A FORWARD BACKWARD APPROACH TO ADAPTIVE SPACE-TIME IDENTIFICATION AND EQUALIZATION OF TIME-VARYING CHANNELS

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ABSTRACT

In this paper, we present an adaptive algorithm for space-time channel identification and equalization. The proposed algorithm performs joint channel estimation and sequence detection by optimizing a least squares cost function iteratively in a forward and backward manner. Simulation results demonstrate the proposed algorithm to be data efficient and fast converging. In addition, good BER performance is achieved in time-varying channels at relatively low SNR and with an extremely short start-up sequence. These attributes render it suitable for wireless mobile communications using short burst data format.

1. INTRODUCTION

The proliferation of wireless communication networks has attracted increasing attention and effort in the research area of blind and semi-blind channel identification and equalization. The main motivation of blindly identifying and equalizing the channel is the offer of higher transmission efficiency and bandwidth conservation through shortening or eliminating the training sequence. This is much desired in TDMA mobile communication networks where short burst data formats are used. For example, out of the 148 bits of the GSM data packet's time-slot, 26 bits are allocated for training the equalizer. Hence, by eliminating or shortening the training length, the spectral efficiency can be significantly improved.

The main technical challenges of developing channel identification and equalization algorithms for wireless mobile TDMA networks using short burst data formats can be succinctly summarized as follow:

1. Ability to track time-varying channel.
2. Only require an extremely short or no training sequence for initial channel estimation.
3. Data efficient and fast converging due to short burst data format.
4. Perform adequately at relatively low SNR.

Several families of blind channel identification and equalization algorithms have been proposed. However, many of the current approaches may not be adequate for such mobile communications applications. For instance, algorithms based on exploiting the higher order statistical information in the channel outputs [1][2] can find limited applications in wireless mobile communications. This is because these algorithms generally exhibit slow convergence and require large number of channel outputs needed to obtain good estimates of the higher order statistics for reliable channel equalization.

The pioneering works of Tong *et. al.*[5] and Moulines *et. al*[6] have led to the developments of several blind channel identification algorithms based on the second-order cyclostationary properties of the channel outputs. In general, these methods achieve blind channel equalization by solving a system of linear equations derived from the covariance matrix of the vectorized channel outputs. These algorithms are block based and can offer the potential of fast and data efficient blind channel identification. However, their explicit assumption of channel's stationarity render them unsuitable when the radio propagation channels are rapidly time-varying.

In most digital communication system, the transmitted symbols are constrained to finite alphabet set $\mathbf{\Omega}$. This will in turn limit the noiseless channel outputs to a finite set. The trellis relationship of channel outputs is determined by the state transitions of the transmitted data and the multipath propagation channel. This trellis relationship not only provide a useful constraint, it also shows that channel identification and equalization of unknown channels can be achieved optimally (in least squares sense) by joint channel estimation and sequence detection. In practice, joint channel estimation and sequence detection is difficult to implement and suboptimal solutions have been proposed. For example, Seshadri[3] proposed a fast blind trellis search algorithm based on Generalized Viterbi Algorithm (GVA) where data detection and channel estimation are performed recursively but separately.

This work is distinct from these previous developments. In this paper, the problem considered here is one of semi-blind identification and equalization of time-varying channels where a short training sequence is used to obtain the initial channel estimates. We approach the problem by exploiting the richness of the inherent structural relationship between channel parameters and data sequence by repeated use of available data through a forward-backward optimization procedure. In the following sections, we describe the data model and the proposed algorithm, and present the simulation results and an analysis of its computational com-

Prof C. Cowan holds a personal research chair supported by the Royal Academy of Engineering and Nortel.

plexity.

2. DATA MODEL AND PROBLEM FORMULATION

Consider an m elements antenna array where each array channel output is oversampled by M times of the symbol period T_b. The channel output vector can be compactly written as [5]

$$\mathbf{x}(kT_b) = \Xi(kT_b)\mathbf{s}_k + \mathbf{n}(kT_b) \quad (1)$$

where $\mathbf{x}(kT_b) = [x_1(kT_b), \ldots, x_1((k+\frac{M-1}{M})T_b), \ldots, x_m(kT_b), \ldots, x_m((k+\frac{M-1}{M})T_b)]^T$ and similarly for $\mathbf{n}(kT_b)$. The symbol vector is given by $\mathbf{s}_k = [s_k, \ldots, s_{k-L+1}]^T$. $x_i(kT_b)$ and $n_i(kT_b)$ are the channel output and observation noise of the i^{th} array channel at time kT_b. $s_k \in \mathbf{\Omega}$ is the transmitted symbol at kT_b and LT_b is the effective length of the channel impulse response.

