VECTOR-SENSOR ARRAY PROCESSING FOR ESTIMATING ANGLES AND TIMES OF ARRIVAL OF MULTIPATH COMMUNICATION SIGNALS

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ABSTRACT

We develop vector-sensor array processing to estimate the angles-of-arrival (AOAs) and time delays of multipath channels in the space-time-polarization domain. A MUSIC-type algorithm for joint angle and delay estimation with a vector-sensor array is derived. Potential applications include multipath channel estimation and mobile localization. Simulation results show that the space-time-polarization parameterization of the multipath channels results in improved accuracy and resolution performance.

1. INTRODUCTION

In wireless communications applications, mobiles emit signals that arrive at a base station with multiple paths, each with its own angle-of-arrival (AOA), path delay, fading and polarization. Estimating these mulitpath channel parameters can offer a number of advantages. Parsimonious parameterization of the multipath channel potentially offers significant accuracy gains in channel estimation, thus resulting in improved sequence detection and tracking performance. In addition, the AOA and TOA (Time Of Arrival) information can provide relevant information for transmit beamforming as well as mobile localization. In [1], using a calibrated antenna array and *a priori* knowledge of the modulation waveform, a subspace formulation to estimate AOAs and delays of multipath channels is proposed. However, the polarization information inherent in the signals is not exploited.

In many mobile applications, the signals impinging on the mobile as well as the basestation antenna often have diverse polarizations. A number of empirical and theoretical studies have been conducted to study the polarization diversity in a number of mobile channels [2],[3]. In applications where space is a premium, exploiting the inherent polarization diversity of the received signals can improve the accuracy and resolution.

In this paper we propose the use of vector-sensor array processing [4] and extend the work of [1] to the space-time-polarization domain. In Section II we present the multipath channel model in the space-time-polarization framework. To illustrate the usefulness of the proposed approach, we derive a MUSIC-type estimator in Section III, and present simulation results to demonstrate its effectiveness in Section IV.

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2. DATA MODEL

We assume the transmitted signal to be linearly modulated and given by

$$s(t) = \sum_i g(t - iT)b_i, \tag{1}$$

where $g(t)$ is the modulation pulse shape, e.g.\ square-root raised cosine, and b_i is the transmitted data symbol which is constrained to a finite alphabet set Ω. For example, in BPSK, $\Omega \ni \{-1, 1\}$. Assuming far-field source and using the results of [4], the received signal at the output of the vector sensor of a single-path source is given by :

$$\mathbf{r}(t) = \sum_{i=1}^{d} \mathbf{a}(\theta_i)\gamma_i(t)s(t - \tau_i) + \mathbf{n}(t) \tag{2}$$

where

$$\begin{aligned} \theta_i &= [\varphi_i, \psi_i, \alpha_i, \beta_i]^T \\ \mathbf{a}(\theta_i) &= \begin{bmatrix} \mathbf{I} \\ \mathbf{u}\times \end{bmatrix} \mathbf{V}_i \mathbf{Q}_i \mathbf{w}_i \end{aligned}$$

$$
\begin{aligned}
\mathbf{u} &= \begin{bmatrix} cos\varphi_i cos\psi_i \\ sin\varphi_i cos\psi_i \\ sin\psi_i \end{bmatrix} \\
(\mathbf{u}\times) &= \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix} \\
\mathbf{V}_i &= \begin{bmatrix} -sin\varphi_i & -cos\varphi_i sin\psi_i \\ cos\varphi_i & -sin\varphi_i sin\psi_i \\ 0 & cos\psi_i \end{bmatrix} \\
\mathbf{Q}_i &= \begin{bmatrix} cos\alpha_i & sin\alpha_i \\ -sin\alpha_i & cos\alpha_i \end{bmatrix} \\
\mathbf{w}_i &= \begin{bmatrix} cos\beta_i \\ isin\beta_i \end{bmatrix}
\end{aligned} \tag{3}
$$

