AN INSTRUMENTAL VARIABLE BASED SUBSPACE TRACKING ALGORITHM BASED ON SUBSPACE AVERAGING

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ABSTRACT

In this paper an instrumental variable (IV) based subspace tracking algorithm is proposed. The basic idea of the algorithm is to reduce the amount of computations using a certain perturbation/approximation strategy. The complexity is reduced to $O(mn^2)$, which should be compared to $O(ml^2)$ for the SVD, where $m, l \gg n$ in general (m denotes the number of sensors, l denotes the number of instruments, and n denotes the number of signals). The proposed algorithm turns out to be related to Karasalo's subspace averaging approach. In a series of simulations we demonstrate that the detection-, stationary estimation-, and tracking performance of the proposed algorithm is essentially equivalent to that achieved by the truncated SVD.

1. INTRODUCTION

The sensor array signal processing problem has received much attention in the literature. Especially, high resolution subspace-based methods for DOA estimation have been in the focus. Typical for these approaches is that a degradation of the performance occurs if the true noise covariance matrix deviates from the assumption of spatial whiteness. When the noise covariance is colored but *known*, this effect can be eliminated by pre-whitening. However, since its estimation requires signal free measurements, pre-whitening is in many cases not a realistic option. An approach that does not require spatially white or known noise covariance matrix is the method of instrumental variables (IV). The applicability of IV methods in sensor array processing has been studied in, for example [2, 4, 5].

Another aspect of the array processing field that has drawn much attention, is the application of subspace-based DOA estimation techniques to non-stationary environments. However, most of the proposed "tracking" algorithms require spatially white noise.

The aim of this paper is to study a computationally efficient IV-based subspace tracking algorithm, that is applicable in non-stationary environments with colored noise.

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2. PROBLEM FORMULATION

Let $\mathbf{x}(t) \in \mathbb{C}^{m\times 1}$ contain the observed samples of an antenna array with m sensors, where it is assumed that n narrow-band plane waves impinge on the array. Hence, the following data model is applicable

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{e}(t). \tag{2.1}$$

The $m \times n$ matrix $\mathbf{A}(\boldsymbol{\theta})$ is deterministic, but possibly time-varying (the vector $\boldsymbol{\theta}$ contains the n DOAs). The unmeasurable signal $\mathbf{s}(t) \in \mathbb{C}^n$ is assumed to be a stationary random process with covariance matrix $\mathbf{P} = E[\mathbf{s}(t)\mathbf{s}^H(t)]$, where $(\cdot)^H$ denotes conjugate transpose. The noise vector $\mathbf{e}(t)$ is assumed to be zero-mean and independent of $\mathbf{s}(t)$. The unknown noise covariance matrix is defined as

$$\mathbf{Q} = E[\mathbf{e}(t)\mathbf{e}^H(t)]. \tag{2.2}$$

Typical for IV-based approaches are the following assumptions. Assume that there exists an IV vector $\boldsymbol{\xi}(t) \in \mathbb{C}^{l\times 1}$, $l \geq n$ such that

A1: $E[\mathbf{e}(t)\boldsymbol{\xi}^H(t)] = \mathbf{0}$

A2: $\text{Rank}\left(E\left[\mathbf{s}(t)\boldsymbol{\xi}^H(t)\right]\right) = \text{Rank}\left(\mathbf{C}_{s\xi}\right) = n.$

The assumption **A2** is imposed to ensure that $\text{Rank}(\mathbf{AC}_{s\xi}) = n$, which implies that $\mathcal{R}(\mathbf{AC}_{s\xi}) = \mathcal{R}(\mathbf{A})$, where $\mathcal{R}(\mathbf{A})$ denotes the space spanned by the columns of $\mathbf{A}$. Note, the DOAs may be identifiable even if **A2** is not fulfilled, see [4]. However, since we in the present paper apply ESPRIT, **A2** is assumed to hold. With these assumptions

$$E[\mathbf{x}(t)\boldsymbol{\xi}^H(t)] = \mathbf{C}_{x\xi} = \mathbf{AC}_{s\xi}. \tag{2.3}$$