Suppose the noise $\mathbf{n}(kT_b)$ is a zero-mean Gaussian random process and P channel output vectors are received, the least squares of estimation of channel parameters and sequence detection can be obtained by solving the mixed continuous-FA parameter optimization problem:

$$\left\{\{\widehat{\Xi}(kT_b)\}_{k=1}^{P}, \widehat{\mathbf{S}}\right\} = \arg\min \sum_{k=1}^{P} |\mathbf{x}(kT_b) - \Xi(kT_b)\mathbf{s}_k|_F^2 . \quad (2)$$

$\{\Xi(kT_b)\}_{k=1}^{P}$ are the continuous parameter and $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_P]$ is a Toeplitz matrix parametrized by $\tilde{\mathbf{s}} = [s_{-L}, \ldots, s_P]^T$. The elements in $\mathbf{S}$ are contrained to the finite alphabet set $\mathbf{\Omega}$.

We can write an equivalent time-reversed version of (2) as

$$\left\{\{\widehat{\Xi}_r(kT_b)\}_{k=1}^{P}, \widehat{\mathbf{S}}_r\right\} = \arg\min \sum_{k=1}^{P} |\mathbf{x}_r(kT_b) - \Xi_r(kT_b)\mathbf{s}_{rk}|_F^2 \quad (3)$$

where $\mathbf{x}_r(kT_b) = \mathbf{x}(lT_b)$ and $\mathbf{s}_{rk} = [s_{l-L+1}, \ldots, s_l]^T$. The time-reversed channel matrix $\Xi_r(kT_b)$ is given by

$$[\Xi_r(kT_b)]_{i,k} = [\Xi(lT_b)]_{i,L-k+1} \quad (4)$$

and $l = P - k + 1$. Similarly, the symbol matrix $\mathbf{S}_r = [\mathbf{s}_{r1}, \ldots, \mathbf{s}_{rP}]$ is a Toeplitz matrix parametrized by $\tilde{\mathbf{s}}_r = [s_P, \ldots, s_{-L}]^T$. $[\mathbf{A}]_{i,j}$ denotes the matrix element located at the i^{th} row and j^{th} column of $\mathbf{A}$.

Under the ideal conditions of perfect initial estimates and perfect tracking, the adaptive Maximum Likelihood Sequence Detection (MLSD) algorithms such as [3][4] can achieve optimal channel identification and equalization. However, in practical radio environment where short data burst formats are used and multipath propagation channels are time-varying, we need to address the following issues:

1. Good initial estimates of the channel are spectrally expensive to achieve. It entails reserving portion of the data sequence for non-blind channel estimation. Due to the presence of noise, reducing the training length will result in noisier initial estimates and induce errors in the sequence detection.
2. Due to the time-variation of the channel parameters, the following inequality holds:
$$\Xi(kT_b) \neq \Xi(iT_b) \text{ for all } i \neq k. \quad (5)$$
When $\Xi(kT_b) = \Xi(iT_b)$ for all $\{i,k\}$ is imposed on time-varying channels, modelling errors will be introduced and can result in poorer sequence detection. One can argue that the time-varying channel can be approximated by contiguous segments of time-invariant channel where each data sub-block is independently processed. This is suboptimal as the full information residing within the data block is not exploited. Moreover, resolving the competing requirements of having large sub-block size and containing modelling errors remain open.
3. Joint estimation of time-varying channels and sequence detection is difficult to implement. Suboptimal approaches estimate the channel parameters and perform data detection recursively but *separately* have been proposed[3][4]. In general, the decoupled optimization of the LS cost function will lead to suboptimal solution.

3. AN ADAPTIVE ALGORITHM FOR LEAST SQUARES SPACE-TIME CHANNEL IDENTIFICATION AND EQUALIZATION

In this paper, we propose an algorithm that can potentially resolve these issues by processing the received data in batch and optimize the LS cost function iteratively in forward-backward manner.