The matrix $\mathbf{I}$ denotes the 3 x 3 identity matrix, φ_i and ψ_i are the azimuth and elevation angles respectively, α_i and β_i embed the polarization information (orientation angle and ellipticity); $\mathbf{u}$ is the unit vector at the sensor pointing towards the source, τ_i is the path delay of the i-th multipath and $\gamma_i(t)$ is a complex scaling variable. In mobile communications, $\gamma_i(t)$ may assume a Rayleigh or Ricean distribution. The variable d is the number of multipaths; $\mathbf{n}$(t) is additive white gaussian noise. We sample the signal in (2) at the symbol rate (i.e. at instants kT). Let P be the number of the symbol-spaced samples of the channel impulse response, $P = 2\delta + M_d$, where $2\delta T$ is the symbol waveform duration and M_d is the maximum integer delay. We then rewrite (2) as

$$\mathbf{r}(k) = \mathbf{H}\mathbf{s}(k) + \mathbf{n}(k) \tag{4}$$

where the j-th element of the P-dimensional vector $\mathbf{s}(k)$ of data is $b(k + j - M_d - \delta)$ and $\mathbf{H}$ captures the effects of the array response, delay, symbol waveform, fading and polarization parameters. We note that

$$\mathbf{H} = \mathbf{A}(\theta)\mathbf{\Gamma}(t)\mathbf{G}(\tau)^T \tag{5}$$

where

$$
\begin{aligned}
\mathbf{A}(\theta) &= [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_d)] \\
\theta &= [\theta_1^T, \theta_2^T, \cdots, \theta_d^T]^T \\
\mathbf{\Gamma}(t) &= \begin{bmatrix} \gamma_1(t) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \gamma_d(t) \end{bmatrix} \\
\mathbf{G}(\tau) &= [\mathbf{g}(\tau_1), \mathbf{g}(\tau_2), \cdots, \mathbf{g}(\tau_d)] \\
\mathbf{g}(\tau_i) &= \begin{bmatrix} g((M_d + \delta - 1)T - \tau_i) \\ g(T - \tau_i) \\ \vdots \\ g((M_d + \delta - P)T - \tau_i \end{bmatrix} \\
\tau &= [\tau_1, \tau_2, \cdots, \tau_d]^T
\end{aligned} \tag{6}
$$

Let h = vec($\mathbf{H}$) be a vector obtained by sequentially stacking the transposed rows of the matrix $\mathbf{H}$. Applying the vec($\cdot$) operation to $\mathbf{H}$ yields

$$\mathbf{h} = \mathbf{A}(\theta) \odot \mathbf{G}(\tau)\mathbf{\Gamma}(t) \tag{7}$$

where $\odot$ denotes the Khatri-Rao product. Extending to the case of an array of multiple vector sensors can be achieved simply by replacing $\mathbf{a}(\theta_i)$ with $\tilde{\mathbf{a}}(\theta_i)$ where $\tilde{\mathbf{a}}(\theta_i) = \mathbf{e}(\theta_i) \otimes \mathbf{a}(\theta_i)$ and $\mathbf{e}(\theta_i)$ is the phase delay vector associated with the DOA θ_i.

3. MUSIC-TYPE ALGORITHMS

As in [1], we assume the angle and time delay of arrival to remain constant over the number of measurement time slots. The fading amplitudes are assumed to remain constant within each time slot but vary independently from each other. The channel matrix can be estimated from the start-up sequence commonly available in most wireless TDMA networks. For example, in GSM (Global System for Mobile Communications) the start-up sequence length spans 27 symbols in each time slot.

To estimate θ and τ, we develop a MUSIC-type algorithm that is an extension of the JADE (Joint Angle and Delay Estimation) -MUSIC technique [1]. Let

$$\mathbf{R_{XX}} = \mathbf{E} < \mathbf{r}(t)\mathbf{r}^H(t) >= \mathbf{E_S}\mathbf{\Lambda_S}\mathbf{E_S}^H + \sigma^2\mathbf{E_n}\mathbf{E_n}^H \tag{8}$$

where $\mathbf{E_S}$ is the signal subspace matrix, $\mathbf{\Lambda_S}$ contains the eigenvalues corresponding to the signal subspace and $\mathbf{E_n}$ is the noise subspace matrix.