Hence, the SVD of $\mathbf{C}_{x\xi}$ can be written as

$$\mathbf{C}_{x\xi} = \mathbf{U}_s\boldsymbol{\Sigma}_s\mathbf{V}_s^H \tag{2.4}$$

where $\boldsymbol{\Sigma}_s = \text{diag}(\sigma_1, \ldots, \sigma_n)$ is diagonal and nonsingular. The key observation is that $\mathcal{R}(\mathbf{C}_{x\xi}) = \mathcal{R}(\mathbf{U}_s)$. That is, the signal subspace is spanned by the n dominating left singular vectors of $\mathbf{C}_{x\xi}$. In practice, $\mathbf{C}_{x\xi}$ is unknown. Hence, the estimate of the signal subspace is typically taken as the n principal left singular vectors of the estimated cross-covariance matrix $\hat{\mathbf{C}}_{x\xi}$:

$$\hat{\mathbf{C}}_{x\xi} = \hat{\mathbf{U}}_s\hat{\boldsymbol{\Sigma}}_s\hat{\mathbf{V}}_s^H + \hat{\mathbf{U}}_n\hat{\boldsymbol{\Sigma}}_n\hat{\mathbf{V}}_n^H, \tag{2.5}$$

where $\hat{\mathbf{\Sigma}}_n \neq \mathbf{0}$, in general. For reasons of statistical optimality, different post and pre weightings of $\hat{\mathbf{C}}_{x\xi}$ may be useful, see [4] for a detailed discussion. However, these weightings are data dependent. Thus, not only $\hat{\mathbf{C}}_{x\xi}$ need to be updated as time goes, the weighting matrices must be updated too. Therefore such weightings tend to increase the overall complexity and are omitted in the present paper. IV-vectors that fulfill **A1-2** can be found as (see for example [4] for further details):

1) *Spatial IVs:* Consider an array that is divided into subarrays. The outputs of one of the subarrays are taken as instruments (this array may be uncalibrated). If the subarrays are sufficiently far apart, the noise in the main subarray is considered to be uncorrelated with the IV-vector.

2) *Temporal IVs:* When a second subarray is not available but the signals are temporally correlated, an IV-vector can be obtained by delaying the sensor outputs. This approach relies on that the temporal correlation length of the signal is longer than that corresponding to the noise.

Implicit in the discussions above is that the signal subspace might be slowly time-varying. By slowly time-varying, it is meant that $\mathbf{z}(t)$ and $\boldsymbol{\xi}(t)$ are approximately stationary in a window of length $1/(1-\mu)$, where $0 \ll \mu < 1$ is the so-called forgetting factor. The cross-covariance matrix may thus be estimated by

$$\hat{\mathbf{C}}_{x\xi}(t) = \mu \hat{\mathbf{C}}_{x\xi}(t-1) + (1-\mu)\mathbf{x}(t)\boldsymbol{\xi}^H(t). \tag{2.6}$$

Our goal is to derive an efficient algorithm which estimates $\mathcal{R}(\mathbf{A}(t))$, given the subspace estimate at time-instant $t-1$ and the snapshots $\mathbf{x}(t), \boldsymbol{\xi}(t)$.

3. IV TRACKING BASED ON NOISE SUBSPACE PERTURBATION

3.1. The Basic Algorithm

The true, but unknown, cross-covariance matrix satisfies (by **A1**)

$$\mathbf{C}_{x\xi} = \mathbf{A}\mathbf{C}_{x\xi} = \mathbf{U}_s \mathbf{\Sigma}_s \mathbf{V}_s^H \tag{3.1}$$

where $\mathbf{U}_s \in \mathbb{C}^{m\times n}$ and, by **A2**, $\mathcal{R}(\mathbf{U}_s) = \mathcal{R}(\mathbf{A})$. At time $t-1$, the estimated cross-covariance and its SVD is given as

$$\hat{\mathbf{C}}_{x\xi}(t-1) = \hat{\mathbf{U}}_s \hat{\mathbf{\Sigma}}_s \hat{\mathbf{V}}_s^H + \hat{\mathbf{U}}_n \hat{\mathbf{\Sigma}}_n \hat{\mathbf{V}}_n^H \tag{3.2}$$

where $\hat{\mathbf{\Sigma}}_n \neq \mathbf{0}$ with probability one. Next, collect new measurements and update the estimate of the cross-covariance:

$$\hat{\mathbf{C}}_{x\xi}(t) = \mu \hat{\mathbf{C}}_{x\xi}(t-1) + (1-\mu)\mathbf{x}(t)\boldsymbol{\xi}^H(t). \tag{3.3}$$

