THE EFFECTS OF LOCAL SCATTERING ON DIRECTION OF ARRIVAL ESTIMATION WITH MUSIC AND ESPRIT

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ABSTRACT

In wireless communication scenarios, multipath propagation from local scatterers in the vicinity of mobile sources may cause angular spreading as seen from a base station antenna array. This paper studies the effects of such local scattering on direction of arrival (DOA) estimation with the MUSIC and ESPRIT algorithms. Previous work has considered rapidly time-varying scenarios, and concluded that local scattering has a minor effect on DOA estimation in such scenarios. This work considers the case in which the channel is time-invariant during the observation period. The distribution of the DOA estimates is derived, and the results show that local scattering has significant impact on DOA estimation in the time-invariant case. In addition, numerical examples are included to illustrate the analysis, and to demonstrate that the results may be used to formulate simple estimators of angular spread.

1. INTRODUCTION

The use of antenna arrays at base stations in wireless communication systems has gained much interest. By using multiple antennas, the idea is to utilize the spatial dimension more efficiently. Among the possibilities are improved range, diversity against fading, interference suppression, and spatially selective transmission to reduce interference in the downlink.

For macro cells in rural and suburban environments with antennas placed above roof-tops, away from potential multipath reflectors, it may be reasonable to assume that most of the energy incident on the array is from local scattering near the mobile. Local scattering models were first used in studies aimed at determining the separation required between two antennas to get effective spatial diversity, i.e., sufficiently low fading correlation [1]. It is assumed that there is no direct line-of-sight between the mobile user and the base station, and the signal is modeled as being due to a large number of independent and identically distributed waves. As the scatterers are local to the mobile, the angular distribution may be relatively narrow. To generalize Rayleigh fading narrowband channels to the multiple antenna case, a similar approach has been taken in [2, 7, 10, 11]. A large number of angular distributions have been proposed [1, 2, 4, 7, 10, 11]. In [4], the radius of the local scattering area is estimated to be of the order of 50 - 100wavelengths in a suburban area.

In [5], the bias and variance of DOA estimates for the case where the channel varies very rapidly with time is studied. The mobile is assumed to travel a distance large enough between each snapshot, so that the channel realizations may be viewed as independent from snapshot to snapshot. However, for a system operating in burst mode, like the well-known GSM system, the observation periods are short, and the array is sampled at symbol rate. The time-variation of the channel between the snapshots may then be neglected, and the analysis in [5] does not apply to this scenario.

This work studies the effects of local scattering on DOA estimation with two conventional algorithms, MUSIC [8] and ES-PRIT [6]. The observation period is assumed short in relation with the mobile speed, so that the channel may be modeled as time-invariant. Finite sample effects and calibration errors are neglected, and only the effect of local scattering is considered, which from a DOA estimation point of view may be regarded as a model error. The angular spread is assumed to be small, and the spatial signature is approximated as a linear combination of the nominal array response due to a plane wave and its derivative. The distributions of the DOA estimates are then determined by invoking the Central Limit Theorem to approximate the coefficients of the linear combination as complex Gaussian random variables. The distribution of the estimates may then be determined.

2. DATA MODEL

It is assumed that d mobile sources are present emitting narrowband signals. The time dispersion introduced by the multipath propagation is assumed to be small in comparison with the reciprocal of the bandwidth of the emitted signals. For an array of mantennas, the observations are modeled as

$$\mathbf{x}(t) = \mathbf{V}\mathbf{s}(t) + \mathbf{n}(t) \,. \tag{1}$$

The *i*th column of **V**, denoted \mathbf{v}_i , represents the spatial signature of the signal $s_i(t)$ transmitted by user *i*. The vector $\mathbf{s}(t) = [s_1(t), \dots, s_d(t)]^T$ contains the signals of all users. The noise, $\mathbf{n}(t)$, is assumed to be spatially white, $\mathbf{E}\{\mathbf{n}(t)\mathbf{n}^*(t)\} = \sigma^2 \mathbf{I}$, and the signals spatially non-coherent in the sense that $\mathbf{E}\{\mathbf{s}(t)\mathbf{s}^*(t)\} = \mathbf{S} > 0$. In contrast to [5], **V** is modeled as constant during the observation period. The spatial signature \mathbf{v}_i is modeled as

