DECOUPLED DIRECTION FINDING: DETECTION

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ABSTRACT

Antenna arrays are likely to be an important feature of future mobile communication systems. With an antenna array, mobile users can be separated by a spatial filtering procedure allowing several users on the same carrier frequency. The uplink part (mobile to base) not only can, but is better solved without using any spatial knowledge in terms of direction of arrival (DOA). However, DOA estimation remains an important issue in the overall system, both for downlink beamforming, as well as channel allocation. Previous results have shown that DOA-estimation is best performed in a post-detection manner, i.e., using the estimated symbol sequence for DOA estimation. In this way, the estimation problem can be decoupled to treat individual users separately. To estimate the number of propagation paths from a specific user, a detection scheme is derived based on the DOA estimation criterion function.

1. INTRODUCTION

Algorithms for separation of multiple signals arriving at an antenna array have been a major research issue during the last couple of years. A key result is that algorithms based on the structure of the signal itself, rather than the spatial structure, by far outperform traditional algorithms based on direction of arrival (DOA) estimation and beamforming ([1]-[3] and the references in [4]). The latter DOA-type algorithms rely on a signal model assuming that the signal arrive at the array from a distinct DOA, or in a narrow cluster. This is not true in a multipath channel with sometimes widely spread scatters. In comparison, signal structure based algorithms make no assumptions about DOA:s and related channel parameters.

The limitation of the signal structure based techniques is that they are only applicable on the *uplink* (*i.e.* transmission from mobile to base). The uplink beamformer provided by the algorithm is likely to be useless in the downlink. This is usually true regardless of the duplexing scheme employed: in a Time-Division Duplex (TDD) system, where the up- and downlink employ the same carrier frequency, fading will cause the channel to change during the uplink processing delay. In a Frequency Division Duplex (FDD) system, with different carrier frequencies for the up- and downlink, the fading will be different even at zero processing delay.

Consequently, estimation of channel parameters, such as the DOA:s, is still a matter of interest. This is not only the case in the context of downlink beamforming. A high-performing estimation scheme is also an important tool to analyse experimental array data in order to acquire knowledge of channels, and models thereof.

An ML-estimator for the DOA:s in a flat fading environment was proposed in [4]. This estimator is based on a multipath propagation model. The approach taken is to *first* apply a signal structure based algorithm to estimate the transmitted symbol sequence, and *then* estimate the DOA:s. This post-detection approach gives a significant improvement in signal to interference ratio compared to the classical pre-detection estimator. It was shown in [4] that the signal estimated by the signal structure based algorithm can be regarded as being exact at low symbol error rates (SER). Also, the estimation problem decouples, meaning that cochannel users can be treated independently. Another interesting result is that the asymptotic performance does not suffer from not knowing the interference covariance [5]. The focus is now turned towards detection of the number of significant propagation paths. The detection scheme to be proposed also applies to the case of delay spread channels.

2. ESTIMATION

The signal received by an m-element base station antenna array in a flat fading environment can be modeled as

$$\boldsymbol{x}(n) = \boldsymbol{h}s(n) + \boldsymbol{j}(n) \tag{1}$$

where $s(n)$ is the signal symbol to be estimated by the signal structure based (blind or semi-blind) algorithm. The term $\boldsymbol{j}(n)$ models the total interference which might have several sources: inter- and intracell cochannel interference and thermal noise. The total interference is modeled as a temporally white circularly symmetric Gaussian process with arbitrary spatial covariance. The vector $\boldsymbol{h}$ is the *spatial signature*, which can be modeled as a sum of responses due to d wave propagation paths:

$$\boldsymbol{h} = \sum_{i=1}^{d} \rho_i \boldsymbol{a}(\theta_i) = \begin{bmatrix} \boldsymbol{a}(\theta_1) & \dots & \boldsymbol{a}(\theta_d) \end{bmatrix} \begin{bmatrix} \rho_1 \\ \dots \\ \rho_d \end{bmatrix} = \boldsymbol{A}(\Theta)\boldsymbol{b} \,. \tag{2}$$

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1. This work was supported by the Swedish National Board for Industrial and Technical Development (NUTEK).