Computing the SVD of $\hat{\mathbf{C}}_{x\xi}(t)$ would require $O(ml^2)$ complex multiplications [1]. The basic question addressed in the following is whether we can reduce this complexity without sacrificing too much accuracy.

Define the following quantities:

$$\mathbf{x}_\mu = \sqrt{1-\mu}\,\mathbf{x}(t), \quad \boldsymbol{\xi}_\mu = \sqrt{1-\mu}\,\boldsymbol{\xi}(t) \tag{3.4a}$$

$$\mathbf{x}_s = \hat{\mathbf{U}}_s^H \mathbf{x}_\mu, \quad \boldsymbol{\xi}_s^H = \boldsymbol{\xi}_\mu^H \hat{\mathbf{V}}_s \tag{3.4b}$$

$$c_1\mathbf{u}_1 = \mathbf{x}_\mu - \hat{\mathbf{U}}_s\mathbf{x}_s, \quad c_2\mathbf{v}_1^H = \boldsymbol{\xi}_\mu^H - \boldsymbol{\xi}_s^H\hat{\mathbf{V}}_s^H \tag{3.4c}$$

where $\mathbf{u}_1^H\mathbf{u}_1 = \mathbf{v}_1^H\mathbf{v}_1 = 1$. Obviously, we have that

$$\hat{\mathbf{U}}_s^H\mathbf{u}_1 = \mathbf{0}, \quad \mathbf{v}_1^H\hat{\mathbf{V}}_s = \mathbf{0}. \tag{3.5}$$

Thus, $\mathbf{u}_1 = \mathbf{\Pi}_{\hat{U}_s}^{\perp}\mathbf{x}(t)/\|\mathbf{\Pi}_{\hat{U}_s}^{\perp}\mathbf{x}(t)\|$, and similarly for $\mathbf{v}_1$, where $\mathbf{\Pi}_{\hat{U}_s}^{\perp}$ denotes the orthogonal projection onto the complement of $\mathcal{R}(\hat{\mathbf{U}}_s)$. Now, write

$$(1-\mu)\mathbf{x}(t)\boldsymbol{\xi}^H(t) = \begin{bmatrix}\hat{\mathbf{U}}_s & \mathbf{u}_1\end{bmatrix} \underbrace{\begin{bmatrix}\mathbf{x}_s \\ c_1\end{bmatrix}\begin{bmatrix}\boldsymbol{\xi}_s^H & c_2\end{bmatrix}}_{\mathbf{T}_1} \begin{bmatrix}\hat{\mathbf{V}}_s^H \\ \mathbf{v}_1^H\end{bmatrix}. \tag{3.6}$$

Rather than computing the expensive SVD of $\hat{\mathbf{C}}_{x\xi}(t)$, we propose to compute the SVD of a perturbed matrix

$$\bar{\mathbf{C}}(t) = \hat{\mathbf{C}}_{x\xi}(t) + \boldsymbol{\varepsilon}(t) \tag{3.7}$$

where $\boldsymbol{\varepsilon}(t)$ denotes some perturbation at time t. A useful class of perturbations is given by

$$\boldsymbol{\varepsilon}(t) = -\mu\hat{\mathbf{U}}_n\hat{\mathbf{\Sigma}}_n\hat{\mathbf{V}}_n^H + \mu\bar{\sigma}_1\mathbf{u}_1\mathbf{v}_1^H + \mu\bar{\sigma}_2\mathbf{N}_1\mathbf{N}_2^H \tag{3.8}$$