$$\mathbf{v}_i = \sum_{k=1}^{N_i} \alpha_{ik} \mathbf{a}(\theta_i + \tilde{\theta}_{ik}) , \qquad (2)$$

where α_{ki} is the (complex) amplitude of the *k*th scattered signal, $\mathbf{a}(\theta)$ is the response of the array to a single unit amplitude signal with DOA θ , and N_i is the total number of local scatterers for the *i*th source. The quantities θ_i and $\theta_i + \tilde{\theta}_{ik}$ represent the nominal DOA of the *i*th user, and the arrival angle of the *k*th scattered signal respectively. The phenomenon is illustrated in Figure 1. The assumption of local scattering near each user means



that Δ_i in Figure 1 is assumed to be small. Define the gradient $\mathbf{d}(\theta) = \partial \mathbf{a}(\theta) / \partial \theta$ so that a first order Taylor series expansion of (2) yields

$$\mathbf{v}_{i} \approx \sum_{k=1}^{N_{i}} \alpha_{ik} \left(\mathbf{a}(\theta_{i}) + \tilde{\theta}_{ik} \mathbf{d}(\theta_{i}) \right) = \gamma_{i} \mathbf{a}(\theta_{i}) + \psi_{i} \mathbf{d}(\theta_{i}) \quad (3)$$

where

$$\gamma_i = \sum_{k=1}^{N_i} \alpha_{ik} , \qquad \psi_i = \sum_{k=1}^{N_i} \alpha_{ik} \tilde{\theta}_{ik} . \tag{4}$$

The framework for additive model errors proposed in in [9] will be used. To fit the approximate spatial signature model of (3) into this framework, the gain γ_i will be associated with the *i*th signal, and the received signal is modeled as

$$\mathbf{x}(t) \approx \left[\mathbf{A}(\boldsymbol{\theta}) + \tilde{\mathbf{A}}\right] \mathbf{\Gamma} \mathbf{s}(t) + \mathbf{n}(t),$$
 (5)

where

$$\tilde{\mathbf{a}}_i = \frac{\psi_i}{\gamma_i} \mathbf{d}(\theta_i) \,. \tag{6}$$

The matrix Γ is a diagonal matrix, $\Gamma = \text{diag}\{\gamma_1, \ldots, \gamma_d\}$, and

$$\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_d)], \qquad \tilde{\mathbf{A}} = [\tilde{\mathbf{a}}_1, \dots, \tilde{\mathbf{a}}_d].$$

3. A FIRST-ORDER ERROR ANALYSIS

In this section, a first-order error analysis is carried out. Finite sample effects and calibration errors are not considered. Thus, only the effects of angular spreading are studied. The perturbation of the covariance matrix caused by the angular spreading is first related to the perturbation of the estimated signal and noise subspaces. These results are then used to find the perturbation of the estimated DOAs. For the case where the local scattering cause no angular spreading, but only variations of the received signal powers (fading), it holds that $\Delta_i = 0$ and $\tilde{\theta}_{ik} = 0$ for all *i*, *k*. The nominal covariance matrix of the observations, $\mathbf{R} = \mathbf{E}\{\mathbf{x}(t)\mathbf{x}^*(t)\}$, is then

$$\mathbf{R} = \mathbf{A}(\boldsymbol{\theta}) \mathbf{\Gamma} \mathbf{S} \mathbf{\Gamma}^* \mathbf{A}^*(\boldsymbol{\theta}) + \sigma^2 \mathbf{I}.$$
 (7)

A basis for the nominal signal subspace may be defined from the eigenvalue decomposition of \mathbf{R} ,

$$\mathbf{R} = \mathbf{E}_s \mathbf{\Lambda}_s \mathbf{E}_s^* + \sigma^2 \mathbf{E}_n \mathbf{E}_n^* \,.$$

