APPLICATION OF SUBBAND ANALYSIS TO ADAPTIVE PREDICTION

James R. Saffle and S. S. Rao

Department of Electrical and Computer Engineering
Villanova University
Villanova, PA 19085, USA

ABSTRACT

A closed-loop adaptive subband prediction architecture is presented by employing an adaptive subband filter in the prediction configuration. Some authors have suggested that applying open-loop prediction methods to subband signals can realize increased prediction gain over fullband prediction. Furthermore, the benefits of applying multirate techniques to adaptive filtering are well understood in terms of reduction of computational complexity and increased convergence speed. Thus, the closed-loop subband adaptive predictor is a novel approach that is expected to exhibit these same benefits along with the advantages of backward adaptation. Results show that the new subband predictor can produce a higher prediction gain than a similar fullband adaptive prediction filter. The proposed architecture is implemented in C++ on the Pentium processor.

1. INTRODUCTION

This paper proposes an adaptive subband prediction architecture that utilizes closed-loop adaptation. Prediction and adaptive filtering have been successfully employed in many applications such as speech coding, time series modeling, and spectrum estimation. Subband techniques which are also popular in speech coding and spectrum estimation have been applied to adaptive filtering in order to improve performance. Recent work has suggested that applying adaptive prediction to subband signals may realize increased prediction gain over fullband prediction under certain conditions; however, this work generally involves open-loop adaptation approaches such as the autocorrelation or covariance method of least squares [4][5][9]. Since subband prediction based on closed-loop adaptation is a novel approach to the prediction problem, an examination of such a predictor is warranted.

Adaptive filtering is applied to prediction because of its ability to track and converge upon the stochastic characteristics of a signal [2]. In nonstationary environments adaptive algorithms provide a frequent coefficient update that follow changes in signal statistics. Such algorithms are generally classified as either open-loop or closed-loop [3]. Open-loop or forward adaptation involves buffering a window of samples and estimating the predictor coefficients for each successive window. Open-loop adaptation introduces buffering delay into the signal path. In communication applications, forward adaptation coefficients are transmitted through a channel requiring additional channel capacity and exposing the coefficients to noise and coding problems. In closed-loop or backward adaptation, the filter coefficients are derived from the previous set of coefficients and a feedback error signal. The backward coefficient update does not require buffering and may be performed as often as desired even on a per-sample basis. In communication applications, the recursive nature of backward adaptation allows a receiver to derive the filter coefficients rather than require a periodic transmission of this information.

Many papers have been written about the combination of multirate techniques and adaptive filtering [1][6][7][8]. The resulting subband adaptive filter can reduce the computational complexity and increase convergence speed with respect to the fullband case. However, multirate analysis and synthesis may introduce delay into the signal path and may cause aliasing in critically downsampled systems. Because the proposed subband prediction structure is based on adaptive filtering in subbands, it inherits the same characteristics.

The remainder of this paper discusses the background and implementation of a subband, closed-loop, adaptive predictor in more detail. Section 2 describes the operation of the closed-loop adaptive prediction filter focusing on some of the possible adaptation algorithms and performance metrics. Section 3 reviews the advantages of adaptive filtering in subbands. Section 4 proposes the closed-loop subband prediction architecture and describes supporting theory. Section 5 provides a computational analysis, section 6 presents simulation results, and section 7 describes the real time implementation. Section 8 summarizes.

2. CLOSED-LOOP ADAPTIVE PREDICTION

In prediction, a filter is used to estimate future values of a signal from prior observations. Figure 1 shows a closed-loop, adaptive predictor where d(n) is the desired signal, u(n) is the adaptive filter input, y(n) is the predicted signal, and e(n) is the prediction error. Although a single delay is shown, the delay could be as many samples as desired. If the predicted signal is the desired output, then this configuration is known as a prediction filter or predictor. If the error signal is the desired output, then the structure is called a prediction error filter.

The Normalized Least Mean Squares (NLMS) and the Recursive Least Squares (RLS) adaptation algorithms are two popular algorithms for updating linear adaptive filter weights [2]. The NLMS algorithm is a stochastic gradient type algorithm that relatively low complexity requiring O(3P) multiplications where P is the adaptive filter order; however, the convergence speed of the NLMS is slowed by input signals with wide eigenvalue spread. The RLS algorithm is more complex than NLMS requiring $O(P^2+5P)$ multiplications, but its convergence is independent of eigenvalue spread.