where $\bar{\sigma}_1, \bar{\sigma}_2 \geq 0$ are user-defined scalar quantities, and $\mathbf{N}_1, \mathbf{N}_2$ denote matrices that have orthonormal columns and fulfill $\mathbf{N}_1 \perp \{\mathbf{u}_1,\ \hat{\mathbf{U}}_s\}, \mathbf{N}_2 \perp \{\mathbf{v}_1,\ \hat{\mathbf{V}}_s\}$. Note, the matrices $\mathbf{N}_1, \mathbf{N}_2$ are only dummies, and are not explicitly calculated. We can then write the matrix $\bar{\mathbf{C}}(t)$ as

$$\bar{\mathbf{C}}(t) = \begin{bmatrix}\hat{\mathbf{U}}_s & \mathbf{u}_1\end{bmatrix}\mathbf{T}(\bar{\sigma}_1)\begin{bmatrix}\hat{\mathbf{V}}_s^H \\ \mathbf{v}_1^H\end{bmatrix} + \mu\bar{\sigma}_2\mathbf{N}_1\mathbf{N}_2 \tag{3.9}$$

where $\mathbf{T}(\bar{\sigma}_1) = \mathbf{T}_1 + \mathbf{T}_2(\bar{\sigma}_1)$, and

$$\mathbf{T}_2(\bar{\sigma}_1) = \begin{bmatrix}\mu\hat{\mathbf{\Sigma}}_s & \mathbf{0} \\ \mathbf{0} & \mu\bar{\sigma}_1\end{bmatrix}. \tag{3.10}$$

Next, compute the $(n+1)\times(n+1)$ SVD of $\mathbf{T}(\bar{\sigma}_1)$

$$\mathbf{T}(\bar{\sigma}_1) = \mathbf{X}\mathbf{S}\mathbf{Y}^H. \tag{3.11}$$

We find that the *exact* SVD of the perturbed matrix $\bar{\mathbf{C}}(t)$ is given by

$$\begin{aligned}\bar{\mathbf{C}}(t) &= \begin{bmatrix}\hat{\mathbf{U}}_s & \mathbf{u}_1\end{bmatrix}\mathbf{X}\mathbf{S}\mathbf{Y}^H\begin{bmatrix}\hat{\mathbf{V}}_s^H \\ \mathbf{v}_1^H\end{bmatrix} + \mu\bar{\sigma}_2\mathbf{N}_1\mathbf{N}_2 \\ &= \begin{bmatrix}\bar{\mathbf{U}}_s & \mathbf{n}_1\end{bmatrix}\mathbf{S}\begin{bmatrix}\bar{\mathbf{V}}_s^H \\ \mathbf{n}_2^H\end{bmatrix} + \mu\bar{\sigma}_2\mathbf{N}_1\mathbf{N}_2.\end{aligned} \tag{3.12}$$

Thus, the estimate of the signal subspace at time t is taken as $\bar{\mathbf{U}}_s$, the estimate of the signal singular values is obtained from a partition of $\mathbf{S}$: $\bar{\mathbf{\Sigma}}_s = \mathbf{S}_{1:n,1:n}$, where we have used a MATLAB-like notation. Note, at time $t+1$ we of course replace $\hat{\mathbf{U}}_s(t)$ with $\bar{\mathbf{U}}_s(t)$ in (3.4). The SVD of $\mathbf{T}(\bar{\sigma}_1)$ is of complexity $O(n^3)$. Due to the matrix-matrix multiplication in (3.12), the overall complexity is $O(mn^2)$. In practical scenarios where $m, l \gg n$, this complexity reduction may be substantial. An advantage with the proposed approach is that only the important parts of the approximation of $\bar{\mathbf{C}}(t)$ is stored, and they are stored in a factored form.

