit error rate of the subspace detector can be represented as

$$P_e^s = Q(\frac{A_1 \sqrt{\kappa^T \mathbf{P}_{\Sigma}^{\perp} \kappa}}{\sigma}). \tag{11}$$

As we have seen above, the subspace-based detection requires only one assumption that all users' spreading codes are known to the receiver.

3.2. Comparison with the Decorrelating Detector

The decorrelating detector was shown in [3]. Different from the chip matched filter output model, a bank of matched filter outputs can be represented as

$$\mathbf{y}_o = \mathbf{RAb} + \mathbf{n}, \tag{12}$$

where $\mathbf{A}$ is the diagonal matrix for the amplitudes of the signals, $\mathbf{b}$ denotes the corresponding bit vector and $\mathbf{n}$ is a Gaussian random vector with zero mean and covariance matrix $\sigma^2\mathbf{R}$. $\mathbf{R}$ is the correlation matrix of signature waveforms, which can be represented as

$$\mathbf{R} = \left[\begin{array}{ccccc} 1 & \rho_{12} & \rho_{13} & \cdots & \rho_{1K} \\ \rho_{21} & 1 & \rho_{23} & \cdots & \rho_{2K} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \rho_{K1} & \rho_{K2} & \cdot & \cdots & 1 \end{array}\right]. \tag{13}$$

ρ_{ij} denotes the inner product of the codes, $\mathbf{c}_i$ and $\mathbf{c}_j$. If we premultiply the matched filter output vector by $\mathbf{R}^{-1}$, then

$$\mathbf{R}^{-1}\mathbf{y}_o = \mathbf{Ab} + \mathbf{R}^{-1}\mathbf{n} \tag{14}$$

In the absence of noise, we can decide the polarity by the following decision mechanism,

$$\hat{b}_k = sgn((\mathbf{R}^{-1}\mathbf{y}_o)_k) = b_k \tag{15}$$

This is the decorrelating detector, which eliminates the multiuser interference present in the matched filter outputs. The decorrelating detector correlates with the projection of $\mathbf{c}_1$ on the subspace orthogonal to the subspace spanned by the interfering signature waveforms.

Now, considering the right side of (8), we can see that

$$\begin{aligned} \left[\begin{array}{cc} \kappa^T\kappa & \kappa^T\mathbf{\Sigma} \\ \mathbf{\Sigma}^T\kappa & \mathbf{\Sigma}^T\mathbf{\Sigma} \end{array}\right] &= \mathbf{R}, \\ \left[\begin{array}{cc} \kappa & \mathbf{\Sigma} \end{array}\right]^T \mathbf{y} &= \mathbf{y}_0. \end{aligned} \tag{16}$$

It is clear that this is the same form as the decorrelating detector in (15). The BER performance of both detectors must be same. Conclusively, the subspace-based detection using oblique projection explains the geometrical structure of the decorrelating detector.

However, it is known that it is preferable to use the conventional detector instead of the decorrelating detector if the interfering users' amplitudes are small enough[8]. This is known as the noise enhancement effect. The same situation appears to the subspace-based detection. In (11), the noise variance becomes enhanced by $1/(\kappa^T \mathbf{P}_{\Sigma}^{\perp} \kappa)$. Note that

$$\kappa^T \mathbf{P}_{\Sigma}^{\perp} \kappa \leq \kappa^T \kappa = 1,$$

where the equality is satisfied when the subspaces $< \kappa >$ and $< \mathbf{\Sigma} >$ are orthogonal.

4. DETECTION IN THE ASYNCHRONOUS CHANNEL

The decorrelating detector in asynchronous case was developed in [4]. The detector, however, assumes that all matched filters exactly match the timings of all signals. Then it can be implemented as a block of independent correlators for each user. In this paper, however, an approach combining the MUSIC algorithm for timing acquisition and the subspace-based detection is presented. Hence, it is meaningless to compare this approach with the decorrelating detector any more.

For the subspace generation in asynchronous CDMA channel, exact information for all users' timings is needed since the timing estimation errors may cause a severe change of the subspace structure. However, in this approach, we need only chip delays for generating interference space by exploiting the convexity of signals in the asynchronous CDMA channel. The computational burden of the MUSIC becomes definitely reduced, since the information for chip delays can be achieved by a simple calculation. There can be various ways generating the spaces. We present one of these in this paper.