The estimates calculated with this covariance matrix will coincide with the nominal DOAs. With angular spread, (5) applies, and the sample covariance matrix is

$$\hat{\mathbf{R}} = \left[\mathbf{A} + \tilde{\mathbf{A}}\right] \mathbf{\Gamma} \mathbf{S} \mathbf{\Gamma}^* \left[\mathbf{A} + \tilde{\mathbf{A}}\right]^* + \sigma^2 \mathbf{I}, \qquad (8)$$

where $\mathbf{A} = \mathbf{A}(\boldsymbol{\theta})$. The estimated basis for the noise subspace is defined from the eigenvalue decomposition of $\hat{\mathbf{R}}$:

$$\hat{\mathbf{R}} = \hat{\mathbf{E}}_s \hat{\mathbf{\Lambda}}_s \hat{\mathbf{E}}_s^* + \hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \sigma^2 \,. \tag{9}$$

To be able to make a perturbation analysis, it is assumed that the angular spreading is small, so that $\|\tilde{\mathbf{A}}\|$ may be regarded as small. As shown in [9], if terms that tend to zero faster than $\|\tilde{\mathbf{A}}\|$ are neglected, then the following holds:

$$\hat{\mathbf{E}}_n \hat{\mathbf{E}}_n^* \mathbf{a}(\theta_i) \approx -\mathbf{E}_n \mathbf{E}_n^* \tilde{\mathbf{a}}_i \,. \tag{10}$$

Similarly, it is possible to show that

$$\hat{\mathbf{E}}_{s}^{*} \mathbf{E}_{n} \mathbf{E}_{n}^{*} \approx \mathbf{E}_{s}^{*} \mathbf{A}^{\dagger *} \hat{\mathbf{A}}^{*} \mathbf{E}_{n} \mathbf{E}_{n}^{*} .$$
(11)

3.1. The MUSIC Algorithm

The MUSIC algorithm [8] calculates the DOA estimates as the d minimizing values of the cost function

$$V(\theta) = \frac{\mathbf{a}^*(\theta)\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^*\mathbf{a}(\theta)}{\mathbf{a}^*(\theta)\mathbf{a}(\theta)}$$

The DOA estimate of the *i*th DOA calculated with MUSIC, $\hat{\theta}_i^{M}$, therefore satisfies

$$0 = V'(\hat{\theta}_i^{\mathrm{M}}), \qquad (12)$$

where $V'(\theta) = \partial V(\theta) / \partial \theta$. An expression for $\hat{\theta}_i^{M} - \theta_i$ is obtained via a first-order Taylor expansion, in which terms that tend to zero faster than $\hat{\theta}_i^{M} - \theta_i$ are neglected. Using (10) and $\mathbf{E}_n^* \mathbf{a}(\theta_i) = 0$,

$$V'(\theta_i) \approx -\frac{2\operatorname{Re}\left\{\mathbf{d}(\theta_i)\mathbf{E}_n\mathbf{E}_n^*\tilde{\mathbf{a}}_i\right\}}{\mathbf{a}^*(\theta_i)\mathbf{a}(\theta_i)}$$

and

$$\lim_{\Delta_i \to 0} V''(\theta_i) = \bar{V}''(\theta_i) = \frac{2 \operatorname{Re} \left\{ \mathbf{d}(\theta_i) \mathbf{E}_n \mathbf{E}_n^* \mathbf{d}(\theta_i) \right\}}{\mathbf{a}^*(\theta_i) \mathbf{a}(\theta_i)}$$

which gives the following first order expression for the error

$$\hat{\theta}_{i}^{\mathrm{M}} - \theta_{i} \approx -\frac{V'(\theta)}{\bar{V}''(\theta)} \approx \frac{\operatorname{Re}\left\{\mathbf{d}(\theta_{i})\mathbf{E}_{n}\mathbf{E}_{n}^{*}\tilde{\mathbf{a}}_{i}\right\}}{\mathbf{d}(\theta_{i})\mathbf{E}_{n}\mathbf{E}_{n}^{*}\mathbf{d}(\theta_{i})}.$$
 (13)