3.2. Preliminary Analysis

Let's discuss some possible alternatives for the user-defined quantities $\bar{\sigma}_1, \bar{\sigma}_1$:

1) $\bar{\sigma}_1 = \bar{\sigma}_2 = 0$: This case corresponds simply to omit the term $\hat{\mathbf{U}}_n \hat{\mathbf{\Sigma}}_n \hat{\mathbf{V}}_n^H$ in (3.2). A theoretical motivation that justifies this ad-hoc step if given by Theorem 8.6.5 in [1]. This Theorem (roughly) states that "$O(\epsilon)$ changes in $\hat{\mathbf{C}}_{x\xi}(t)$ can alter a singular subspace by an amount $O(\epsilon/\delta)$ where δ measures the separation of the relevant singular values". In the scenario considered herein,

$$\epsilon \sim \|\mu \hat{\mathbf{U}}_n \hat{\mathbf{\Sigma}}_n \hat{\mathbf{V}}_n^H\|_F = \mu \sqrt{\sum_{i=n+1}^{\min(m,l)} \hat{\sigma}_i^2} \tag{3.13}$$

tends to be "small", due to the low-rank structure of (2.3), and there will be a "large gap" between the signal and noise singular values. Thus, the error in the approximated signal subspace will be "small". Furthermore, the error in the signal singular values will also be small, since from corollary 8.6.2 in [1] it holds that

$$|\sigma_k(\bar{\mathbf{C}}(t)) - \sigma_k(\hat{\mathbf{C}}_{x\xi}(t))| \leq \sigma_{n+1}(\hat{\mathbf{C}}_{x\xi}(t)). \tag{3.14}$$

2) $\bar{\sigma}_2 = 0$: In this case we include a term $\mu\bar{\sigma}_1 \mathbf{u}_1 \mathbf{v}_1^H$ which may be characterized as an instantaneous approximation of the "noise matrix" $\hat{\mathbf{U}}_n \hat{\mathbf{\Sigma}}_n \hat{\mathbf{V}}_n^H$. This since $\mathbf{u}_1 \in \mathcal{R}(\hat{\mathbf{U}}_n), \mathbf{v}_1 \in \mathcal{R}(\hat{\mathbf{V}}_n)$. Now, minimize the norm of the perturbation with respect to $\bar{\sigma}_1$, and we find that

$$\bar{\sigma}_1^{opt} = Re\left(\mathbf{x}^H(t)\hat{\mathbf{U}}_n \hat{\mathbf{\Sigma}}_n \hat{\mathbf{V}}_n^H \boldsymbol{\xi}(t)\right) \tag{3.15}$$

$$\epsilon^{opt} = \mu \sqrt{\sum_{i=n+1}^{\min(m,l)} \hat{\sigma}_i^2 - \mu(\bar{\sigma}_1^{opt})^2}. \tag{3.16}$$

Consequently, we may choose an $\bar{\sigma}_1$ so that the norm of the perturbation is smaller than in the previous case. However, there is no obvious implication that the statistical quality of $\bar{\mathbf{U}}_s$ is improved with this "optimal" choice. A better approach would be to choose $\bar{\sigma}_1$ so that the distance, see [1], between $\bar{\mathbf{U}}_s$ and its SVD counterpart is minimized. Such an analysis is however a topic for future research. In the simulations we heuristically have chosen $\bar{\sigma}_1(t) = \mathbf{S}_{n+1,n+1}(t-1)$, which in our simulations have not degraded the performace compared to (3.15).

3) $\bar{\sigma}_1 = \bar{\sigma}_2 \neq 0$: Compared to case 2, this choice of perturbation does not influence any of the computations. However, this choice brings up the possibility to interpret the proposed algorithm as an IV counterpart of Karasalo's subspace averaging algorithm [3]. In fact, it can be shown that if $\boldsymbol{\xi}(t) = \mathbf{x}(t)$, this approach corresponds exactly to [3]. These interpretations lead us to define

$$\bar{\sigma}_1(t) = \frac{1}{\tilde{n}} \mathbf{S}_{n+1,n+1}(t) + \frac{\tilde{n}-1}{\tilde{n}} \mu \bar{\sigma}_1(t-1), \tag{3.17}$$

where $\tilde{n} = \min(m, l)$. For this to be meaningful, at every time t, it should hold that

$$\bar{\sigma}_1(t) \leq \mathbf{S}_{n,n}(t). \tag{3.18}$$

In words this mean that the estimated "noise level" should be smaller than the smallest signal singular value. This can be proven to hold, but for the proof of this fact we refer to the full version of this paper.