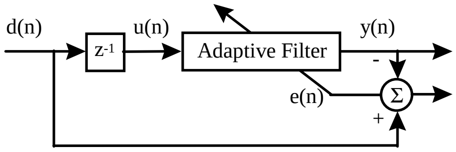

Figure 1. Single-step adaptive prediction filter.

The performance of a predictor is measured using a quantity known as prediction gain [3]. The prediction gain is given by the ratio

$$G_p = \frac{\sigma_d^2}{\sigma_e^2}$$

where σ_d^2 is the variance of the predictor input, d(n), and σ_e^2 is the variance of the prediction error, e(n). Thus, given the same input signal, the better predictor produces a smaller error variance yielding a larger prediction gain. The error variance may be estimated using the time average

$$\sigma_e^2 = \frac{1}{M}\sum_{n=1}^{M} e(n)^2$$

and the input variance may be estimated using

$$\sigma_d^2 = \frac{1}{M}\sum_{n=1}^{M} (d(n) - \bar{d})^2$$

where $\bar{d}$ is the mean of the input signal.

Another popular metric, the spectral flatness measure (sfm) gauges the flatness or whiteness of a power spectral density (psd) [3]. The sfm may take values between zero and unity where a unity spectral flatness measure represents a white process. The inverse of the sfm measures the waveform predictability of a process where the value for an unpredictable (white) process is unity and the value for a totally predictable process is infinity. For linear prediction, the prediction gain is upper bounded by the waveform predictability.

3. ADAPTIVE FILTERING IN SUBBANDS

The subband adaptive filter merges the concept of subband analysis and adaptive filtering by applying an adaptive filter to each subband signal [1][7][8]. Figure 2 shows the structure of the subband adaptive filter where only one of the adaptive filters is shown for brevity. (The necessity of either synthesis bank is determined by the application.) The adaptive filter order for each subband is usually chosen to be much smaller than a comparable fullband adaptive filter although the total number of coefficients may be comparable.

Adaptive filtering in subbands has the potential to reduce computational complexity compared to the fullband case. Decimation through subband analysis reduces the required computation rate [3]. Since the power spectral density of each subband signal is flatter than the psd of the fullband signal, the

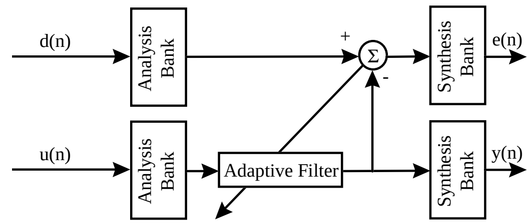

Figure 2. Adaptive filtering in subbands.

adaptive filter of each subband requires fewer filter coefficients than the fullband filter for similar performance. The ability to reduce filter order is particularly important for adaptation algorithms such as the RLS where complexity is proportional to the square of the adaptive filter order. For a fullband filter of sufficiently large order, the computational savings of adaptive filtering in subbands absorbs the complexity of the analysis and synthesis banks.

Adaptive filtering in subbands can increase convergence speed. Subband signals have a smaller eigenvalue spread than the corresponding fullband signal which increases convergence speed for stochastic gradient based adaptation algorithms such as the NLMS. Thus, the composite subband adaptive filter convergence speed is enhanced.

In cases where the subbands are critically decimated, aliasing problems may occur. Due to the adaptive nature of this structure, perfect reconstruction filters do not correct aliasing as they do for standard subband analysis and synthesis. Methods for alleviating aliasing problems include using adaptive cross filters for adjacent bands which slow the convergence rate by coupling adjacent subbands; nonoverlapping subbands which leave spectral gaps; and nonoverlapping subbands with auxiliary filter banks which require additional analysis and synthesis filter banks [1][6][7][8]. A more in-depth discussion of these methods is beyond the scope of this paper; however, these references are made for completeness.

4. SUBBAND PREDICTION ARCHITECTURE

Some studies have shown that applying prediction to subband signals can realize increased prediction gain over prediction of the fullband signal under certain conditions. This research applies AR and ARMA models to subband signals using open-loop adaptation techniques such as Prony’s method and the method of least squares. Similar to the subband coding rate allocation problem, Rao and Pearlman [5] show that prediction coefficients can be optimally allocated among the subbands in order to form a subband predictor that is superior to the fullband predictor given a fixed prediction order and stationary input. Using the spectral flatness measure, they reason that the combined prediction error power spectral densities of such an optimally formed subband predictor is whiter than the psd of the prediction error from the fullband predictor. However, the fullband predictor is superior when it is able to capture all of the prediction gain, i.e., when the prediction order approaches or exceeds the order of the input process. Other experimental results