The vector function $\boldsymbol{a}(\theta_i)$ (the array manifold) represents the array response to a unit signal from the DOA θ_i, whereas ρ_i is the complex amplitude of path *i*. The distinct DOA:s are collected in a parameter vector $\Theta = [\theta_1 \ \dots \ \theta_d]$. The ML-estimator of Θ and ***b***, according to the model (2) including the effect of modeling errors, was considered in [4].

It is easy to extend the above model to cover for delay spread (letting L+1 be the filter length):

$$\boldsymbol{x}(n) = \begin{bmatrix}\boldsymbol{h}_0 & \dots & \boldsymbol{h}_L\end{bmatrix}\begin{bmatrix}s(n)\\ s(n-1)\\ \dots\\ s(n-L)\end{bmatrix} + \boldsymbol{j}(n) = \boldsymbol{H}\boldsymbol{s}(n) + \boldsymbol{j}(n). \quad (3)$$

A better physical model is given by reformulating in terms of DOA:s, path amplitudes and delays as

$$\begin{aligned}\boldsymbol{x}(n) &= \begin{bmatrix}\boldsymbol{a}(\theta_1) & \dots & \boldsymbol{a}(\theta_d)\end{bmatrix}\begin{bmatrix}\boldsymbol{p}_1(\rho_1,\tau_1)\\ \dots\\ \boldsymbol{p}_d(\rho_d,\tau_d)\end{bmatrix}\boldsymbol{s}(n) + \boldsymbol{j}(n) \\ &= \boldsymbol{A}(\Theta)\boldsymbol{B}\boldsymbol{s}(n) + \boldsymbol{j}(n)\end{aligned} \quad (4)$$

where $\boldsymbol{p}_i(\rho_i,\tau_i)$ is a (L+1) $\times$ 1 row vector describing the effects of pulse shaping, amplitude, as well as time delay for path *i*. Collecting *N* array snapshots, a general block-model for both (2) and (4) is given by

$$\boldsymbol{X} = \boldsymbol{A}(\Theta)\boldsymbol{B}\boldsymbol{S} + \boldsymbol{J} = \boldsymbol{H}\boldsymbol{S} + \boldsymbol{J} \quad (5)$$

where $\boldsymbol{X} = [\boldsymbol{x}(1) \ \dots \ \boldsymbol{x}(N)]$ etc.

An approximate ML-estimate of the DOA vector Θ is given as the minimizing argument of [5]

$$\boldsymbol{V}(\Theta) = \left\|\hat{\Pi}_A^{\perp}\hat{\boldsymbol{Q}}^{-1/2}\hat{\underset{\sim}{\boldsymbol{H}}}\right\|_F^2 = \left\|\hat{\Pi}_A^{\perp}\hat{\underset{\sim}{\boldsymbol{G}}}\right\|_F^2 \quad (6)$$

where

$$\hat{\Pi}_A^{\perp} = \boldsymbol{I} - \hat{\boldsymbol{Q}}^{-1/2}\boldsymbol{A}(\boldsymbol{A}^H\hat{\boldsymbol{Q}}^{-1}\boldsymbol{A})^{-1}\boldsymbol{A}^H\hat{\boldsymbol{Q}}^{-1/2} \quad (7)$$

with $\hat{\boldsymbol{Q}}^{-1/2}$ being a Hermitian square-root of the inverse of the estimated interference covariance matrix

$$\hat{\boldsymbol{Q}} = \hat{\boldsymbol{R}}_X - \hat{\underset{\sim}{\boldsymbol{H}}}\hat{\underset{\sim}{\boldsymbol{H}}}^H. \quad (8)$$

$\hat{\underset{\sim}{\boldsymbol{H}}}$ is given as

$$\hat{\underset{\sim}{\boldsymbol{H}}} = \frac{1}{N}\boldsymbol{X}\boldsymbol{S}^H\boldsymbol{R}_{SS}^{-1/2} \quad (9)$$

where $\boldsymbol{R}_{SS}^{-1/2}$ serves to decorrelate the signal basis. 'Hats' on ***S*** and $\boldsymbol{R}_{SS}^{-1/2}$ are omitted since the estimated signal can be regarded as being exact at low symbol error rates [4]. Note that in the case L = 0, (6) coincides with the exact ML-estimator [5].