There are several problems with designing an IV counterpart of Karasalo's algorithm. First of all, Karasalo's algorithm results from minimizing a very natural least squares criterion. For the above IV version, we have not found such a strong motivation. Another problem that occurs when attempting to find an IV version, lies in the freedom of choosing noise eigenvectors utilized by Karasalo. In the present case we can write

$$\mathbf{u}_1 = \hat{\mathbf{U}}_n \mathbf{t}_1, \text{ and } \mathbf{v}_1 = \hat{\mathbf{V}}_n \mathbf{t}_2, \tag{3.19}$$

for some unity norm vectors $\mathbf{t}_1, \mathbf{t}_2$. The difficulty is that $\mathbf{t}_1 \neq \mathbf{t}_2$ in the IV-case, whereas

$$\mathbf{t}_1 = \mathbf{t}_2, \text{ and } \hat{\mathbf{U}}_n = \hat{\mathbf{V}}_n \tag{3.20}$$

in Karasalo's algorithm. In conclusion, even if we have no well-motivated criterion as the basis for the proposed algorithm, the fact that $\boldsymbol{\xi}(t) = \mathbf{x}(t)$ leads to Karasalo's algorithm, allow us to interpret the proposed algorithm as an IV based subspace averaging algorithm.

Even though we should except $\bar{\sigma}_1(t) \approx 0$, compared to $\bar{\sigma}_1(t) \approx \sigma^2$ in the "non-IV" spatially white noise case, (3.17) handles the possibility that n is time-varying. Furthermore, we believe that this is a more accurate description of the fact that $\hat{\mathbf{\Sigma}}_n \geq \mathbf{0}$, with probability one. In the examples, we demonstrate that $\bar{\sigma}_1(t)$ is as good in detecting changes in n as are the noise singular values of the SVD. In case 1 and 2 above, $\mathbf{S}_{n+1,n+1}$ is used as such an indicator. The main advantage with the averaging approach is that then $\bar{\sigma}_1(t)$ has a built in smoothing, thus they are less noisy. Which one of the proposed perturbation strategies is the best, in terms of accuracy of $\bar{\mathbf{U}}_s$, is a topic of further research.

A non-trivial problem is to analyze how the errors introduced by the proposed algorithm propagates. Since the errors propagate through non-linear and time-variant filtering, this is difficult. So far we have no results on the convergence properties of the proposed algorithm, and we have to resort to simulations.

4. EXAMPLES

In this section the results of the proposed algorithm is compared with the estimates obtained from the truncated SVD of $\hat{\mathbf{C}}_{x\xi}$. In a sense, given $\hat{\mathbf{C}}_{x\xi}$ the truncated SVD is the best possible way to find the subspace estimate. It is therefore interesting to compare the SVD-based estimates with those of the proposed algorithms. The DOA estimates are found using the ESPRIT algorithm. A ULA of 12 elements is studied. The first $m = 6$ sensors form a calibrated subarray, whereas the outputs of the last $l = 6$ sensors are used as IVs. Both subarrays are corrupted by additive zero-mean temporally white Gaussian noise, independent from subarray to subarray. The noise covariance matrix of both subarrays have (k, l):th element

$$[Q]_{k,l} = \sigma^2 0.9^{|k-l|} e^{j\frac{\pi}{2}(k-l)}. \tag{4.1}$$

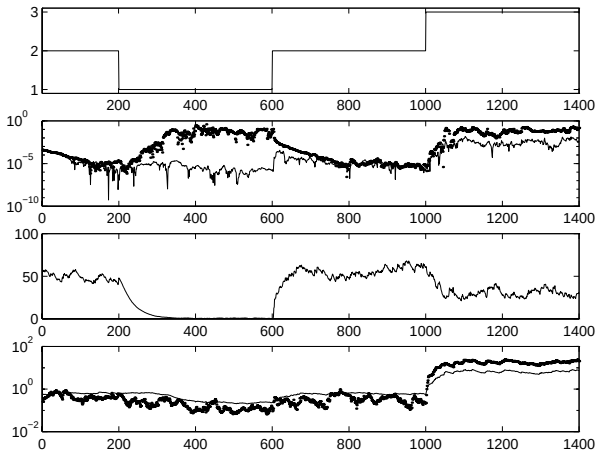

Figure 1: a) #sources present. b) Relative error in the SV. Solid: largest SV, Dotted: Second largest SV. c) Second largest SV of the proposed approach d) Dotted: sum of noise singular values, Solid: Estimate from proposed approach (case 3). $\mu = 0.98$. SNR=10 dB.