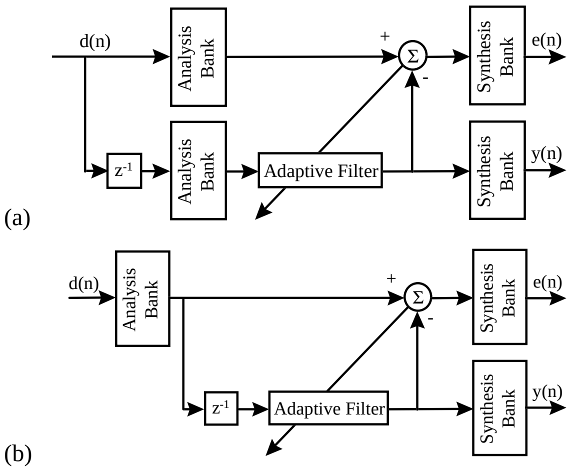

Figure 3. Two subband adaptive predictor structures. The top form (a) shows an adaptive subband filter in the prediction configuration. The bottom form (b) shows adaptive prediction in each subband.

show that the subband prediction gain can exceed the fullband prediction gain for nonstationary signals such as speech [9].

A closed-loop adaptation structure may show a similar increase in subband prediction gain while providing a per-sample model of the predictor input. Additionally, a subband adaptive filter applied in the prediction configuration directly inherits the performance advantages in reduced computation and increased convergence speed as described in the previous section. Figure 3(a) shows the structure of the closed-loop subband adaptive predictor where the desired signal leads the adaptive filter input by one fullband sample. Figure 3(b) shows an alternative structure for the subband predictor with one less analysis bank where the desired signal leads by one subband sample or N fullband samples. Since the delay between the desired and input signals is smaller, a higher correlation between the two signals is expected for figure 3(a) indicating a potential for better performance.

5. COMPUTATIONAL ANALYSIS

The fullband predictor requires one adaptive filter with P_f coefficients. For N fullband samples, the NLMS algorithm requires $3NP_f$ operations, and the RLS algorithm requires on the order of $N(P_f^2 + P_f)$ operations. The corresponding subband predictor has N subbands and requires N adaptive filters. Assuming that the total number of subband predictor coefficients equals the fullband predictor order and that these coefficients are evenly distributed across the subbands, each subband adaptive filter has order $P_s = P_f / N$. For N fullband samples (or one subband sample per band), the NLMS algorithm requires $3NP_s$ or $3P_f$ operations. The RLS algorithm for all N subbands requires $N(P_s^2 + P_s)$ or $P_f^2 / N + 5P_f$ operations. Thus, the subband adaptive filter provides a factor of N improvement in computation for the NLMS algorithm and a factor of N^2 for the RLS algorithm. For cases where the total number of subband adaptive filter coefficients is less than the fullband adaptive filter order, the reduction in complexity can absorb the computation required by efficient analysis and synthesis algorithms.

Table 1. Prediction gain using a total of 10 prediction coefficients for (a) a dual tone signal, (b) real speech, and (c) a 15th order AR process.

(a)

	Fullband	2 Subbands
NLMS	2.3×10^{13}	1.3×10^{6}
RLS	2.2×10^{22}	2.0×10^{19}

(b)

	Fullband	2 Subbands
NLMS	2.3779	3.9794
RLS	4.4787	9.8583

(c)

	Fullband	2 Subbands
NLMS	0.8508	2.1220
RLS	1.9726	6.1866

Table 2. Prediction gain using a total of 12 prediction coefficients for (a) a dual tone signal, (b) real speech, and (c) a 15th order AR process.

(a)

	Fullband	4 Subbands
NLMS	1.4×10^{26}	2.0×10^{6}
RLS	2.2×10^{22}	4.7×10^{6}

(b)

	Fullband	4 Subbands
NLMS	2.5384	2.1336
RLS	4.4675	12.5896

(c)

	Fullband	4 Subbands
NLMS	0.8927	1.0153
RLS	1.9785	6.5701

6. SIMULATION RESULTS

The subband adaptive predictor from Figure 3(a) is simulated for the two subband and four subband cases. The prototype analysis filter is a 29th order lowpass filter designed using the window method. Adaptive filter coefficients are divided equally among the subbands although better performance is expected for optimally allocated coefficients. The structure is tested using three signals: a deterministic dual tone signal, a short sample of real speech, and a 15th order AR process. The prediction gain is estimated over 500 samples and compared against results for the fullband case.

