SPATIO-TEMPORAL CODING FOR RADAR ARRAY PROCESSING

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ABSTRACT

The aim of this paper is to present a new method allowing radar digital beamforming (DBF) with only one receiver channel. The key point is the use of a particular spatio-temporal waveform transmitted by an active phased array antenna. By properly choosing the temporal modulation applied on each of the transmission elements, one can transmit different signals in different directions, allowing (under certain hypotheses) angular localisation with only one receiver channel. The major advantage is the design of a low cost system providing DBF capabilities.

1. INTRODUCTION

Over the last past few years, numerous studies in the field of array processing have shown that the use of an antenna array (instead of only one antenna) and a suitably designed processing can lead to major improvements in radar system performances. Such advantages include jammer cancellation, high angular resolution capabilities, dwell time and update time decoupling, ... A good overview about such topics and others can be found in [1] and [2].

The main drawback when considering DBF radar is their prohibitive cost, which strongly depends on their number of channels (or subarrays). This paper describes a new technique, allowing the application of DBF processing with a single receiver channel. This new technique is based on the transmission of a particular waveform, which when properly designed, sends orthogonal signals in different spatial directions.

The paper is organized as follows : in a first part, the signal model is defined. In a second part, the matched filter structure is derived, and the ambiguity function is calculated. The last part of the paper describes several examples, involving different choices for the spatio-temporal waveform.

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2. HYPOTHESES AND SIGNAL MODEL

We consider an N-element uniform linear antenna. For sake of simplicity, we consider identical elements with omnidirectional gain. On each of those elements, a pulse train of period Tr is transmitted. Each pulse is composed of M adjacent subpulses of time duration T. The m^{th} subpulse on the n^{th} element is characterized by the complex number w_{nm}. This particular waveform, which can be considered as a spatio-temporal amplitude-phase modulation, can therefore be characterized by a complex coding matrix **W**, with N rows and M columns.

Figure 1 illustrates an example of such a principle.

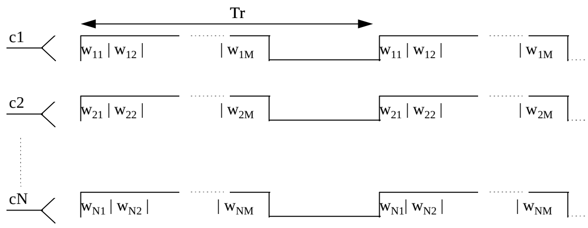

Figure 1 : Transmission principle

For the rest of the paper, we will focus on the transmission of only one pulse, characterized by a coding matrix **W**. The goal is then to estimate two target parameters (two-way time delay and direction of arrival) with a single receiver channel. The generalization of all the results to the pulse train case is straightforward, and needs only the addition of the Doppler parameter in the signal model.

In the one pulse case, the signal $s_n(t)$ transmitted by the n^{th} antenna element can be written :

$$s_n(t) = \sum_{m=0}^{M-1} w_{nm} v_T(t - mT)$$

where $v_T(t)$ is a rectangular signal of amplitude one and duration T, centered around T/2.

The signal r(t) received on one sensor (for a target direction u_0 and a two-way time-delay τ_0) can then be written :

$$r(t) = \alpha\, g(t-\tau_o, u_o) + b(t)$$

$$\text{with } g(t,u) = \mathbf{a}^H(u)\,\mathbf{W}\,\mathbf{h}(t)$$

where :

W is the coding matrix of size [N,M]

a(u) is the N-element vector defined by :

$$\mathbf{a}(u) = \left[\exp\{j\beta x_0 u\} \cdots \exp\{j\beta x_{N-1} u\}\right]^t$$

where x_n stands for the abscissa of the n^{th} element, and with $\beta = 2\pi/\lambda$

h(t) is the M-element vector defined by :

$$\mathbf{h}(t) = \left[v_T(t)\, v_T(t-T) \ldots v_T(t-(M-1)T)\right]^t$$

α is an unknown complex scalar.

b(t) is an additive white Gaussian noise.