μ / SNR	-10	-5	0	5
0.95	7.74	2.69	1.01	0.488
0.97	6.44	1.85	0.720	0.384
0.99	3.12	1.01	0.434	0.224

Table 1: RMSE (in $(\cdot)^\circ$) for SVD.

This noise is reminiscent of a signal cluster at $\theta = 30^\circ$. The noise level σ^2 is adjusted to give a desired SNR defined as, SNR=$1/\sigma^2$. In Fig.1, we plot the estimated signal singular values and the singular values of $\hat{\mathbf{C}}_{x\xi}(t)$ (and the relative error) in a scenario where the number of signals is time-varying, the DOAs are $[0^\circ\ 20^\circ\ 10^\circ]$. The emitter signals are zero-mean white and Gaussian, $\mathbf{P} = \mathbf{I}$, and SNR=10 dB. The algorithms are run with the hypothesis that $n = 2$. Based on some threshold strategy, the conclusion is that the proposed algorithm would decide the same changes in n as does the SVD. Note, the large relative error in the interval 300-600 is due that the singular values are close to zero.

Consider next the stationary performance. In this scenario, two planar wavefronts arrive at DOAs $[0^\circ\ 20^\circ]$. In Tables 1 and 2 we summarize the outcome from 500 Monte Carlo simulations. The conclusion is that the proposed algorithm has essentially equivalent estimation accuracy as has SVD for high to medium SNR's, and slightly worse for low SNR's.

Finally, consider a tracking example, see Fig. 2. In this simulation there are two sources present, and the DOA of one of the sources is time-varying. The emitter signals are once again zero-mean, temporally white, and Gaussian, $\mathbf{P} = \mathbf{I}$ with SNR=10 dB. Study especially the relative error for the time-varying source!

μ / SNR	-10	-5	0	5
0.95	8.22	2.75	1.02	0.488
0.97	6.96	1.91	0.725	0.385
0.99	3.54	1.05	0.436	0.224

Table 2: RMSE (in $(\cdot)^\circ$) for the proposed algorithm (case 1)

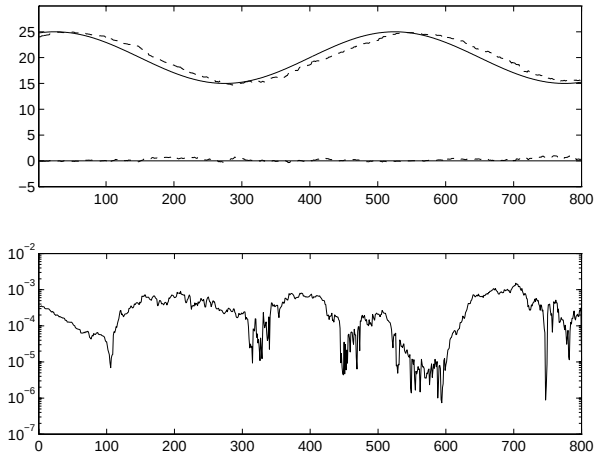

Figure 2: Tracking performance. Top: The DOA trajectories for the proposed case 1 algorithm (dashed) and the true DOAs (solid). Bottom: The relative error compared to the SVD. $\mu = 0.97$, SNR=10 dB.

5. CONCLUSIONS

In this paper we have proposed an $O(mn^2)$ IV-based subspace tracking algorithm. This low complexity is achieved by applying a certain perturbation strategy. Based on the simulation results, the conclusion is that we significantly have reduced the amount of computations at very little cost in terms of estimation and detection accuracy.

6. REFERENCES

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