3. DETECTION

The previously presented estimator presumes the number of propagation paths, *d*, to be known. In practice, this is not the case, and a simple detection scheme can be formulated based on the statistics of the criterion function (6), in analogy with the WSF detection scheme in [6].

3.1 General Detection Statistic

Theorem 1: *The ML-criterion (6), normalized by the multiplicative factor 2N, is asymptotically chi-squared distributed with* $2(L+1)(m-d)$ *degrees of freedom, when evaluated at* Θ_o. *When evaluated at the estimate* $\hat{\Theta}$, *the asymptotic distribution is still chi-squared, but with* $2(L+1)(m-d)-d$ *degrees of freedom.*

Proof: The estimated interference covariance matrix is estimated based on sample moments, implying that

$$\hat{\boldsymbol{Q}} = \boldsymbol{Q} + O_p(N^{-1/2}). \quad (10)$$

The error in the estimated weighting matrix $\hat{\boldsymbol{W}} = \hat{\boldsymbol{Q}}^{-1/2}$ will be of the same asymptotic order:

$$\hat{\boldsymbol{W}} = \boldsymbol{W} + \tilde{\boldsymbol{W}} = \boldsymbol{W} + O_p(N^{-1/2}). \quad (11)$$

From (9), the weighted channel matrix estimate is

$$\hat{\underset{\sim}{\boldsymbol{G}}} = \hat{\boldsymbol{W}}\frac{1}{N}\boldsymbol{X}\boldsymbol{S}^H\boldsymbol{R}_{SS}^{-1/2} = \hat{\boldsymbol{W}}\frac{1}{N}(\boldsymbol{H}\boldsymbol{S} + \boldsymbol{J})\underset{\sim}{\boldsymbol{S}}^H. \quad (12)$$

The rows of the transformed signal matrix $\underset{\sim}{\boldsymbol{S}}^H$ are now a set of orthonormal basis functions.

Each column of $\hat{\underset{\sim}{\boldsymbol{G}}} = [\hat{\underset{\sim}{\boldsymbol{g}}}_0 \ \dots \ \hat{\underset{\sim}{\boldsymbol{g}}}_L]$ is given by

$$\begin{aligned}\hat{\underset{\sim}{\boldsymbol{g}}}_i &= \hat{\boldsymbol{W}}\frac{1}{N}(\boldsymbol{H}\boldsymbol{S} + \boldsymbol{J})\underset{\sim}{\boldsymbol{s}}_i^H = \hat{\boldsymbol{W}}\underset{\sim}{\boldsymbol{h}}_i + \hat{\boldsymbol{W}}\frac{1}{N}\boldsymbol{J}\underset{\sim}{\boldsymbol{s}}_i^H \\ &= \hat{\boldsymbol{W}}\underset{\sim}{\boldsymbol{h}}_i + (\boldsymbol{W} + \tilde{\boldsymbol{W}})\frac{1}{N}\boldsymbol{J}\underset{\sim}{\boldsymbol{s}}_i^H \\ &= \hat{\boldsymbol{W}}\underset{\sim}{\boldsymbol{h}}_i + \boldsymbol{W}\frac{1}{N}\boldsymbol{J}\underset{\sim}{\boldsymbol{s}}_i^H + O_p(N^{-1})\end{aligned} \quad (13)$$