Note that the underlying hypotheses allowing the derivation of this model are :
- Narrow band assumptions (BL/c << 1, where B is the frequency band, L the total length of the array antenna, and c the light speed).
- Doppler effect negligeable over the time duration of the coded pulse ($2v_{MAX}/\lambda << 1/MT$, where v_{MAX} is the maximal radial velocity and λ the wavelength).

In most applications, those hypotheses are valid (as an example, in X band, if the duration T is 100 ns, and if there are M=32 subpulses, L shoud be smaller than 30 m and v_{MAX} smaller than 4600 m/s).
The model being defined, the next step consists in deriving the matched filter allowing detection and parameters estimation.

3. MATCHED FILTER

Under the hypothesis of additive white Gaussian noise, the maximum likelihood method leads to the following parameter estimation criterion :

$$\{\hat{u}, \hat{\tau}\} = \arg\max_{u,\tau}\{f(u,\tau)\}$$

where f(u,τ) is given by :

$$f(u,\tau) = \frac{\left|\int_{-\infty}^{+\infty} r(t)\, g^*(t-\tau,u)\,dt\right|^2}{\int_{-\infty}^{+\infty} |g(t,u)|^2 dt} = \frac{\left|\mathbf{a}^H(u)\;\mathbf{W}\;\mathbf{s}(\tau)\right|^2}{\left|\mathbf{W}^H \mathbf{a}(u)\right|^2}$$

$$\text{with } \mathbf{s}(\tau) = \int_{-\infty}^{+\infty} r^*(t)\,\mathbf{h}(t-\tau)\,dt$$

Figure 2 illustrates a possible implementation for this filter, where the theoretical continuous search over τ is replaced by a discrete one (sampling with period T).

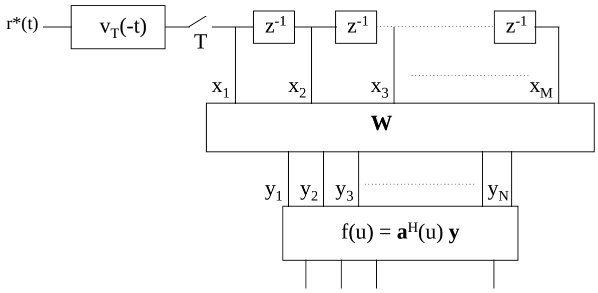

Figure 2 : Scheme of Matched Filter

It can be observed that the matched filter is actually constituted by :
1) A first filter matched to the subpulse enveloppe, whose output is sampled with period T
2) A time delay line of length M (number of subpulses)
3) A temporal to spatial linear transformation (via matrix **W**)
4) A classical beamformer

A useful tool for analyzing the performances of this kind of filtering consists in calculating the ambiguity function (output of the matched filter in the absence of noise, for an hypothesis (u,τ), when the true parameters are u_0 and τ_0). In the case considered here, the calculation gives:

$$\chi(u,\tau,u_o,\tau_o) = \frac{\left|\mathbf{a}^H(u)\mathbf{W}\;\mathbf{H}(\tau-\tau_o)\;\mathbf{W}^H\mathbf{a}(u_o)\right|^2}{\left|\mathbf{W}^H\mathbf{a}(u)\right|^2}$$

$$\text{with } \mathbf{H}(\tau) = \int_{-\infty}^{+\infty} \mathbf{h}(t-\tau)\,\mathbf{h}^H(t)\,dt$$

It is worth noting that in general, this function depends on u and u_0, and not only on their difference. This property is however highly desirable, since it means that the performances do not depend on the direction u_0. Keeping this in mind, a particularly interesting choice for the spatio-temporal waveform consists in selecting a coding matrix **W**, such that :

$$\mathbf{W}\mathbf{W}^H = \mathbf{I}_N$$

This orthogonality condition ensures that :

1) The matched filter output depends only on the difference between u and u_0, when τ and τ_0 are equal (that is to say when the matched filter is perfectly synchronous with the received signal).
2) The transmission principle is perfectly omnidirectional (the mean energy radiated in the direction u is constant over u. Note that this also implies that the denominator term in the matched filter expression can be dropped).

