DETECTION OF UNCERTAIN MULTIPLE CISIOID MODELS

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ABSTRACT

The problem of detection of multiple complex sinusoidals, with uncertain parameters, is addressed in this paper. It is shown that uncertainties in amplitude and small uncertainties in frequency can be handled analytically, while unknown phases must be handled numerically. Robust detectors for some or all of the uncertainties are formulated. Performance in noise, and robustness are evaluated through simulations. Finally the applicability of the detectors for the problem of radar target recognition is discussed, and some results are presented.

1. INTRODUCTION

Multiple complex sinusoidal, or cisoid, models are common in signal processing. Estimation methods for such models are abundant and well known (see e.g. [1] and references therein). In this paper we study detection, i.e. the decision of which of a number of models that best describe the observed data. Optimal detection is based on the probability of the data for the considered models, in the binary case through the classical likelihood ratio test. Evaluation of the likelihood of the data for a given cisoid model is straight forward. The problem is that this simple approach can not be used for applications that have models with uncertain parameters, as is the case in many practical situations. Unfortunately, integration of the parameterized likelihood over the prior probabilities for the uncertain parameters, “marginalization”, is not analytically tractable in the general case. Approximate solutions must be found. In the present work two kinds of approximations are employed: Firstly, small uncertainties are linearized and a Gaussian prior assumed, then the integral can be solved analytically. Secondly, the integral may be approximated by the maximum of the integrand. Detection using this approach is usually referred to as a generalized likelihood ratio test (GLRT). We use this method to handle the phase uncertainty, and show that a Newton search algorithm can be used to effectively find the maximum.

In the following sections we derive the likelihood functions for models with different kinds of uncertainties. The detection performance for some simple cases are studied through simulations. Finally, we describe an example of application to radar target recognition, where this method should be very beneficial.

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2. MODEL FORMULATION

To establish notation we first state the likelihood for an exact model. Then in following subsections we develop the likelihood for different kinds of uncertainties.

2.1. Exact Model

The matrix form of a cisoid model in Gaussian noise is:

$$x = s(\omega, a, \phi) + n = MA\psi + n \qquad n \in CN(0, R) \tag{1}$$

where $x=[x(1) \ldots x(N)]^T$ is the observed signal, $\omega=[\omega_1 \ldots \omega_L]^T$ the frequencies, $a=[a_1 \ldots a_L]^T$ the amplitudes and $\phi=[\phi_1 \ldots \phi_L]^T$ the phases of the model. R is the covariance matrix for n, and

$$M = \begin{bmatrix} m_1 \ldots m_L \end{bmatrix} \qquad m_k = \begin{bmatrix} 1 \; e^{i\omega_1} \ldots e^{i(N-1)\omega_L} \end{bmatrix}^T$$
$$A = \mathrm{Diag}(a) \qquad \psi = \begin{bmatrix} e^{i\phi_1} \ldots e^{i\phi_L} \end{bmatrix}^T$$

We can then state the likelihood of observing x:

$$p(x|\omega, a, \phi) = \pi^{-N}|R|^{-1}e^{-\mu}$$
$$\mu = (x - MA\psi)^H R^{-1}(x - MA\psi) \tag{2}$$

where ^{H} denotes conjugate transpose. When the noise is white, $R=\sigma^2 I$, then

$$\mu = \frac{1}{\sigma^2}(x^H x + \psi^H AGA\psi - 2\mathrm{Re}[\psi^H AM^H x])$$
$$G = M^H M \tag{3}$$

where we have introduced the matrix G. When the cisoids have well separated frequencies $G \approx N I$, and the likelihood can be factored into a product of the likelihoods of the individual cisoids.