There are two important facts for generating interference spaces. First, two different symbols can be involved in one symbol interval. Both symbols may have same polarities or not. These two cases should constitute different bases in subspace. Secondly, a user's signal is always the linear combination of the delayed codes $\mathbf{c}_k(p_k)$ and $\mathbf{c}_k(p_k+1)$, which are the cyclically shifted versions of $\mathbf{c}_k$ with the chip delays indexed. This means that a signal is always included in the space constructed by the two delayed codes. Based on these facts, we can construct the interference subspaces $<\mathbf{\Sigma}>$ as

$$\mathbf{\Sigma} = [\mathbf{ac}_2(\hat{p}_2)\ \mathbf{bc}_2(\hat{p}_2)\ \mathbf{ac}_2(\hat{p}_2+1)\ \mathbf{bc}_2(\hat{p}_2+1)\cdots \\ \mathbf{ac}_K(\hat{p}_K)\ \mathbf{bc}_K(\hat{p}_K)\ \mathbf{ac}_K(\hat{p}_K+1)\ \mathbf{bc}_K(\hat{p}_K+1)]. \quad (17)$$

The $\mathbf{ac}_k(\hat{p}_k)$ denotes the shifted version of $\mathbf{c}_k$ for the chip delay $\hat{p}_k$ with consecutive same bits, while $\mathbf{bc}_k(\hat{p}_k)$ denotes the same shifted version with consecutive different bits. For the signal space, we need two definitions,

$$\begin{aligned} \kappa_1 &= [\mathbf{ac}_1(\hat{p}_1)], \\ \kappa_2 &= [\mathbf{bc}_1(\hat{p}_1+1)]. \end{aligned} \quad (18)$$

Recall that two consecutive time intervals should be considered for the detection at time i. We need some modified process for (9). At the first step, we perform

$$\mathbf{r}_i = \mathbf{P}_{\Sigma}^{\perp}\mathbf{y}_i, \quad (19)$$

which is the projection of $\mathbf{y}_i$ onto the orthogonal complement of the interference space $<\mathbf{\Sigma}>$. The null-steering operator eliminates the interference signal component in the received signal vector.

Next, we need to define a series of $(1 \times N)$ vector sets,

$$\begin{aligned} \mathbf{q}_{11} &= [(\hat{p}_1-1)\ zeros,\ r_{i-1}(\hat{p}_1),\cdots,\ r_{i-1}(N)], \\ \mathbf{q}_{12} &= [r_i(1),\cdots,\ r_i(\hat{p}_1-1),\ (N-\hat{p}_1+1)\ zeros], \\ \mathbf{q}_{21} &= [(\hat{p}_1)\ zeros,\ r_{i-1}(\hat{p}_1+1),\cdots,\ r_{i-1}(N)], \\ \mathbf{q}_{22} &= [r_i(1),\cdots,\ r_i(\hat{p}_1),\ (N-\hat{p}_1)\ zeros]. \end{aligned} \quad (20)$$

The zero insertion in the above vector sets is for rejecting the effects of the previous symbol which may be included in the current sample. Using these vectors, we can obtain two 1×2 vectors based on oblique projections,

$$\begin{aligned} \hat{\mu}_a &= (\kappa_1^T\mathbf{P}_S^{\perp}\kappa_1)^{-1}\kappa_1^T[\mathbf{q}_{11}^T\mathbf{q}_{12}^T], \\ \hat{\mu}_b &= (\kappa_2^T\mathbf{P}_S^{\perp}\kappa_2)^{-1}\kappa_2^T[\mathbf{q}_{21}^T\mathbf{q}_{22}^T]. \end{aligned} \quad (21)$$

The sum of two components in each vector corresponds to the total oblique projection part of the received signal onto the signal space, κ_1 or κ_2,

$$\begin{aligned} \hat{\mu}_1(i) &= \hat{\mu}_a(1)+\hat{\mu}_a(2), \\ \hat{\mu}_2(i) &= \hat{\mu}_b(1)+\hat{\mu}_b(2). \end{aligned} \quad (22)$$

Therefore, the final projection must be the convex combination of the above two measurements. Using the estimation value for interchip delay of user 1,

$$\hat{\mu}_i = (1-\hat{\delta}_1)\hat{\mu}_1(i)+\hat{\delta}_1\hat{\mu}_2(i) \quad (23)$$

The decision is simply based on that $\hat{b}_1(i) = sgn\{\hat{\mu}_i\}$.

5. EXPERIMENTAL RESULTS

Experiments were performed on the subspace-based detection in synchronous and asynchronous CDMA channels. The simulation system was a 6-user case with 31 chips per bit maximal length sequences. All interference powers were set to be equal. Figs.1 and 2 show the BER performances of the subspace-based detection, the conventional detector and the decorrelating detector in a synchronous CDMA channel. Each Monte-Carlo run represents a particular realization of the noise and data sequence. As we expected, in any cases, the subspace-based detection has the same performance as the decorrelating detector and those are immune to MAI. Fig.1 shows the result when MAI equals 40dB. The subspace-based detection clearly outperforms the conventional detector. However, when MAI equals -20dB, i.e, when the background noise dominates the interferences, the conventional detector outperforms the subspace-based detection. This means that we need a switching mechanism based on the signal powers.