In the last part of the paper, several solutions for the choice of the matrix **W** are analyzed.

4. EXAMPLES

4.1 SAR effect

Perhaps the first idea coming in mind for the choice of **W** is the identity matrix. Physically, it means that the antenna elements radiate sequentially a short pulse of duration T. This transmission principle can then be viewed as a fast displacement of the antenna phase center. That is the reason why we call it « SAR effect ». The normalised ambiguity function of this waveform is depicted in figure 3. Note that we use a linear scale for easier figure legibility. A cut at the right time-delay ($\tau=\tau_0$) shows that the angular ambiguity function is a sinus cardinal, which mainlobe width is λ/L (the whole transmission antenna directivity is thus obtained with a single receiver channel). For other values of the time delay, angular cut changes smoothly, meaning that this waveform is tolerant regarding the exact sampling time. The main drawback of this waveform is a matter of power budget. Each antenna element transmits indeed during only a small fraction of the total pulse duration, and in the case of an active antenna, this is clearly a rather poor solution. A simple way ensuring the waveform to be energy efficient is to impose the following new constraint :

$| w_{nm} | = 1, \forall$ m and n

It means that we limit our investigations to pure phase codes. Note thas this constraint is not only a way to be energy efficient, but is also desirable to ensure good hyperfrequency characteristics.

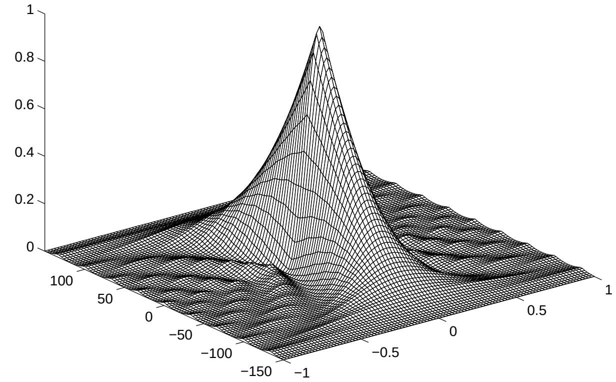

Figure 3 SAR effect (N = 9, M = 9, u_0 = 0, τ_0 = 0, MT = 1 μs, λ = 3 cm, d = λ/2)

4.2 Energy constraint codes

4.2.1 Fast beamscanning

Very well known pure phase orthogonal codes are given by the Fourier matrix. The element w_{nm} of this matrix is :

$$w_{nm} = e^{2j\pi nm/N}$$

It can then be shown that, during the m^{th} subpulse, the transmission antenna is steered towards a direction u_m. This pointing direction changes linearly from subpulse to subpulse, such that, during the whole pulse duration, all the visible domain is scanned. The normalised ambiguity function of this waveform is illustrated in figure 4. This figure clearly shows a very strong coupling between direction and time-delay, meaning that any bias on time sampling gives bias on angular localisation. This kind of waveform is thus not adapted to the one channel case (note that this would not be true if several receiver channels were available).

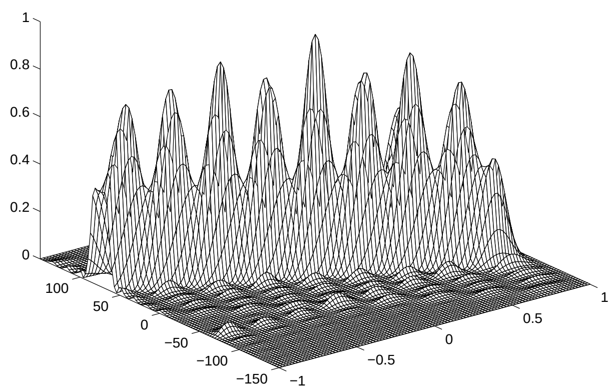

Figure 4 Fast Beamscanning (N = 9, M = 9, u_0 = 0, τ_0 = 0, MT = 1 μs, λ = 3 cm, d = λ/2)