The projection matrix (7) can be factorized as

$$\hat{\Pi}_A^{\perp} = \hat{\boldsymbol{U}}(\Theta)\hat{\boldsymbol{U}}^H(\Theta) \quad (14)$$

where $\boldsymbol{U}(\Theta)$ is a unitary matrix of size $m \times (m\text{-}d)$. The L+1 vectors $\hat{\underset{\sim}{\boldsymbol{g}}}_i$ are transformed into a new set of size (m-d) $\times$ 1 vectors

$$\hat{\boldsymbol{u}}_i = \hat{\boldsymbol{U}}^H(\Theta)\hat{\underset{\sim}{\boldsymbol{g}}}_i \quad (15)$$

and the criterion (6) is rewritten as

$$\boldsymbol{V}(\Theta) = \sum_{i=1}^{L+1}\hat{\boldsymbol{u}}_i^H\hat{\boldsymbol{u}}_i. \quad (16)$$

At $\Theta = \Theta_o$, $\hat{\boldsymbol{U}}^H(\Theta_o)\hat{\boldsymbol{W}}\underset{\sim}{\boldsymbol{h}}_i = \hat{\boldsymbol{U}}_o^H\hat{\boldsymbol{W}}\underset{\sim}{\boldsymbol{h}}_i = \boldsymbol{0}$, and combining (13) with (15) yields

$$\hat{\boldsymbol{u}}_i = \hat{\boldsymbol{U}}_o^H \hat{\boldsymbol{g}}_i = \boldsymbol{U}_o^H \boldsymbol{W} \frac{1}{N} \boldsymbol{J} \underset{\sim}{\boldsymbol{s}}_i^H + O_p(N^{-1}) \tag{17}$$

In view of the central limit theorem, the asymptotic distribution of $\hat{\boldsymbol{u}}_i$ at Θ_o will be Gaussian with moments

$$E[\hat{\boldsymbol{u}}_i] = \mathbf{0} + O(N^{-1}) \tag{18}$$

$$\begin{aligned} E[\hat{\boldsymbol{u}}_i \hat{\boldsymbol{u}}_j^H] &= \boldsymbol{U}_o^H \frac{1}{N^2} \boldsymbol{W} E[\boldsymbol{J} \underset{\sim}{\boldsymbol{s}}_i^H \underset{\sim}{\boldsymbol{s}}_j \boldsymbol{J}^H] \boldsymbol{W}^H \boldsymbol{U}_o + O(N^{-3/2}) \\ &= \boldsymbol{U}_o^H \frac{1}{N} \boldsymbol{W} \boldsymbol{Q} \boldsymbol{W}^H \delta_{ij} \boldsymbol{U}_o + O(N^{-3/2}) \\ &= \frac{1}{N} \boldsymbol{I}_{m-d} \delta_{ij} + O(N^{-3/2}) \end{aligned} \tag{19}$$

$$\begin{aligned} E[\hat{\boldsymbol{u}}_i \hat{\boldsymbol{u}}_j^T] &= \boldsymbol{U}_o^H \frac{1}{N^2} \boldsymbol{W} E[\boldsymbol{J} \underset{\sim}{\boldsymbol{s}}_i^H \underset{\sim}{\boldsymbol{s}}_j^* \boldsymbol{J}^T] \boldsymbol{W}^T \boldsymbol{U}_o^* + O(N^{-3/2}) \\ &= \mathbf{0} + O(N^{-3/2}) \end{aligned} \tag{20}$$

where the latter follows from the assumption $\boldsymbol{J}$ being circularly symmetric[2]. The rest of the proof follows from [6].

Theorem 1 leads to the following result.

A Criterion-based Detection Scheme:

1. Set $\hat{d} = 1$.
2. Null hypothesis, H_0: $\hat{d} = d$
3. Set a threshold γ based on the tail area of the chi-square distribution with $2(L+1)(m-\hat{d})-\hat{d}$ degrees of freedom.
4. If $2N\boldsymbol{V}(\hat{\Theta}) < \gamma$ accept H_0 and stop, else set $\hat{d} = \hat{d} + 1$ and return to 3.