Fig. 3 shows the BER performance of subspace-based detection with its lower bound in an asynchronous CDMA channel. This was performed when MAI equals 40dB. It was assumed that the desired user's timing delay converges after at least 300 iterations and other users' chip delays are estimated after 100 iterations for lessening the running time of the MUSIC algorithm. The lower bound is the result when all interfering users' timing except the desired user's are assumed to be exactly estimated. Therefore, the gap between the detectors with estimation errors and the lower bound means the uncertainty of the subspaces generated from the timing estimation by the MUSIC algorithm.

Fig.4 shows the BER performance of subspace-based detection and the blind adaptive demodulator which was proposed in [5] when MAI equals 40dB. It is known that both algorithms show reliable performance against near-far effect for the code timing acquisition. It is clear that the subspace-based detection outperforms the blind adaptive demodulator as the SNR increases.

6. CONCLUSION

It is known that the subspace-based approach, MUSIC, is robust to the near-far effect for timing acquisition in an asynchronous CDMA channel. Based on this property, a subspace-based detection approach for CDMA was proposed. It was shown that the subspace-based detector using the idea of oblique projection represents the geometrical form of the decorrelating detector. The BER performance of the subspace-based detection is immune to the near-far effect. In an asynchronous CDMA channel, different from the decorrelating detector, the subspace-based detection assumes no knowledge of the code timings. In this paper, we suggested an approach of combining the subspace-based detection and the MUSIC algorithm. We have shown that it may be well adapted to asynchronous CDMA channels. The reduction of the performance gap of this detector with a lower bound based on known timing values, and modifications to account for possible noise magnification are under study.

7. REFERENCES

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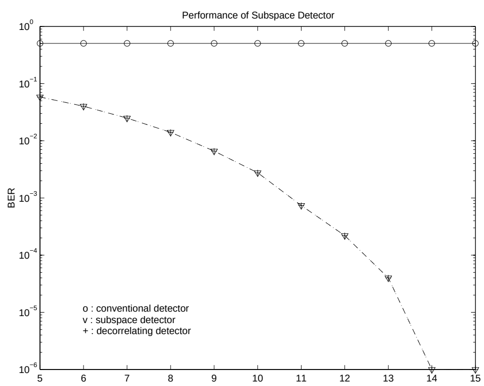

Figure 1: Synchronous Subspace Detector at MAI= 40dB

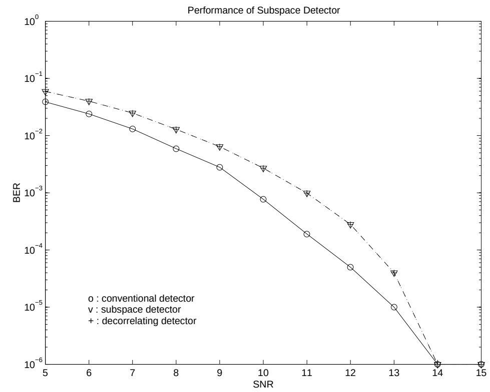

Figure 2: Synchronous Subspace Detector at MAI= -20dB

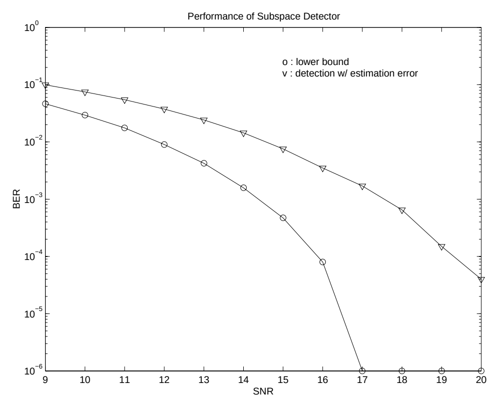

Figure 3: Asynchronous Subspace Detector at MAI= 40dB

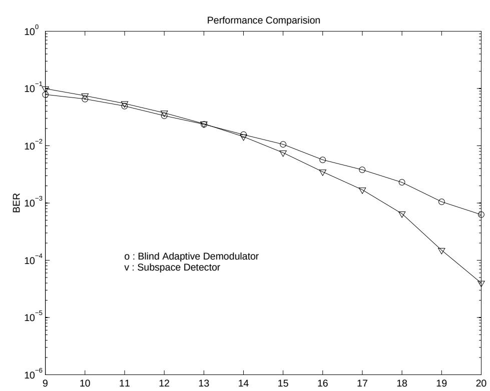

Figure 4: Asynchronous Subspace Detector at MAI= 40dB