Define the spatial-temporal-polarizational steering vector as

$$
\begin{aligned}
\breve{\mathbf{a}}(\theta, \tau) &= \begin{bmatrix} \mathbf{I} \\ \mathbf{u}\times \end{bmatrix} \mathbf{VQw} \otimes \mathbf{g}(t - \tau) \\
&= \mathbf{a}'(\varphi, \psi)\mathbf{k}(\alpha, \beta) \otimes \mathbf{g}(t - \tau)
\end{aligned} \tag{10}
$$

where

$$\mathbf{a}'(\varphi, \psi) = \begin{bmatrix} \mathbf{I} \\ \mathbf{u}\times \end{bmatrix} \mathbf{V} \tag{11}$$

$$\mathbf{k}(\alpha, \beta) = \mathbf{Qw} \tag{12}$$

We express (9) as

$$\breve{\mathbf{a}}(\theta, \tau) = [\mathbf{a}'(\varphi, \psi) \odot [\mathbf{g}(t - \tau), \mathbf{g}(t - \tau)]] \begin{bmatrix} k_1(\alpha, \beta) \\ k_2(\alpha, \beta) \end{bmatrix} \tag{13}$$

Defining

$$\tilde{\mathbf{A}} = [\mathbf{a}'(\varphi,\psi) \odot [\mathbf{g}(t-\tau), \mathbf{g}(t-\tau)]] \quad (14)$$

and

$$\mathbf{k} = \begin{bmatrix} k_1(\alpha,\beta) \\ k_2(\alpha,\beta) \end{bmatrix} \quad (15)$$

the space-time-polarization spectrum of the MUSIC estimator is given by

$$f(\theta,\tau) = \frac{\mathbf{k}^{\mathbf{H}}\tilde{\mathbf{A}}^{\mathbf{H}}\mathbf{E_n}\mathbf{E_n^H}\tilde{\mathbf{A}}\mathbf{k}}{\mathbf{k}^{\mathbf{H}}\tilde{\mathbf{A}}^{\mathbf{H}}\tilde{\mathbf{A}}\mathbf{k}} \quad (16)$$

where sup H is the conjugate transpose of a matrix. A direct approach to reduce the search space in (16) is to constrain $||\mathbf{k}|| = 1$ and the value of the space-time spectrum for each θ and τ set is obtained from the minimum eigenvalue of $\tilde{\mathbf{A}}^{\mathbf{H}}\mathbf{E_n}\mathbf{E_n^H}\tilde{\mathbf{A}}$. An alternative approach is to use the linear constraint $k_1 = 1$ as in [6]. This is a reasonable assumption since $k_1 \approx 0$ is likely to be zero measure in many situations. The MUSIC function then becomes

$$f(\theta,\tau) = [1, k_2^H]\tilde{\mathbf{A}}^{\mathbf{H}}\mathbf{E_n}\mathbf{E_n^H}\tilde{\mathbf{A}} \begin{bmatrix} 1 \\ k_2 \end{bmatrix} \quad (17)$$

By equating the derivative of (17) to zero, we have $\widehat{k}_2 = -\frac{M_{21}}{M_{22}}$ where

$$\tilde{\mathbf{A}}^{\mathbf{H}}\mathbf{E_n}\mathbf{E_n^H}\tilde{\mathbf{A}} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad (18)$$

and the resulting concentrated MUSIC function is given by

$$\tilde{f}(\varphi,\psi,\tau) = [1, -\frac{M_{21}}{M_{22}}^{*}]\tilde{\mathbf{A}}^{\mathbf{H}}\mathbf{E_n}\mathbf{E_n^H}\tilde{\mathbf{A}} \begin{bmatrix} 1 \\ -\frac{M_{21}}{M_{22}} \end{bmatrix} \quad (19)$$

Clearly the space-time spectrum computed based on (17) is computationally more efficient than computing the minimum eigenvalues of (16) since the need for eigendecomposition of (18) for each pair of τ and θ is alleviated. It suffices to note that M_{11}, M_{21}, M_{22} and M_{12} are functions of φ , ψ and τ.