The proposed algorithm begins with a noisy (but of *sufficiently* accuracy to avoid divergence) initial channel estimates. Then it seeks to optimize the LS cost function alternatingly and recursively with respect to the channel parameters and data sequence. Towards the end of the forward iteration, the corresponding channel parameters are likely to converge nearer to the true estimates and the symbol vectors will have a lower probability of error. In the backward iteration, (3)(the time reversed version of (2)) is similarly minimized based on the initial state extracted from the last symbol vector in $\mathbf{S}$ and its corresponding time-reversed channel estimates. The improved initial estimates can lead to more accurate estimates $\mathbf{S}$ and $\{\Xi(kT_b)\}_{k=1}^{P}$. Hence, with each forward-backward optimization iteration, the residual in (2) will be further reduced with the improved estimates of $\mathbf{S}$ and $\{\Xi(kT_b)\}_{k=1}^{P}$. The cyclic refinement of the channel estimates and sequence detection will monotonically decrease the LS cost function and converge finally to a minima.

In this paper, the channel estimation and sequence detection are performed recursively but separately as follow. The symbol vector at time kT_b are decided based on the last updated channel estimates. Then, the channel estimates are adaptively updated in a decision directed manner. One way to exploit the inherent Toeplitz structure of symbol matrix $\mathbf{S}$ recursively is to apply the Generalized Viterbi algorithm (GVA)[3]. GVA retains a number of "locally best" survivors entering each state and updates the channel estimates associated with each survivor independently. However, the computational complexity of employing GVA in the proposed approach can become prohibitive. In this development, we adopt a simpler VA implementation where only one survivor is retained in each state and the channel parameters are updated based on the survivor with the lowest accumulated metric.

The channel parameters are adaptively updated by the computationally simple Least Means Squares (LMS) implementation of

$$\widehat{\Xi}(kT_b) = \widehat{\Xi}((k-1)T_b) + \mu \left(\mathbf{1}_{mM} \otimes \widehat{\mathbf{s}}_i^H\right) \odot \left(\eta(kT_b) \otimes \mathbf{1}_L^T\right) \quad (6)$$

and similarly for its time-reversed version. $\eta(kT_b) = \mathbf{x}(t) - \widehat{\Xi}((k-1)T_b)\widehat{\mathbf{s}}_k)$ and $\widehat{\mathbf{s}}_k$ is the tentative estimates of the k^{th} symbol vector. $\mathbf{1}_L$ denotes a vector of 1 of length L and μ is the adaptation step-size. $\otimes$ and $\odot$ are the Kronecker and Hadamard operator, respectively.

The algorithm proposed herein can be summarized as follows.

The Algorithm

Compute the initial estimates of $\Xi(T_b)^{(0)}$ and derive the initial state $\mathbf{s}^{(l)}(0)$ from the start-up sequence. Set $l = 0$.

- Repeat
  - Optimize (2) by joint channel matrix estimation using (6) and sequence detection by VA.
  - Perform Time-reversal to obtain $\{\widehat{\Xi}_r(kT_b)^{(l)}\}_{k=1}^P$ and $\widehat{\mathbf{S}}_r^{(l)}$ from $\{\widehat{\Xi}(kT_b)^{(l)}\}_{k=1}^P$ and $\widehat{\mathbf{S}}^{(l)}$, respectively.
  - Set
  $$\mathbf{s}_r^{(l)}(0) = \begin{bmatrix} [\mathbf{S}_r^{(l)}]_{2\cdots L,1} \\ \kappa \end{bmatrix} \quad (7)$$
  where $[\mathbf{A}]_{i\cdots k,l}$ is a vector extracted elements in l^{th} column and i^{th} to k^{th} row of the matrix $\mathbf{A}$ and κ is any element in $\mathbf{\Omega}$.
  - Optimize (3) by joint channel matrix estimation using the time-reversed version of (6) and sequence detection by VA.
  - Perform Time-reversal to obtain $\{\widehat{\Xi}(kT_b)^{(l)}\}_{k=1}^P$ and $\widehat{\mathbf{S}}^{(l)}$ from $\{\widehat{\Xi}_r(kT_b)^{(l)}\}_{k=1}^P$ and $\widehat{\mathbf{S}}_r^{(l)}$, respectively.
  - $l = l + 1$
- Until convergence.