2.2. Phase Uncertainty

In order to extend the model in the previous subsection to the case were the phases are unknown, we have to evaluate:

$$p(x|\omega,a) = \int p(x|\omega, a, \phi)p(\phi)d\phi \tag{4}$$

for $p(\phi_k)=1/2\pi$. We have

$$p(x|\omega,a) = \pi^{-N}|R|^{-1}(2\pi)^{-L}\int e^{-\mu(\phi)}d\phi$$

$$\mu(\phi) = x^H R^{-1} x + \psi^H A M^H R^{-1} M A \psi - 2\mathrm{Re}[\psi^H A M^H R^{-1} x] \tag{5}$$

The function $\mu(\phi)$ has one minimum at $\phi = \hat{\phi}$. Second order approximation around that minimum gives:

$$p(x|\omega,a) \approx \pi^{-N}|R|^{-1}\ \nu\ e^{-\mu(\hat{\phi})} \tag{6}$$

$$\nu = (2\pi)^{-L}\int e^{-\frac{1}{2}(\phi-\hat{\phi})H(\phi-\hat{\phi})}d\phi$$

$$\hat{\phi} = \underset{\phi}{\mathrm{Arg\,Min}}\ \mu(\phi) \qquad H = \left.\frac{\partial^2\mu}{\partial\phi\partial\phi^T}\right|_{\phi=\hat{\phi}} \tag{7}$$

If the integration interval can be extended to the whole $\mathbf{R}^N$ we have $\nu = (2\pi)^{-L/2}|H|^{-1/2}$. This is applicable if the minimum eigenvalue of H is large enough. Another acceptable approximation would be to set $\nu = 1$, its upper bound. The exact value is of minor interest since (6) is dominated by μ rather than ν. To find $\hat{\phi}$ (and H) we have used a numerical method and the same approach as in [2], with:

$$\left.\frac{\partial\mu}{\partial\phi}\right|_{\phi=\alpha} = \mathrm{Im}(\mathrm{diag}(\Upsilon ww^H - ww^H\Upsilon) + 2w^* \bullet AM^H R^{-1}x)$$

$$\left.\frac{\partial^2\mu}{\partial\phi\partial\phi^T}\right|_{\phi=\alpha} = 2\,\mathrm{Re}((ww^H - I)\bullet\Upsilon + \mathrm{Diag}(w^*\bullet AM^H R^{-1}x))$$

$$\Upsilon = AM^H R^{-1} MA \qquad w = \left[e^{i\alpha_1} \dots e^{i\alpha_L}\right]^T \tag{8}$$

where “diag” denotes taking the diagonal of a matrix as a vector, “Diag” denotes making a diagonal matrix from a vector, * is complex conjugation, and • is an component-wise product.

We use a Newton minimization to find $\hat{\phi}$. Some additions are made to handle points where the Hessian is not positive definite (in which case we use the steepest decent direction).

2.3. Amplitude and Phase Uncertainties

To handle uncertainties in amplitude we use the following signal model instead of (1):

$$x = s(\omega, a, \phi) + n = M\theta + n \quad \theta \in CN(A\psi, \Xi) \quad n \in CN(0, R) \tag{9}$$

where Ξ is the covariance of the complex amplitudes. Since the phases are unknown this model can be used for uncertainties in ‘a’ too. It is easy to show, by integration over the nuisance parameter θ, that the likelihood of this model can be found if we substitute R in the previous subsections with a covariance T formed like:

$$T = R + M\Xi M^H \tag{10}$$

2.4. Frequency and Phase Uncertainties

For small cisoid frequency errors, we can use a linearized model:

$$e^{i(\omega_k + \varepsilon_k)n} \approx (1 + i\varepsilon_k n)\ e^{i\omega_k n} \tag{11}$$