3.2. The ESPRIT-Algorithm

In this section, DOA estimation with a uniform linear array (ULA) with elements separated δ wavelengths and ESPRIT [6] is studied. Let $\hat{\lambda}_i$ be the *i*th eigenvalue of the matrix

$$\hat{\mathbf{\Psi}} = \hat{\mathbf{E}}_{s1}^{\dagger} \hat{\mathbf{E}}_{s2}$$
,

where $\hat{\mathbf{E}}_{s1}^{\dagger} = (\hat{\mathbf{E}}_{s1}^{*} \hat{\mathbf{E}}_{s1})^{-1} \hat{\mathbf{E}}_{s1}^{*}$. The matrices $\hat{\mathbf{E}}_{s1}$ and $\hat{\mathbf{E}}_{s2}$ are given by $\hat{\mathbf{E}}_{s1} = \mathbf{J}_{1} \hat{\mathbf{E}}_{s}$ and $\hat{\mathbf{E}}_{s2} = \mathbf{J}_{2} \hat{\mathbf{E}}_{s}$, with $\mathbf{J}_{1} = [\mathbf{I}_{m-1} \ 0]$ and $\mathbf{J}_{2} = [0 \ \mathbf{I}_{m-1}]$. The *i*th DOA estimate, $\hat{\theta}_{i}^{E}$, is

$$2\pi\delta\sin\hat{\theta}_i^{\rm E} = \arg\hat{\lambda}_i \,. \tag{14}$$

Define λ_i as $\lambda_i = \exp(2\pi\delta\sin\theta_i)$, and recall that λ_i is the *i*th eigenvalue of $\Psi = \mathbf{E}_{s_1}^{\dagger}\mathbf{E}_{s_2}$. As $\|\tilde{\mathbf{A}}\|$ tends to zero, $\hat{\mathbf{E}}_s$ tends to \mathbf{E}_s , and as shown in [3], for small $\hat{\lambda}_i - \lambda_i$, the following holds:

$$\hat{\theta}_i^E - \theta_i \approx \frac{1}{2\pi\delta\cos\theta_i} \operatorname{Im}\left\{\frac{\hat{\lambda}_i - \lambda_i}{\lambda_i}\right\}.$$
 (15)

In [3], it is also shown, that if terms that tend to zero faster than the retained terms as $\|\tilde{\mathbf{A}}\|$ tends to zero are neglected, then

$$\hat{\lambda}_i - \lambda_i \approx \boldsymbol{\rho}_i^* \mathbf{E}_n \mathbf{E}_n^* \hat{\mathbf{E}}_s \mathbf{E}_s^* \mathbf{A} \mathbf{e}_i \,, \tag{16}$$

where \mathbf{e}_i is a column vector with the *i*th element being one, and the other zero. The vector $\boldsymbol{\rho}_i^*$ is defined as

$$\boldsymbol{\rho}_{i}^{*} = \mathbf{e}_{i}^{*} \left(\mathbf{J}_{1} \mathbf{A} \right)^{\dagger} \left(\mathbf{J}_{2} - e^{j 2 \pi \delta \sin \theta_{i}} \mathbf{J}_{1} \right) \,. \tag{17}$$

Using (11) and the facts that $\tilde{\mathbf{A}}\mathbf{A}^{\dagger}\mathbf{E}_{s}\mathbf{E}_{s}^{*}\mathbf{A}\mathbf{e}_{i} = \tilde{\mathbf{a}}_{i}$, and $\rho_{i}^{*}\mathbf{E}_{n}\mathbf{E}_{n}^{*} = \rho_{i}^{*}$, (16) becomes $\hat{\lambda}_{i} - \lambda_{i} \approx \rho_{i}^{*}\tilde{\mathbf{a}}_{i}$. Substituting this into (15), finally yields

$$\hat{\theta}_{i}^{\mathrm{E}} - \theta_{i} \approx \frac{\mathrm{Im}\left\{\mathbf{e}_{i}^{*}\left(\mathbf{J}_{1}\,\mathbf{A}\right)^{\dagger}\left(e^{-j2\pi\delta\sin\theta_{i}}\mathbf{J}_{2}-\mathbf{J}_{1}\right)\tilde{\mathbf{a}}_{i}\right\}}{2\pi\delta\cos\theta_{i}}.$$
 (18)