The threshold γ gives the conditional probability of detection $P_{Dd} = P(H_o|H_o)$. The overall unconditional probability of a correct detection depends on the probability of underestimating d, and is given as

$$P_D = \prod_{i=1}^{d-1} (1 - P_{FAi}) P_{Dd} \tag{21}$$

The proposed scheme does not give consistent detection, i.e., $P_D \to 1$ as $N \to \infty$ [6]. However, consistency is not an important issue here as the channel parameters are time-varying, and processing will always be performed on finite block lengths (N). The scheme can be compared to the one in [8], but differs in a major aspect: the latter does not allow the detection problem to decouple. The decoupling is a consequence of the exploitation of the estimated symbol sequence. An advantage of the above scheme as compared to detectors based on information theoretic criteria, such as MDL [9], is that the latter requires DOA estimation also for one excess path, i.e. $\hat{d}+1$.

3.2 Alternative Weighting

In some applications it might be advantageous to weight data with $\hat{\boldsymbol{R}}_X^{-1/2}$ in place of $\hat{\boldsymbol{Q}}^{-1/2}$, in order to reduce the computational complexity. An example is if several users' channels are to be estimated from the same array data. The operations $\boldsymbol{WX}$ and $\boldsymbol{WA}$, $\boldsymbol{A}$ being the array manifold, then only have to be carried out once. From an estimation point of view, this does not affect the asymptotic performance. It is easy to show that the same holds for detection:

Theorem 2: *If $\hat{\boldsymbol{Q}}^{-1/2}$ in (6) and (7) is replaced with $\hat{\boldsymbol{R}}_X^{-1/2}$ Theorem 1 still holds.*

Proof: With $\hat{\boldsymbol{W}} = \hat{\boldsymbol{R}}_X^{-1/2}$, equation (19) will be

$$\begin{aligned} E[\hat{\boldsymbol{u}}_i \hat{\boldsymbol{u}}_j^H] &= \boldsymbol{U}_o^H \frac{1}{N} \boldsymbol{R}_X^{-1/2} \boldsymbol{Q} \boldsymbol{R}_X^{-1/2} \delta_{ij} \boldsymbol{U}_o + O(N^{-3/2}) \\ &= \boldsymbol{U}_o^H \frac{1}{N} \boldsymbol{R}_X^{-1/2} (\boldsymbol{R}_X - \underset{\sim}{\boldsymbol{H}} \underset{\sim}{\boldsymbol{H}}^H) \boldsymbol{R}_X^{-1/2} \delta_{ij} \boldsymbol{U}_o + O(N^{-3/2}) \\ &= \boldsymbol{U}_o^H \frac{1}{N} (\boldsymbol{I} - \boldsymbol{R}_X^{-1/2} \underset{\sim}{\boldsymbol{H}} \underset{\sim}{\boldsymbol{H}}^H \boldsymbol{R}_X^{-1/2}) \delta_{ij} \boldsymbol{U}_o + O(N^{-3/2}) \\ &= \frac{1}{N} \boldsymbol{I}_{m-d} \delta_{ij} + O(N^{-3/2}) \end{aligned} \tag{22}$$

as $\boldsymbol{R}_X^{-1/2} \underset{\sim}{\boldsymbol{H}}$ lies in the nullspace of $\boldsymbol{U}_o^H$. The statistics of $\hat{\boldsymbol{u}}_i$ remain unchanged.

One can add that in a situation where the true $\boldsymbol{R}_X$ is ill-conditioned (high signal to noise ratio (SNR)), and N is small, it is wise to regularize $\hat{\boldsymbol{R}}_X$, to avoid statistical instability.

4. EXAMPLES

The first example is similar to the one in [4]. Two mobile cochannel users transmit QPSK-modulated signals in bursts of $N = 200$ symbols simultaneously. An 8-element uniform linear array (ULA) is used for reception. The two signals are observed in additive spatially and temporally white Gaussian noise. The channels linking the mobiles to the base station are frequency flat 2-ray ($d = 2$) channels with parameters (2) $\Theta_1 = [2^o\ 10^o]$ and $\Theta_2 = [-2^o\ -10^o]$, relative to the array broadside, and $\boldsymbol{b}_1 = \boldsymbol{b}_2 = [0.5\ \ 1]^T$. The proposed estimation/detection scheme was employed to estimate d and Θ_1. The alternating projections algorithm of [10] was used to minimize (6).