Using a Gaussian prior for ε we replace the signal model in (1) with:

$$x = s(\omega, a, \phi, \varepsilon) + n = MA\psi + DMA\Psi\varepsilon + n$$

$$\varepsilon \in N(0, \sigma_\varepsilon^2 I) \qquad n \in CN(0, R) \tag{12}$$

where $\varepsilon=[\varepsilon_1 \dots \varepsilon_L]^T$, $\Psi = \mathrm{Diag}(\psi)$ and $D=\mathrm{Diag}([0, i, 2i \dots (N-1)i])$. The standard deviation of the frequency must be $\sigma_\varepsilon \ll \pi/2N$ for the approximation (11) to apply. The nuisance parameter ε can now be integrated over, resulting in:

$$p(x|\omega, a, \phi) = \pi^{-N}|R|^{-1}\left|R^{-1}\Lambda\right|^{-\frac{1}{2}} e^{-\mu}$$

$$\mu = (x - MA\psi)^H\Lambda^{-1}(x - MA\psi)$$

$$\Lambda = R + 2\sigma_\varepsilon^2 D^H M A^2 M^H D \tag{13}$$

The method from section 2.2. can now be applied, using Λ from (13) instead of R in (8) and (6).

2.5. Combined Uncertainties

The final method to handle uncertainties in all the parameters can now be stated as follows: Substitute R in (13) by T from (10), and use Λ from (13) instead of R in (8) and (6).

When the noise is white, $R=\sigma^2 I$, and after some algebraic manipulations we get:

$$\mu(\phi) = \sigma^{-2}(\vartheta + \psi^H\Upsilon\psi - 2\mathrm{Re}[\psi^H A b])$$

$$\vartheta = x^H x - b_0^H\Gamma b_0 - (b_1 - G_1\Gamma b_0)^H\Theta(b_1 - G_1\Gamma b_0)$$

$$\Upsilon = AG_0A - AG_0\Gamma G_0A - A(I - \Gamma G_0)^H G_1^H\Theta G_1(I - \Gamma G_0)A$$

$$b = b_0 - G_0\Gamma b_0 - A(I - \Gamma G_0)^H G_1^H\Theta(b_1 - G_1\Gamma b_0)$$

$$\Theta = \frac{2\sigma_\varepsilon^2}{\sigma^2}A\left(I - \frac{2\sigma_\varepsilon^2}{\sigma^2}A(G_2 - G_1\Gamma G_1^H)A\right)^{-1}A$$

$$\Gamma = \Xi(G_0\Xi + \sigma^2 I)^{-1}$$

$$G_0 = M^H M \qquad G_1 = M^H D M \qquad G_2 = M^H D^H D M$$

$$b_0 = M^H x \qquad b_1 = M^H D x \tag{14}$$

Note that the sufficient statistic for the detection problem is $[b_0, b_1]$.

3. PERFORMACE EVALUATION

3.1. Detection Performance

Monte Carlo simulation has been done in order to evaluate detection performance in a binary case. The first hypothesis consisted of the first two of four identically spaced, unit amplitude, cisoids; and the second hypothesis of the last two. The spacing was 0.7 DFT bins, and 200 realizations was drawn from the first model with a uniform distribution of phases and additive white Gaussian noise at different signal to noise ratios ($(a_1^2 + a_2^2)/\sigma^2$). The number of data points was 100. In Figure 1. the probability of detection of the first hypothesis, using a logarithmic detector and zero threshold, are plotted for the different detectors in the previous section. The frequency uncertainties was set to $\sigma_\varepsilon = 0.2/N$ and the amplitude uncertainties $\Xi = 0.02\ I$. The detection probability for a pair of well separated cisoids has also been added as a comparison. (The latter was computed by a Monte Carlo simulation of the detector obtained from the exact solution of (4).)

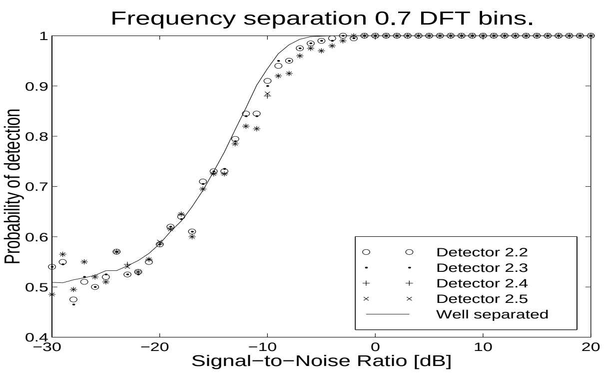

Figure 1. Detection probability for detecting one pair of cisoids separated 0.7 DFT bins against another pair also separated 0.7 DFT bins and 0.7 DFT bin away from the first.