4. DISTRIBUTION OF ESTIMATES

So far, the model error, $\tilde{\mathbf{a}}_i$, has been considered deterministic but small. As in [7, 10, 11], the number of scattered signals, N_i , is assumed to be relatively large, and the rays are assumed to be independent and identically distributed (iid) with phases uniformly distributed over $[0, 2\pi[$. It is then reasonable to approximate γ_i and ψ_i in (4) as complex Gaussian random variables. Assume that the power is normalized so that all rays have equal power, $1/N_i$, and, as in [7] assume that $\tilde{\theta}_{ik}$, is uniformly distributed over the interval $[-\Delta_i, \Delta_i]$. Then $\alpha_{ik} = \frac{1}{\sqrt{N_i}}e^{j\phi_{ik}}$ with all ϕ_{ik} iid uniformly over $[0, 2\pi[$. Using (4), it is straightforward to verify that γ_i and ψ_i are approximately independent complex Gaussian random variables with mean zero and variances

$$E\{|\gamma_i|^2\} = 1, \qquad E\{|\psi_i|^2\} = \Delta_i^2/3.$$

A reasonable modeling assumption is that the rays carrying different source signals are independent. For this case, $\tilde{\mathbf{a}}_i$ and $\tilde{\mathbf{a}}_k$ are independent for $i \neq k$, and from (13) and (18) it then follows that the DOA estimates also to first order are independent. Combining the expression for the model error $\tilde{\mathbf{a}}_i$ as given in (6) with the expression for MUSIC in (13) gives

$$\hat{\theta}_i^{\mathrm{M}} - \theta_i \approx \operatorname{Re}\left\{\frac{\psi_i}{\gamma_i}\right\}.$$
 (19)

For the ESPRIT algorithm, combining (6) with (18) gives

$$\theta_i^{\rm E} - \theta_i \approx \frac{\operatorname{Im}\left\{\mathbf{e}_i^* (\mathbf{J}_1 \mathbf{A})^{\dagger} \left(e^{-j2\pi\delta\sin\theta_i} \mathbf{J}_2 - \mathbf{J}_1\right) \mathbf{d}(\theta_i) \frac{\psi_i}{\gamma_i}\right\}}{2\pi\delta\cos\theta_i}.$$
(20)

The estimates calculated with both algorithms will have the same distribution, since the real and imaginary part of the ratio of two independent complex Gaussian random variables have the same distribution. Define the estimation error as $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$. Using (19) and (20), it is then straightforward to derive the probability density function (pdf) of the estimation error $\tilde{\theta}_i$ as

$$f_{\tilde{\theta}_{i}}(\theta) = \frac{\beta_{i}^{2}}{2\left(\theta^{2} + \beta_{i}^{2}\right)^{3/2}}.$$
(21)

For the MUSIC-algorithm, the parameter β_i is

$$\beta_i^M = \Delta_i / \sqrt{3} \,, \tag{22}$$

and for the ESPRIT algorithm, the parameter β_i is

$$\beta_i^{\rm E} = \frac{\Delta_i}{\sqrt{3}} \left| \frac{\mathbf{e}_i^* (\mathbf{J}_1 \mathbf{A})^{\dagger} \left(e^{-j2\pi\delta\sin\theta_i} \mathbf{J}_2 - \mathbf{J}_1 \right) \mathbf{d}(\theta_i)}{2\pi\delta\cos\theta_i} \right|$$

For a ULA and a single source it is easily shown that the parameters are equal, $\beta_i^{\mathrm{M}} = \beta_i^{\mathrm{E}} = \Delta_i / \sqrt{3}$. To first order, the local scattering will introduce no bias as $\mathrm{E}\{\hat{\theta}_i^{\mathrm{E}}\} \approx \mathrm{E}\{\hat{\theta}_i^{\mathrm{M}}\} \approx \theta_i$. Further, it is possible to calculate $\mathrm{E}|\tilde{\theta}_i|$, as

$$E[\hat{\theta}_i] \approx \beta_i \,. \tag{23}$$

The second moment is infinite since the approximations give a distribution whose tail does not decay sufficiently fast. Any practical DOA estimator restricts it search over DOAs between $[0, 360^{\circ}]$ and will of course not have infinite variance. The behavior is probably explained by the Rayleigh fading.