Figure 1 shows the resulting unconditional probability of a correct detection P_D (21) for various SNR:s. The conditional probability of detection (P_{Dd}), determining the threshold γ, was varied from 95% - 99%. A vertical line is included in the plot to indicate at what SNR the estimation rms-error is 4^o, i.e. half of the angular separation of the two rays.

2. From relations (18) to (20), the residuals of (6) are white, which shows that (6) is the (asymptotically) best weighted subspace fitting criterion for known signals in unknown noise.

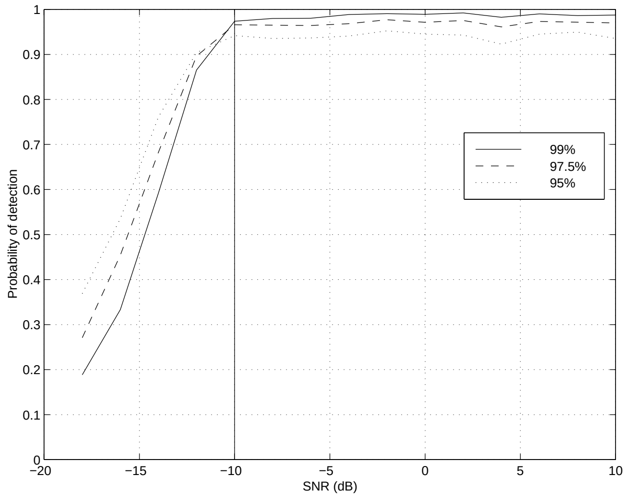

Figure 1. Frequency flat channel. Probability of correct detection (P_D), versus SNR, for different conditional detection probabilities.

The second example involves a delay spread channel with L+1 = 4. Again two users transmit QPSK signals in bursts of 200 symbols. The channels are 3-ray channels with parameters $\Theta_1 = [5^o\ 10^o\ 20^o] = -\Theta_2$ and

$$\boldsymbol{B}_1 = \boldsymbol{B}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{23}$$

The detection performance for this setting is shown in Figure 2. The vertical line is now drawn where the largest rms estimation error is 2.5^o.

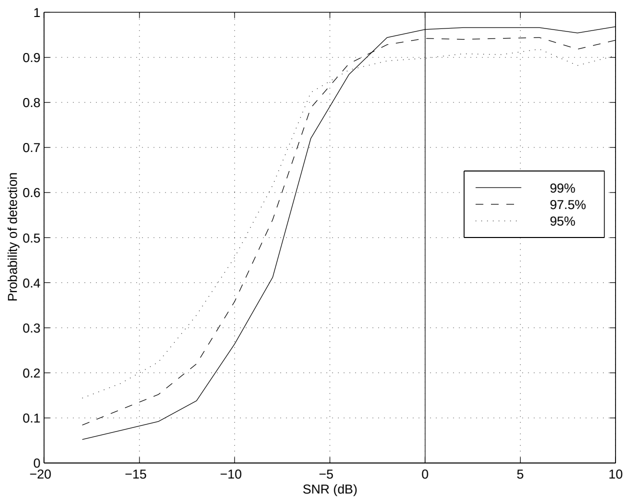

Figure 2. Channel with delay spread. Probability of correct detection (P_D), versus SNR, for different conditional detection probabilities.

Two things can be seen from the Figures. First, the detection threshold (expressed in SNR) is somewhat lower than the estimation threshold. This means that the detection scheme provides reliable detection in the region where the DOA:s can be estimated with good accuracy. Second, it can be seen that the false alarm probabilities P_{FAi} in (21) are small. The unconditional detection probability P_D can be kept in control by setting an appropriate threshold γ.

5. CONCLUSIONS

A detection scheme to estimate the number of signal paths arriving at an antenna array in a multipath scenario has been proposed. The detection scheme is based on the statistics of the criterion function employed by the DOA estimator. As the estimator is decoupled, *i.e*. treating different user signals separately, the same holds for the detector. Numerical experiments indicate good detection performance: the detection scheme works well also at low SNR:s where the estimator lose performance.

6. REFERENCES

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