4.2.2 Discrete chirp

Other pure phase orthogonal codes can be constructed using results derived from studies on sequences with perfect periodic correlations. Such sequences include for instance Franck codes, Chu codes, ... (see [3])
Considering as an example the case of a discrete chirp, we obtain the ambiguity function illustrated in figure 5. It can be shown that this function is very close to the one obtained in the « SAR effect » case, but with the major advantage that it is energy efficient. This kind of spatio-temporal waveform appears therfore to be a very good choice in the class of polyphase orthogonal codes.
For hardware simplicity considerations, it can also be interesting to investigate binary codes (in this case, the transmission principle can indeed be applied with a low cost antenna).

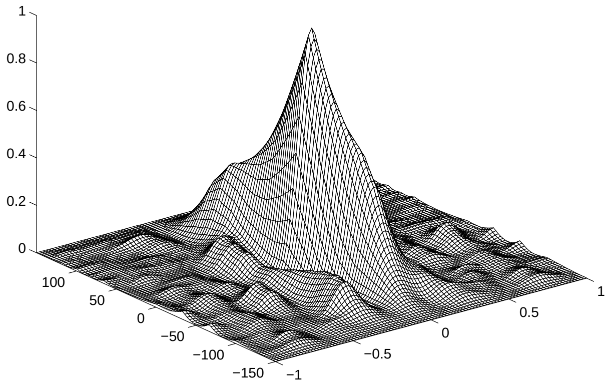

Figure 5 Discrete Chirp (N = 9, M = 9, $u_0 = 0$, $\tau_0 = 0$, MT = 1 µs, λ = 3 cm, d = λ/2)

4.2.3 Gold sequences

The Hadamard matrix is an example of a binary orthogonal coding matrix, but it can be shown that the corresponding ambiguity function is useless for radar applications. A closer insight into the analytical expression of the ambiguity function reveals that its shape strongly depends on the autocorrelation and intercorrelation properties of the code.
Nearly orthogonal binary codes, with good autocorrelation and intercorrelation properties, are given by the so-called Gold sequences.
An example of ambiguity function corresponding to such sequences is depicted in figure 6. It can be observed that this waveform gives good angular and time-delay resolutions, and that there is no coupling between direction and time-delay. Note however that the average sidelobe level is higher than in previously considered cases (1/M on average). Moreover this level can not be reduced by any windowing. It is thus preferable to use this kind of waveform when the number of subpulses is high (with the restriction that Gold sequences exist only for M being a power of two minus one).
In [4], it has been shown that other binary sequences can be found by numerical optimization methods, and that they lead to interesting properties for spatio-temporal applications.

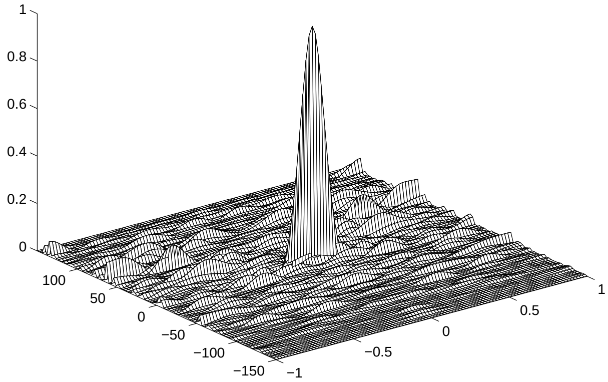

Figure 6 Gold Sequences (N = 9, M = 31, $u_0 = 0$, $\tau_0 = 0$, MT = 1 µs, λ = 3 cm, d = λ/2)

5. CONCLUSION

In this paper, a new formulation for array processing spatio-temporal coding has been introduced. The major advantage of such a coding is to give more flexibility in DBF radars design, allowing for example the use of a single receiver channel. Of course, as soon as anti-jamming is needed, it is necessary to use more than one receiver channel, so that future works will be devoted to combination of spatio-temporal coding with DBF applied on subarray receiver channels. It opens new fields of investigations, such as simultaneous application of high resolution methods on transmission and reception.

6. REFERENCES

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