Table 1 shows the prediction gain results for the two subband case. The total number of prediction coefficients is ten with five coefficients allocated to each subband. Table 1(a) shows that the fullband predictor is superior for the dual tone test signal. Tables

1(b) and 1(c) show that subband predictor can outperform the fullband predictor.

Table 2 shows the prediction gain results for the four subband case. The total number of prediction coefficients is twelve with three coefficients allocated to each subband. Table 1(a) shows again that the fullband predictor is better for the dual tone signal. In table 2(b), the fullband NLMS predictor slightly surpasses the corresponding subband predictor while the subband RLS predictor outperforms the fullband RLS predictor. Table 2(c) shows that the subband predictor performs better for the AR process input.

7. IMPLEMENTATION

The two-band NLMS subband predictor is implemented in C++ with the Standard Template Library (STL) on a 120 MHz Pentium processor. The analysis and synthesis filter banks use a 29th order lowpass prototype. Figure 4 shows the average execution time to process 32,000 samples for varying prediction order. For instance, the resulting code consumes approximately 1.43 seconds in order to process 32,000 samples with a 5th order predictor in both subbands. Assuming an 8 kHz sampling rate, the code executes roughly 2.8 times better than real time for this case. In fact, Figure 4 shows that the code executes under real time for more than 90 prediction coefficients in both subbands at 8 kHz.

8. CONCLUSION

This paper has presented a prediction architecture based on closed-loop, adaptive filtering in subbands. The closed-loop structure maintains a per-sample model of the input and allows a coefficient update as often as desired. The subband configuration can increase convergence speed and considerably reduces the computational complexity of the structure. Results show that the proposed predictor can yield increased prediction gain over the fullband case especially for nonstationary input for both the NLMS and RLS algorithms. A real time NLMS subband predictor has been implemented in C++ for the 120 MHz Pentium processor.

9. REFERENCES

[1] André Gilloire and Martin Vetterli. "Adaptive Filtering in Subbands with Critical Sampling: Analysis, Experiments, and Application to Acoustic Echo Cancellation", *IEEE Transactions on Signal Processing*, vol. 40, no. 8, pp. 1862-1875, August 1992.

[2] Simon Haykin. *Adaptive Filter Theory*, Englewood Cliffs, NJ: Prentice-Hall, 1991.

[3] S. Jayant and P. Noll. *Digital Coding of Waveforms*, Englewood Cliffs, NJ: Prentice-Hall, 1984.

[4] Sung-won Park. "Speech Compression Using ARMA Model and Wavelet Transform", *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. I, pp. 209-212, 1994.

[5] Sudhakar Rao and William A. Pearlman. "Spectral Estimation From Subbands", *Proceedings of the IEEE-SP International Symposium on Time-Frequency and Time-Scale Analysis*, pp. 69-72, 1992.

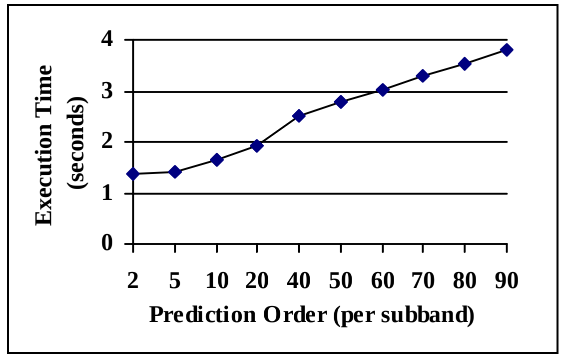

Figure 4. Execution time versus prediction order averaged over 32,000 samples.

[6] John J. Shynk. "Frequency Domain and Multirate Adaptive Filtering", *IEEE Signal Processing Magazine*, pp. 14-37, January 1992.

[7] S. Somayazulu, S. K. Mitra, and J. J. Shynk. "Adaptive Line Enhancement Using Multirate Techniques", *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 928-931, 1989.

[8] M. Tahernezhadi, J. Liu, G. Miller, and X. Kong. "Adaptive Echo Cancellation Using Subband Technique For Teleconferencing Applications", *IEEE Globecom*, pp. 243-247, November 1994.

[9] Siew-Lian Tan and Thomas R. Fischer. "Linear Prediction of Subband Signals", *IEEE Journal on Selected Areas in Communications*, vol. 12, no. 9, pp. 1576-1583, December 1994.