4. SIMULATION RESULTS

In this section, we describe some simulation results using the proposed algorithm. We consider a two element antenna array with temporal oversampling 2 times the symbol rate. The carrier frequency used is 900MHz. The data symbols are drawn from $\mathbf{\Omega} \ni \{-1, 1\}$ and transmitted at GSM data rate of 277kps in packets 100 symbols each. The combined transmit and receive filter frequency response is a raised cosine with roll-off factor of 35%. We simulate the Rayleigh multipath environment based on the TU channel from the ETSI recommendations[7] with paths arriving with a uniform spatial distribution. The local spread of each path is 30 degrees. In this study we restrict the channel length to $L = 3$.

Figure 1 shows the bit error rates for the proposed algorithm and GVA as a function of SNR. The step-size used for LMS update is 0.0025 and the initial channel estimates are obtained from an extremely short training sequence of 5 symbols using direct matrix inversion. In GVA, $K = 4$ locally best survivors entering each state are retained. The results are averaged directly from 500 independent trials. The relative speed between the transmitter and receiver is 300km/hr. This results in a doppler frequency of 250Hz. The result shows that at an error rate of 10^{-3}, the proposed algorithm suffers a loss of 3dB against the known channel bound. On the other hand, GVA suffers 9dB. Figure 2 plots a typical example of the convergence trajectory of proposed algorithm and its corresponding number of erroneous symbol detection as a function of forward-backward iterations. Note that the LS cost function and its corresponding number of symbol detection errors reduces monotonically with iteration number. Moreover, the LS cost function after each forward(backward) optimization is always lower than the preceeding backward(forward) optimization. Typically, convergence is achieved within 3 iterations.

5. ANALYSIS OF COMPUTATIONAL COMPLEXITY

We analyze the relative computational complexity of GVA and proposed algorithm based on the number of complex multiplications and additions and is denoted by $(\alpha_{mul}, \alpha_{add})$. The number of computations involved for each LMS channel update and metric computation are $(3mML + mM, 2mML + mM)$ and $(mML + mM, mML + 2mM)$, respectively. The number of metric computations at each time instant by GVA and proposed algorithm are $K \times 2^L$ and 2^L, respectively. The corresponding number of channel updates are $K \times 2^L$ and 1. Hence the total number of computations involved in GVA and the proposed algorithm are $(K \times 2^L \times (4mML + 2mM), K \times 2^L \times (3mML + 3mM))$ and $(2\rho \times (2^L \times (mML + mM) + 3mML + mM), 2\rho \times (2^L \times (mML + 2mM) + 2mML + mM))$, respectively. ρ is the number of forward-backward iterations to achieve convergence. In this study, the number of computations per symbol for GVA and the proposed algorithm ($\rho = 3$) are $(1792, 1536)$ and $(1008, 1128)$, respectively.

6. CONCLUSIONS

In this paper, we have presented an adaptive semi-blind channel identification and equalization algorithm that jointly estimate the time-varying channel and perform sequence detection in a least squares framework. The simulation results are encouraging. They have demonstrated the proposed algorithm to be data-efficient, fast converging and capable of achieving good BER performance in time varying channel environment at relatively low SNR and with an extremely short training sequence. These features render this algorithm to be potentially suitable for short burst data format communications where only extremely short training sequence can be made available in order to conserve bandwidth.

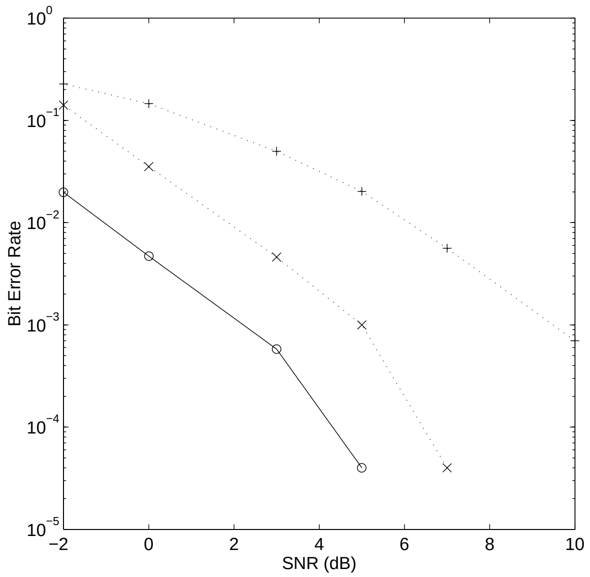

Figure 1: BER vs. SNR: $\cdot \times \cdots$: proposed algorithm. $-o-$: VA with exact knowledge of channel parameter matrix. $\cdots + \cdots$: GVA

7. REFERENCES

[1] Y. Sato, "A Method of Self-Recovering Equalization for Multilevel Amplitude-Modulation Systems", *IEEE Trans. Comms.*, Vol. 23, 1975, pp.679-682.