4. NUMERICAL STUDY AND DISCUSSION

We present results of computer simulations to compare the performance of our algorithm with JADE-MUSIC. We assume a single user that results in 3 multipaths. 3 vector sensors are used for our proposed algorithm and 3 scalar sensors for the JADE-MUSIC. The 3 sensors in each case are arranged in a triangular configuration. The azimuths of the 3 multipaths are [80, 50, 20] degrees and the corresponding path delays are $[1.0, 0.7, 0.3]T$ seconds, where T is normalized to 1. The elevation angles are assumed to be known. The modulation waveform is a raised cosine pulse with excess bandwidth of 0.35 and assumed duration of $2\delta = 6$ (i.e. is zero outside the interval [-3,3)). The collected data is corrupted by noise with SNR (defined as the power of the path fading to the variance of the noise) ranging from 0 to 15 dB. We sample at a rate of $\frac{T}{2}$ (the purpose of the oversampling is to provide improved definition of the delay manifold). Data is collected over 20 time slots and at each time slot, the channel is estimated via least squares using 27 training bits. The experimental variance of the AOA and delay estimates is computed from 100 runs. The results are summarized in Figure 1 and it can be seen that using our proposed algorithm with 3 vector sensors gives significantly better performance than JADE-MUSIC with 3 scalar sensors. Table 1 shows the results for the time delays. Probability of resolution curves are shown in Figure 2 for closely spaced angles of arrivals with different polarizations. We considered two signals to be resolved if the estimated angle separation is greater than half of the actual separation.

SNR(dB)	STD for JADE(T)	STD for Proposed Algorithm(T)
0	0.0159	0.0099
5	0.0082	0.0000

Table 1: Comparison of Standard Deviations of Time Delay Estimates

5. CONCLUSIONS

We presented a MUSIC-type algorithm for estimating AOAs and TOAs of multipath communication signals. Compared with JADE, it exploits the polarization diversity of signals in addition to the space-time diversity. It was shown to resolve sources with angles of arrivals that cannot be resolved by the original scalar sensor version of JADE. If no training sequence is available, blind equalization techniques can then be used.

6. REFERENCES

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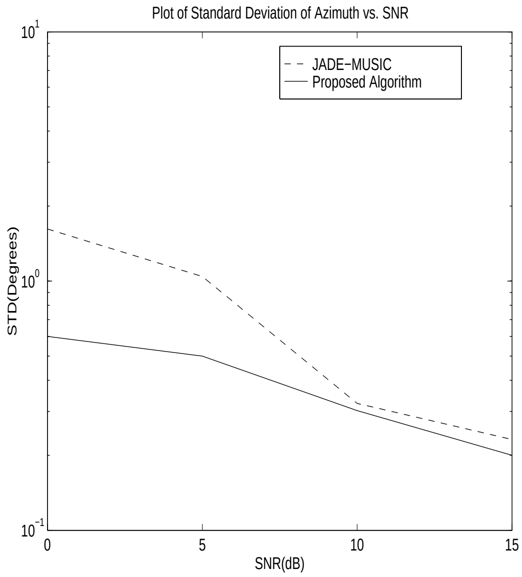

Figure 1: Comparison of JADE-MUSIC and Proposed Algorithm

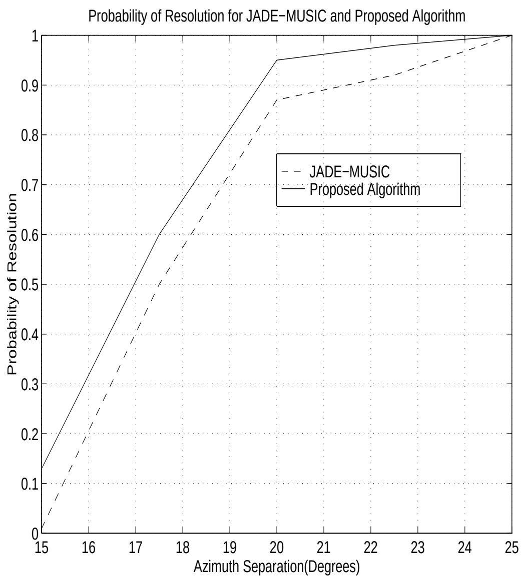

Figure 2: Probability of Resolution for JADE-MUSIC and Proposed Algorithm

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