We note that the different methods from the previous section perform similarly. The performance is nearly as good as it is in the case when the cisoids are well separated, even though the separation is less then the fourier resolution.

3.2. Robustness against uncertainties

To evaluate the robustness against model uncertainties we computed the mean of the log likelihood in a number of Monte Carlo simulations. One parameter in the model that generated the data was altered from the model used in the likelihood calculation. The log likelihood was calibrated in such a way that the likelihood for the undistorted model corresponds to zero. One hundred realizations was drawn with a uniform distribution of phases and additive white Gaussian noise at 10 dB signal to noise ratio. The signal model was two unit amplitude cisoids separated by 0.7 DFT bins. In Figure 2. the amplitude of one of the cisoids was increased various amounts, and in Figure 3. the frequency of one of the cisoids was altered in such a way that the separation increased. The frequency uncertainties where set to σ_ε=0.2/N and the amplitude uncertainty in the element of Ξ corresponding to the altered cisoid was set to 0.02.

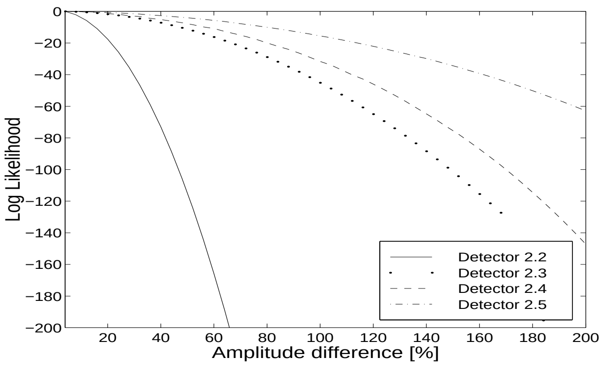

Figure 2. Log likelihood when the amplitude of one cisoid is different from the model.

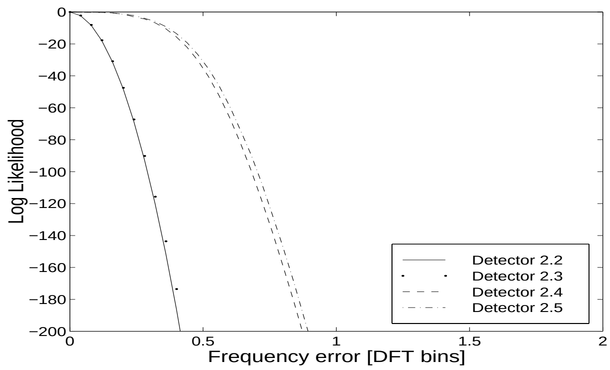

Figure 3. Log likelihood when the frequency of one cisoid is different from the model.

In Figure 3. one recognizes the anticipated robustness to small frequency errors for the methods of section 2.4. and 2.5.

From Figure 2. one can conclude that the estimator in section 2.4. originally designed to handle frequency errors in itself is quite tolerant to amplitude errors. The combined method in section 2.5. may find its utility in that it can represent different uncertainties for different cisoids.

4. RADAR APPLICATION

Automatic target recognition (ATR) based on high range resolution (HRR) has for some time been of considerable interest [3],[4]. The principle behind this kind of ATR is to identify a target by using its impulse response [3]. This can be measured directly with a short radar pulse, or equivalently in frequency domain by using a number of measurements at different frequencies, i.e. sampling the transfer function of the target over a wide enough bandwidth. It has been shown that the target transfer function often can be modelled as a sum of cisoids at typical radar frequencies, at least over a moderate bandwidth (see [5] and references therein). Hence the methods presented in this paper can be applied. Although the frequency takes the place of "time", and ω_i is the positions for a number of scattering centers.