[2] D. Hatzinakos and C.L. Nikias, "Blind Equalization using a Trispectrum based Algorithms", *IEEE Trans. Comms.*, Vol. 39, No.5, 1991, pp. 312-321.

[3] N. Seshadri, "Joint data and channel estimation using fast blind trellis search techniques", *Proc Globecom*, 1991, pp. 1653-1659.

[4] R. Raheli, A. Polydoros and C.K. Tzou, "Per-Surviour Processing: A General Approach to MLSE in Uncertain Environment", *IEEE Trans. Communications*, Vol. 43, Feb./Mar./April 1995, pp.354-364.

[5] L. Tong, G.Xu and T. Kailath,"Blind identification and equalization based on second-order statistics: a time domain approach", *IEEE Trans. Inform. Theory*, Vol. 41, Mar. 1994, pp. 329-334.

[6] E. Moulines, P. Duhamel, J. Cardoso and S. Mayrarque, "Subspace Methods for the Blind Identification of Multichannel FIR Filters", *IEEE Trans. Signal Processing*, Vol. 43, Feb. 1995, pp.516-525.

[7] ETSI, European Digital Cellular Telecommunication System (Phase 2), European Telecom Standard Institute, 1994.

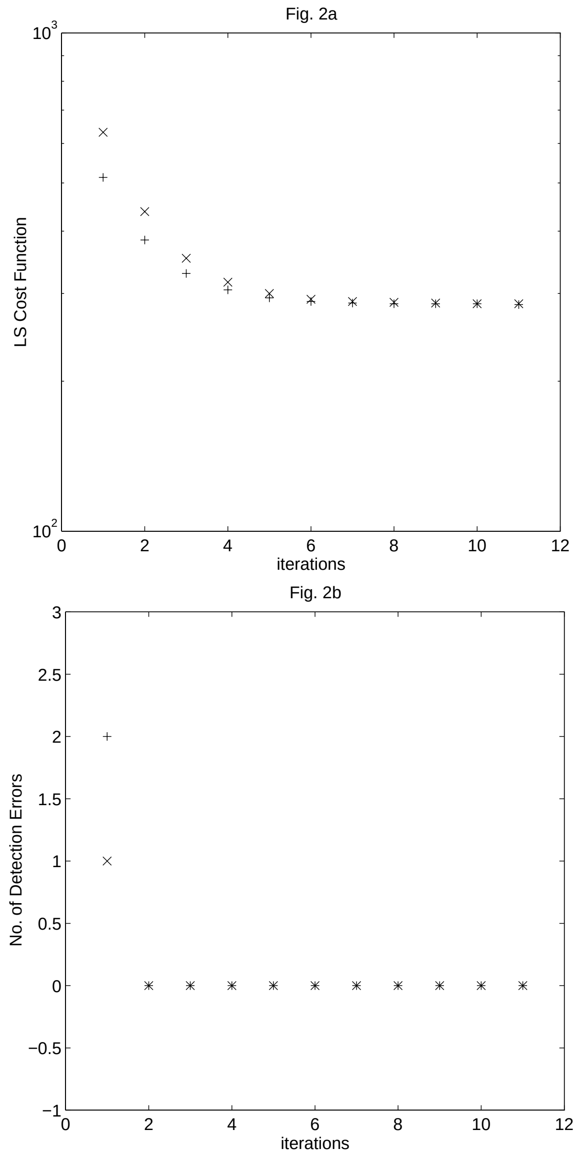

Figure 2: (a)LS Cost Function and (b)Number of Symbol Detection Errors vs. Iterations. SNR: 3dB. The LS cost function after the forward and backward optimization at i^{th} iteration are denoted by '$\times$' and '$+$', respectively.