5. NUMERICAL EXAMPLES

In the examples, each source was modeled with multipath propagation from 30 iid local scatterers. Each scatterer had a fixed amplitude, a random phase uniformly distributed over $[0, 2\pi[$, and an angular perturbation from a uniform distribution of width $2\Delta_i$.

In the first example, a ULA with six elements and a single source with nominal DOA 0° was considered. For this case the estimated signal subspace is simply the spatial signature. The DOA was estimated with MUSIC and ESPRIT for 2000 realizations of the spatial signature. In Figure 2, the average value of $|\tilde{\theta}_1| = |\hat{\theta}_1 - \theta_1|$ is plotted for different values of Δ_1 . The solid line is the approximate expression for $E\{|\tilde{\theta}_1|\}$ given by (23), which for this case is $\Delta_1/\sqrt{3}$ for both estimators. The empirical values obtained with MUSIC and ESPRIT agree well with theory for small angular spread. The simulations indicate that ESPRIT gives smaller estimation errors.



In Figure 3, a histogram with 5000 DOA estimates calculated with MUSIC is plotted. A six element ULA was considered, and a single source with nominal DOA 30° and angular spread $2\Delta_1 = 5^{\circ}$ was present. The solid line represents the relative frequencies



Figure 3: Histogram of estimates compared with approximation.

predicted by the expressions in (21) and (22). The derived expressions give a relatively good prediction of the pdf of the estimates.

In the last example, a noisy scenario was considered. Three sources with nominal DOAs -45° , 0° and 45° were present. All sources had an average SNR of 10 dB measured at a single sensor. A ULA with eight elements was used, and bursts of length 100 snapshots were collected. For each burst the spatial signatures were generated as above and held constant during the observation period. The angular spreads, as seen from the array, were $2\Delta_1 = 5^\circ, 2\Delta_2 = 1^\circ$ and $2\Delta_3 = 15^\circ$. The MUSIC algorithm was used, and the MUSIC spectrum was only calculated for a region $\pm 20^{\circ}$ around each nominal DOA. In this way, some of the outliers which appear during fading dips are removed. A total of 5000 bursts were simulated, and the averaged DOA estimates were -45.0° , 0.0° , and 45.0° , which agrees with our previous results. Also, (23), states that $2\Delta_i \approx \sqrt{12} \mathbb{E}\{|\hat{\theta}_i - \theta_i|\}$. The sample mean of the absolute value of the angular perturbations may be used to form an estimate of the angular spread $2\Delta_i$. Using all 5000 bursts, the estimated spreads calculated in this way were $4.8^{\circ}, 1.1^{\circ}$, and 13.54°. Finite sample effects probably add to the variations of the DOA estimates, and this partially compensates for the discrepancy between theory and experiments for larger spreads. Finally, a window of length 100 was considered, meaning that only the estimates from the last 100 bursts were used. The sample mean was subtracted from the estimates to estimate the angular perturbations. The mean of the absolute value of the angular perturbations was then calculated. By multiplying this sample mean by $\sqrt{12}$, an estimate of $2\Delta_i$ was calculated. In Figure 4, the transient behavior is neglected, and the estimated angular spreads are plotted. The solid lines represent the true values for $2\triangle$. The DOA estimates may thus be used to calculate a rough estimate of the angular spread. Also, this agrees with the analysis of the MUSIC algorithm that the DOA estimates to first order are uncorrelated, as the presence of multiple sources does not affect the angular spread estimates.

6. SUMMARY

The effects of local scattering on DOA estimation with the MUSIC and ESPRIT algorithms were studied. The analysis considered only the effect of the local scattering and neglected finite sample effects and calibration errors. The angular spread was assumed to be small, so that the spatial signature could be approximated as a linear combination of the nominal array response due to a plane wave and its derivative. The coefficients of the linear combination



were approximated as complex Gaussian, and the distributions of the DOA estimates were then derived. The results show that local scattering has significant impact on DOA estimation for timeinvariant scenarios. Numerical examples were included to illustrate the analysis, and to demonstrate that the results may be used to formulate simple estimators of angular spread as well.

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