Previously published methods for detection based on HRR data have not recognized the potential of the scattering centre model. The following approaches seems to be prevailing: Firstly, the traditional method seems to be, to use the correlation of "magnitude only" range profiles (e.g. [4]). Secondly, many results concerning estimation of scattering centre model parameters have been reported (e.g. [5],[6]). But, to our knowledge, little have been reported in the open literature about identification using these parameters as features. Thirdly, there is a multitude of "ad hoc" methods, including neural networks, that rely on some transformation of data to reduce its dimension, and the availability of large training sets. It is our belief, that the knowledge of a correct model for the data (the scattering centers), must be utilized in order to solve this genuinely difficult problem.

In the case of well separated scattering centers the likelihood can be factored into a product of the likelihoods of individual cisoids, and the integral (4) can be solved analytically. We get a phase independent detector that is based on the magnitude of $M^H x$. This detector is similar to magnitude correlation in the time domain, which explains early results of the traditional method [4]. Although, later work has reported the inadequacy of using this method on real data [7].

To give an indication of the utility of the detectors we present an example using predicted RCS data. The data was generated from a CAD model of a trainer aircraft by the EPSILON program. EPSILON is a commercially available program from Roke Manor Research Ltd. (a well known similar program is XPATCH). Data was generated at 90 frequencies on the radar X-band over a 625 MHz bandwidth, at 100 different aspects over a 10 degree range. Data from one aspect (0.9˚) was used to estimate a cisoid model of order six using the RELAX algorithm [6]. To simulate measurements, 43 frequency samples (0.5 m nominal resolution) was used and white Gaussian noise was added to achieve a SNR of 10 dB. Finally, we computed the log likelihood for the measured data from different aspects. The results are presented in Figure 4. where we have normalized the likelihood in such a way that the maximum over the range of aspects for each detector is zero.

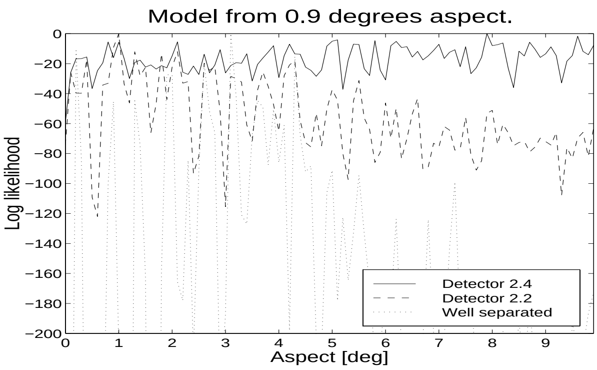

Figure 4. Log likelihood of radar data using an order 6 cisoid model.

From Figure 4. we note the poor performance of the detector based on the assumption of "well separated" cisoids (corresponding to the traditional approach above). This is of course due to that the "well separated" assumption is invalid. Secondly, we note that our detector from section 2.2. does not generalize to other aspects as good as the one from section 2.4. This was even more accentuated in tests done with the whole bandwidth.

5. DISCUSSION AND CONCLUSIONS

We have shown how uncertainties in the parameters of cisoid models can be handled when constructing detectors for such models.

In the case of small frequency uncertainties it was shown that a linearization enabled us to develop a detector with the desired characteristics. The use of a detector with such robustness is essential in practical situations, especially when the model has to be estimated from reference data.

The fundamental characteristic of our detectors is their ability to handle unknown phases. This is done through a numerical minimization algorithm. Further work has to be done to make this algorithm as efficient as possible under various conditions.

We have also demonstrated how the detectors can be applied to the problem of radar target recognition. We feel confident that the "measure of match" asked for in [7] can be accomplished in this manner.

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